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Constrained PID Control

Učební texty k semináři

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TENTO STUDIJNÍ MATERIÁL JE SPOLUFINANCOVÁN EVROPSKÝM SOCIÁLNÍM FONDEM A STÁTNÍM ROZPOČTEM ČESKÉ REPUBLIKY

Contents

Chapter 1

Basic Notions of Robust Constrained PID Control

This chapter is aimed as introduction to the prepared textbook on the Robust constrained PID control. It makes you familiar with historical development of PID control, its basic components and structures, problems and motivations and with basic terminology used within this area. After studying this chapter you should be better able to describe phases in the technology development of the PID control, to characterize basic existing types of PID controllers and to explain, why the development of PID control cannot be considered as finished, to characterize different performance specifications and related terms as ε-nonovershooting, ε-nonundershooting, ε-monotonic and ε-n-pulse (nP) functions and their use in deriving the so called closed loop performance portrait, to explain notion of dynamical classes (DCs) of control and their relation to the Feldbaum's theorem about n-intervals of the relay minimum time control (MTC), to explain impact of DCs on performance and design of PID control, to explain notion of fundamental solutions for setpoint tracking and disturbance rejection and to explain and characterize basic elements of the extended table of fundamental PID controllers. Within this publication, this introduction is followed by the next chapter bringing the simplest structures of the DC0 and the new robust method based on the Performance Portrait used for the controller tuning. More advanced problems from higher dynamical classes are treated by separate papers in the associated workbook and in Preprints of the NIL workshop (Huba et al, 2011a,b)

1.1 Introduction

This text is devoted to developing new approach to the robust constrained PID control of simple single-input-single output (SISO) plants. It starts by showing and arguing, where and why new alternative solutions were proposed to the traditional ones to extend mainstream of the contemporary development and how the traditional problems may be treated more efficiently. Despite the strong emphasis on comparing with the already existing works, the overview of references given is surely not complete. With

respect to this, but also in other points, we will welcome any comments and proposals for improvements. For mathematically oriented reader the text may seem to be not sufficiently covered by proofs of basic conclusions. And conversely, for people from practice it may seem to be mathematically too demanding: We spent a lot of space by trying to fit the academic and engineering control methods together to get approaches matching optimally needs of real time control.
PI controllers operate about 90% control systems. They represent the core module of PID controllers that cover about 95% Åström and Hägglund (1995), or even 98% (Datta et al, 2000) of all control systems in practice. By nearly century of its existence and by its impact on practice it is related to personal experience of huge amount of people. We are trying to address this experience by stressing importance of controlling simple plants and by comparing different approaches and results.
As each control design, also the design of PID controllers must be based on some model of the plant behavior and the resulting controllers will necessarily depend on information embedded into this model. Since the model represents just an abstract approximation of chosen features of real systems, it is never complete and always it is to some degree uncertain. Its uncertainty is expected to influence quality of achieved control results that usually depends on factors as:

- measurement noise in identification and control,
- disturbances acting on the plant during the identification and control,
- numerical errors and other imperfection of the methods used in the identification, controller design and control,
- plant nonlinearities relevant to larger deviations from fixed operating points,
- plant nonlinearities relevant to the vicinity of the operating point (as e.g. hysteresis),
- non-modeled (high frequency) plant dynamics, i.e. dynamics not considered in deriving controller equations/structure,
- time related changes of the plant dynamics.

From the beginning of control design, each method used in practice was somehow be able to cope with impact of all these factors. In the last decades, the robustness aspects related to model uncertainty are treated more rigorously and many works and publication on robust process control based e.g. on H_2 and H_∞ norms proposed, discovered and analyzed a lot of useful features and methods. However, the huge number of newly appearing papers devoted to the robust control of simple SISO systems that try to optimize traditional solutions, or propose new ones (Skogestad, 2003; Baños and Vidal, 2007; Johnson and Moradi, 2005; Keel et al, 2008; O'Dwyer, 2006; Seok et al, 2007) indicate that there still exist features of PID control that are expected to be improved. The high number of appearing publications has also drawbacks as that it is practically impossible to follow all streams of ideas and methods of the development and to offer a unifying presentation giving explanation of their internal relations. And it is not enough to deal just with the newest development. As it is documented by many examples from science history, not every time the mostly spread opinions guarantee further progress and it can happen that some already forgotten ideas finally show to play the key role in achieving new generation of solutions.

Newer approaches of robust process control trace their origins (Morari and Zafiriou, 1989) to the "analytical" design by Newton, Gould, and Kaiser (1957) based on optimizing the ISE (Integral Square Error) performance index. Morari and Zafiriou denoted the earlier approaches as the "Trial and Error" ones. It can be, however, shown that already the older approaches involved some features of the robust design. And, on the other side, also the modern "robust&analytical" approaches mostly require some iterative modifications until the best compromise between the usually conflicting objectives is reached. It may e.g. be caused by the fact that the ISE based design is not primarily motivated by practical requirements but by the mathematical convenience and it is known to lead to slightly oscillatory behavior. Therefore, in this text design based on minimal IAE (Integral of Absolute Error) values will be preferred (Shinskey, 1990). For practical use, requirements of the fastest possible transients giving minimal IAE values will be extended by requirements of nonovershooting (NO), or monotonic (MO) control responses of the output variable that should be achieved by a reasonable excurse of the manipulated (control) variable giving minimal Total Variance (TV) values (Skogestad, 2003). So, in the control design reported in this text we will try to get as fast as possible MO control responses by respecting both the plant model uncertainties, the control signal constraints and demand on the total excurse of the manipulated variable.

As the 2$^{\text{nd}}$ most important pillar of the robust process control give Morari and Zafiriou the work by Youla et al (1976) on parameterizing all stable controller transfer functions possible to given (linear) plant and specified problem. In this way their primarily aim, to search for a good controller, was greatly simplified. From this point of view it may be, however, noted that the same aim (to parameterize all stable controllers and so to simplify search for a good controller) was partially achieved already by the pole placement (pole assignment) control design. This (when choosing stable closed loop poles) is also giving continuum of stable parameterized controllers. According to Åström and Wittenmark (1984) pole assignment approach was firstly treated by J. Bertram in 1959 and the first published solution was given by Rissanen (1960). Despite this (older) approach is not as general as the parameterization by Youla, it is broadly used within different “modern” approaches. The first design step, the determination of parameterized nominal controllers, was, however, up to now not sufficiently completed by the second step, the robustification of the controller. This requires choosing appropriate closed loop poles. Instead of working with the closed loop poles (that are specified by negative numbers) it may be simpler to use positive parameters denoted as bandwidth, or their reciprocal values having meaning of time constants. However, up to now there do not exist proven techniques for robust performance design relating simply given control specifications with the closed loop poles and with the uncertainty information, nonmodelled dynamics and measurement noise. This step is therefore still mostly done by the trial and error method, whereby the choice of the closed loop poles is not only influenced by the system uncertainty and the nonmodelled dynamics but also by the constraints put on the control and state variables. This text shows how the “trial and error” procedures can be automatized and replaced by a systematic computer based qualitative and quantitative analysis appropriately taking into account both role of constraints, plan-model mismatch and different performance specifications. At least for the simplest loops with dominant dynamics up to the second order the problem of the control signal constraints may be eliminated by the generalized constrained pole assignment control (Huba et al, 1999; Huba, 2006). In connection with the computer based analysis, all developed controllers can be used also for achieving specified robustness degree.

Another broadly accepted approach to general parametrized solutions related to the robust control and building on the sensitivity functions, or the complementary sensitivity functions, was introduced byÅström and Hägglund (1995). By trying to have clear-cut physical interpretation of the effect of such tuning parameters and clear picture of their appropriate default values, the tuning should be relatively easily adjusted (Skogestad, 2003) to a particular situation and so to be much simpler and reliable. However, from the point of view of the robust constrained pole assignment control the sensitivity and complementary sensitivity functions do not always represent an effective and efficient solution. They e.g. do not match the natural expectation that when requiring the fastest possible monotonic output transients by decreasing:

- range of possible parameter fluctuations,
- effect of the nonmodelled dynamics (parasitic delays) and
- amplitude of the measurement noise,

the achieved solutions should converge to the MTC. Using the pole assignment method, such a requirement was systematically followed by Glattfelder and Schaufelberger (2003). The anti-windup PI controllers they have analyzed were very close to give ideal control signal step reactions converging to one pulse of the MTC.
The other important handicap of the development – the gap between the classical state space approach and the newer robust control was formulated byMorari and Zafiriou (1989) as "*no smooth transition from the established proven techniques and tools (PID controllers, Smith Predictor) to the new ones*" - may only be eliminated by modifications done from both sides. We will try to resonse this comment by requiring smooth transition of the new robust approach to the PID control up to the Relay MTC. Such attempts have already been done e.g. by works of Glattfelder and Schaufelberger (2003) (who analyzed achieved PID solutions both from the point of view of robust control and MTC). Compatibility of different approaches and their relevance for practice was approached from different points of view also by many other authors. E.g. Rivera et al (1986), or Skogestad (2003) tried to combine theory with practice and stressed importance of the manipulated variable in evaluating achieved control performance. In this text we are going to look for compatibility and to explore different structures of PID control from the point of view of the state-space approach to controller design and to reconstruction and compensation of disturbances by using disturbance observer (DOB). Simultaneously, the achieved constrained loop dynamics will be confronted with result of the MTC. Thereby, it will not be related just to a fixed nominal operating point but to larger areas of loop parameters enabling to keep chosen loop dynamical properties under every time present uncertainties of the plant model. In order to introduce

an effective controller classification, it is further important to introduce new notions like *n-pulse function, fundamental controllers* and *dynamical classes of control.* Before coming with these new definitions, let us briefly review basic notions of PID control.

Definition 1.1 (PID controller). Under the notion of PID control we will include all controllers for setpoint tracking and disturbance rejection in systems with the dominant dynamics up to the 2^{nd} order described by the transfer functions

$$S(s) = \frac{K_s(1+T_0 s)}{s^2 + a_1 s + a_0} e^{-T_d s} \tag{1.1}$$

An alternative previously used definition could speak about controllers dealing with the reference and with the output signal (control error), its derivative and integral given by the transfer function

$$R(s) = K_P\left(1 + \frac{1}{sT_I} + sT_D\right) \tag{1.2}$$

or by its realizable modifications characterized by following definitions.

Definition 1.2 (ISA PID controller). According to the ISA standard, the two-degree-of-freedom PID controllers can be described as

$$U(s) = k_P\left\{bW(s) - Y(s) + \frac{1}{T_I s}[W(s) - Y(s)] + \frac{T_D s}{1 + sT_D/N}[cW(s) - Y(s)]\right\} \tag{1.3}$$

whereby

- $Y(s), W(s), U(s)$ represent Laplace transforms of the controller output, setpoint and process output variable,
- k_P is the controller gain,
- T_I and T_D the integral and derivative time constants,
- b and c are the weighting coefficients of the proportional and derivative action and N describes filtration of the derivative action.

By setting $T_I \to \infty$ one achieves PD controller, for $T_D = 0$ one gets PI controller and for $T_I \to \infty$ and $T_D = 0$ the P controller.

Definition 1.3 (Series PID controller). As an alternative to the previous description one can consider serial controller form:

$$U(s) = k'\left\{\left[b + \frac{1}{sT_I'}\right]\frac{1 + scT_D'}{1 + T_D'/N}W(s) - \left[1 + \frac{1}{sT_I'}\right]\frac{1 + scT_D'}{1 + T_D'/N}Y(s)\right\} \tag{1.4}$$

Definition 1.4 (Parallel PID controller). The third basic controller form is given by equation

$$U(s) = K\left[bW(s) - Y(s)\right] + \frac{K_I}{s}\left[W(s) - Y(s)\right] \frac{K_D s}{1 + sK_D/(NK)}\left[cW(s) - Y(s)\right] \tag{1.5}$$

These 3 basic PID controllers can yet be completed by the I-controller:

Definition 1.5 (I-controller). I-controller may be defined as

$$U(s) = \frac{1}{T_I s}\left[W(s) - Y(s)\right] = \frac{K_I}{s}\left[W(s) - Y(s)\right] \tag{1.6}$$

I-controller can be simply derived just from the parallel form by setting $K = K_D = 0$.

For decades, these linear controllers represent building stones of the vast majority of solved problems. As a standard option they yet include a constant output signal (bias) and structures for switching from manual to automatic regime.

By analyzing their possibilities many authors have finally come to conclusion that it would be oversimplified to consider just controllers (1.2)–(1.6). This shift from transfer function (1.2) to more complex structures is not just the invention of this publication. It results from a longer historical development reflecting needs of practice. Even when remaining within the scope of linear control, since 1960s controllers (1.2) characterized by a triple of parameters (K_P, T_I, T_D) are being replaced by more complex structures of the *two-degree-of-freedom* controller (1.3)–(1.5) with *setpoint weighting* (Horowitz, 1963; Åström and Hägglund, 2005). These are characterized by 5 or 6 parameters (with filters of the derivative action). Controllers used in practice take forms of more or less complex structures that e.g. always consider also constraints given on the controller output. Despite to the linear character of basic equations, the industrially produced controllers are usually equipped with control constraints and blocking of an abundant integration known as *anti-windup* (aw), or *anti-reset-windup* (arw), by the possibility of on-off (pulse width modulated) control, or with other nonlinear options (as e.g. error squared controllers). As we show later, even all these advanced possibilities are not enough to cover needs on a reliable and high quality control of systems (1.1) and despite to respect to traditions it shows to be necessary to extend this basis by new elements and to introduce internal differentiation of all existing solutions. It is also to note that despite speaking about PID control as being derived for the dominant second order dynamics this does not restrict its applications to controlling much more complicated systems.

1.2 Innovation versus Conservativeness

It is not easy to quote the first application of PID control. Integrated with other parts of controlled processes, controllers with proportional and integral action were used for ages. But as the first mathematical formulation of the proportional and integral action one can mention the analysis of the speed control of a steam engine "On Governors" by Maxwell (1868). As devices independent from sensors, actuators and controlled plants PI controllers (denoted as automatic reset) appeared at the end of the World War I. Controllers with the derivative (D) action (denoted as pre-act) came around 1935. The development in this area relates to the legendary firms as Bristol, Fisher, Foxboro, Honeywell, Leeds & Northrup, or Taylor Instruments. The first methods for an optimal tuning of controllers appeared few years later (see e.g. Ziegler and Nichols (1942); Oldenbourg and Sartorius (1951)). But, when now, after more than one century of study and development of the relatively simple concept of PI and PID control one can still find several open questions of their reliable use and tuning, it is necessary to ask "why"? What are the reasons for inflation of different forms and realizations (series, parallel, non-interactive – ISA and interactive – series (Åström and Hägglund, 1995) different quasi-continuous realizations, etc.)?

What are the reasons for inflation of "optimal" tuning rules? Just O'Dwyer (2000, 2006) reports in his works 154 tuning rules for PI control and practically each control conference devoted to the control design brings new ones.

Also some other points are not clearly explained: why it is e.g. sometime necessary to use setpoint weighting - it means to modify coefficients b and c in Eqs. (1.3)–(1.5) - and sometimes not?

Why it is sometime necessary to use anti-windup measures and sometimes not? Why do we have inflation of aw – circuitry, when just Glattfelder and Schaufelberger (2003) report and analyze 10 different schemes for PI control (see also Kothare et al (1994))? But, can we expect something else, when there does not exist a generally accepted unique definition of the windup phenomenon?

When we start to analyze reasons for this multi-dimensional inflation, we can identify several possible sources and points to discuss:

- PID control is not a closed and unique solution, but result of a not yet finished development,
- conservatism of practice and tendency to work with older (may be out

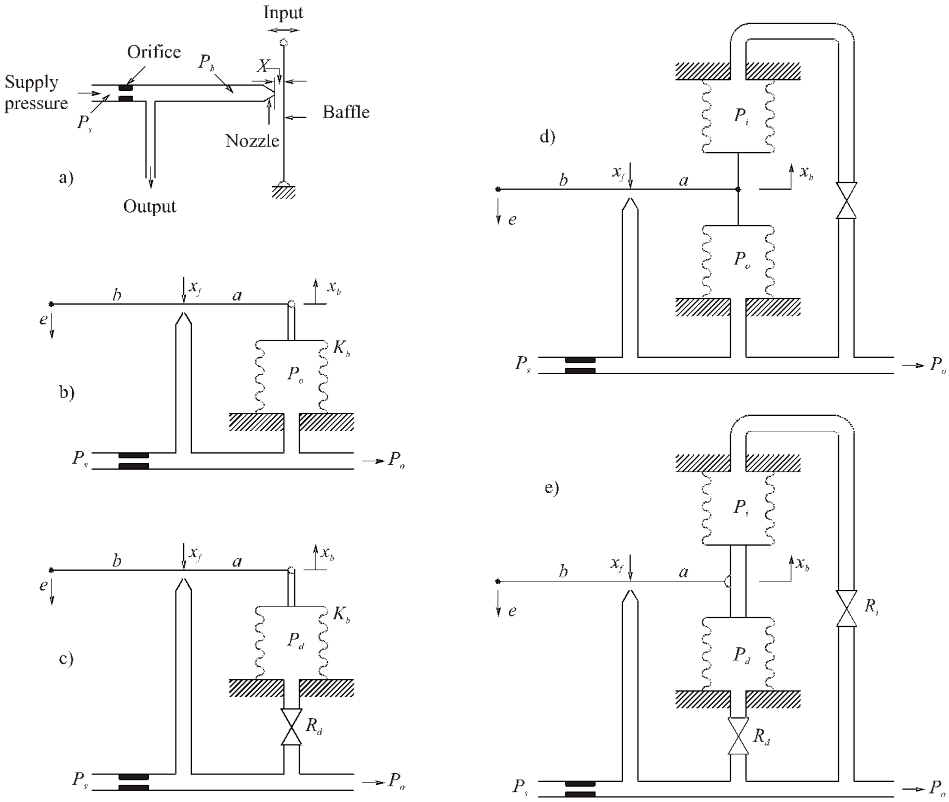

Figure 1.1: Modular concept of pneumatic PID Control. a) Flapper-Nozzle high gain nonlinear amplifier, b) P controller, c) PD controller, d) PI controller and e) PID controller (see e.g. Ogata (1997); Van de Vegte (1994))

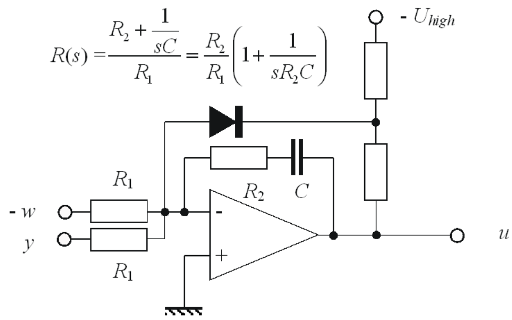

Figure 1.2: Analog electronic PI controller

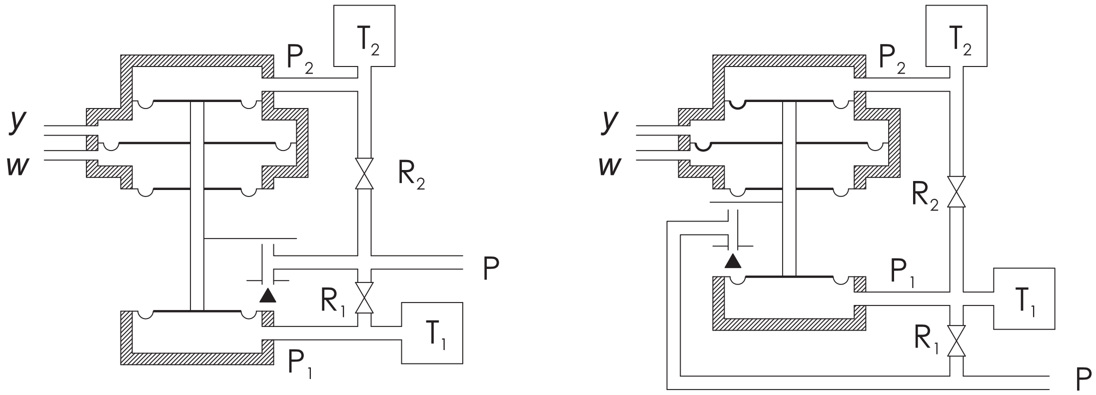

Figure 1.3: Modifications of the pneumatic PID controller with a parallel (left) and a serious feedback (right) enabling to achieve different ranges of adjustable parameters and different tuning properties (Ogata, 1997)

of data) solutions,

- existence of alternative solutions to the specified problems offering different performance,
- failure to analyze the physical essence of the solved problems,
- absence of reliable controllers and their tuning for some typical situations, e.g. for systems with large dead-time, or for unstable systems,
- absence of controllers respecting given control constraints for some typical situations,
- not yet finished development of methods for a reliable (self-) tuning of controllers.

Conservativeness of users is closely related to the historical development of the PID controller technology. In the initial period, large amount of different pneumatic, hydraulic electrical and electro-mechanical devices were spontaneously developed mostly on an experimental bases. Theoretical studies of derived controllers started just later, when practice required a deeper understanding of their optimal tuning and when it was necessary to replace older devices by newer electronic controllers (to the end of 1950s) and digital ones (since 1980s).
After 1960, due to the invention of transistors, the older pneumatic controllers started being replaced by newer electronic devices based on high gain operational amplifiers. Their dynamical properties, determined by the feedback impedances are much more transparent and can easily be mathematically described.

After 1960, new wave of digital controllers started. Around 1980 it is already to observe fast invasion of digital quasi-continuous microprocessor based controllers. They work with relatively short sampling periods that can frequently be neglected and the controllers may be considered as the continuous-time ones. However, it is to remember that by sampling a high frequency noise signals at the input of the analogue to digital (A/D) converters, a low frequency signals may appear. In the older analogue controllers the controller inertia naturally filtered these. The study of digital controllers brought to light necessity of introduction of new anti-aliasing filters.

Comparing with the analogue control, the fundamental and up to day not fully used feature of digital control is its flexibility and broad functionality. One of its exceptional features is an easy implementation of dead time that is very important for its compensation in control loops. It was required by solutions as the Smith predictor (Smith, 1957), or a bit older controller by Reswick (1956). For the analogue controllers, implementation of the dead time required in its compensation represented a serious technical problem. Due to this, practice has motivation to use simpler PI controllers instead of them. Reaction to the new situation still does not correspond to the well-known fact that the majority of processes can be approximated by the first order models with dead time!

Introduction of digital controllers gave birth to new phenomenon called *windup*. Why it was not recognized earlier? May this fact be explained so that in the digitalization phase the older solutions robust against windup were not described fully correctly? Effect of the waves of innovation on the anti-windup control circuitry is in a catching way described in the book "Control Systems with Input and Output Constraints" by Glattfelder and Schaufelberger (2003). They show several examples from the field of power control, by which they demonstrate problems arising by replacing older generations of controllers by newer. They show that it is sometimes simpler to imitate by new solutions the old pneumatic controllers than to invest into reengineering of the whole technological complex. Related back to the already existing solutions, this argumentation can be understood. However, it should not be acceptable for newly designed solutions, when the new controllers give much broader possibilities!

From the application point of view it has to be noted that the first generation of controllers was mostly designed to compensate effect of disturbances acting in the vicinity of fixed operating points. When a transition to a new operating point was required, it was either done under manual control, or by special units. So, the first tuning rules and strategies were devoted to optimal compensation of disturbances. This category is e.g. represented by the most popular tuning rules by Ziegler and Nichols (1942). The optimal behavior corresponding to the setpoint changes was focused just later. Furthermore, the first pneumatic controllers were constructed in such way that they did not initiate excessive controller windup. This started to be dominating just for newer generation of digital controllers, what resulted also in corresponding research work (see e.g. Fertik and Ros (1967); Kramer and Jenkins (1971)), when more important results appear just around 1970.

Development of the technology of PID control has, of course, influenced also the development of the control theory. The today frequently ventilated gap between the theory and practice has several resources: e.g. the generally accepted internal classification of PID control does not reflect all basic situations occurring in practice. It seems that this gap reasonably increased after replacing the first generation of experimentally designed controllers by more transparent and easily describable electronic and digital ones, when some important construction details were neglected and forgotten. The other point is that the control theory developed into an independent discipline what has brought also several self-centered features and artificial problems that do not respond to real needs. Failures in solving real problems lead many researchers to leave the traditional analytical controllers and to look for new solutions based e.g. on fuzzy control, neural networks, genetic algorithms or optimization based predictive control. Although these new solutions bring many new interesting and useful features and options, it does not mean that a theory describing PID control becomes obsolete. Many fictitious advantages of the new approaches represent, in fact, just the not sufficient knowledge of the possibilities of the traditional ones. But, what should be improved in the traditional approach? At first, we should understand more deeply motivations that gave birth to these structures. This will also need to introduce performance specification that will be used for evaluating, if the controller design is meeting as close as possible practical requirements.

1.3 Advanced Modifications of PID Control

Next we will briefly show some newer modifications of the PI and PID controllers to illustrate broad spectrum of existing solutions that will be stepwise explained in this book. Going back to the first pneumatic PI controllers, their structure may be represented by feedback from the controller

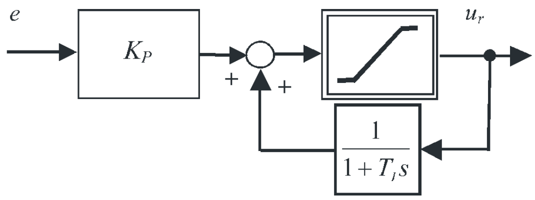

Figure 1.4: Serial implementation of the PI controller considering control signal constraints

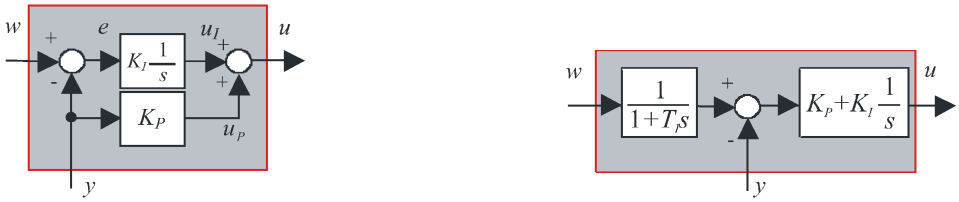

Figure 1.5: I-P controller (left) and the equivalent PI controller with prefilter (right), $K_I = K_P/T_I$

output through a low pass filter (Åström and Hägglund, 1995) in Fig. 1.4. In the proportional zone of control, when the saturation limits are not active, the controller transfer function becomes

$$R(s) = K_P \frac{1}{1 - \frac{1}{1+T_I s}} = K_P \left(\frac{1}{1 + T_I s} \right) \tag{1.7}$$

Another broadly used modification of the PI controller denoted usually as the I-P controller uses the proportional feedback acting just on the plant output (Fig. 1.5 left). It may be shown to be a special case of setpoint weighting (with $b = 0$, $T_D = 0$ in (1.3), or to be equivalent to the PI controller with the input filter (prefilter) in Fig. 1.5 right.

Similarly, by using PD terms acting on the output only may give the I-PD, or PI-PD controllers.

For controlling stable first order systems with long dead time T_D, the Predictive PI controller (PPI, Hägglund (1996); Fig. 1.6 left) is used. This may be extended by a PD in the feedback path (Fig. 1.7 right) to the PID τd controllers (Shinskey, 2000). Both may be extended by the IMC filter, or by a prefilter, when one e.g. gets the structure of the Model Driven PID controller (Shigemasa et al, 2002; Yukitomo et al, 2002) with the IMC filter and the prefilter in Fig. 1.7 (above) that is equivalent to the PPI-PD controller in Fig. 1.7 (below)

All above mentioned structures may be covered by the 2 degree of freedom (2DOF) MD-PID controller with the 2nd order IMC filter and the prefilter in Fig. 1.8 (Shigemasa and Yukitomo, 2004; Yukitomo et al, 2004). They document that the PID control developed is far from to be finished and

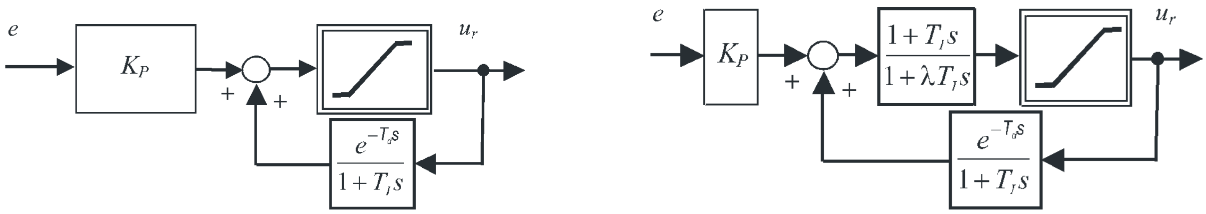

Figure 1.6: Predictive PI controller (PPI) (left) and extended by the IMC controller (right)

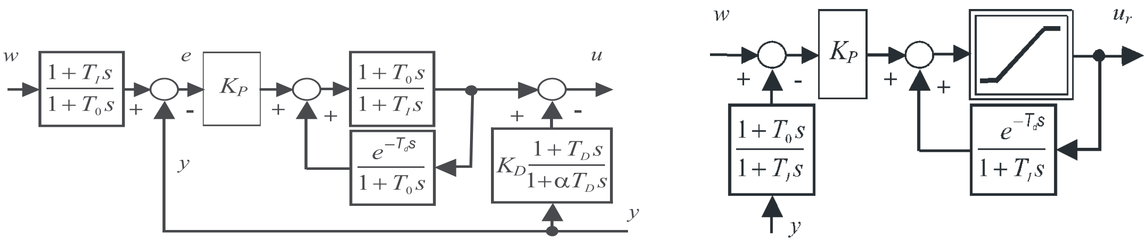

Figure 1.7: Model Driven PID controller (above) (Shigemasa et al, 2002; Yukitomo et al, 2002) that is equivalent to the PPI-PD controller (below)

be interpreted just by the transfer function (1.2). Obviously, systems with long dead-time are being integrated as a part of the general PID control.
As it is obvious from the title "Model Driven" PID controller, the plant model played an important role in derivation of previous controllers. One may speak about approaches using the plant model at least from late 1950s, when the first schemes for dead time compensation by Reswick (1956) and Smith (1957) appeared. Both were based on reconstruction of an output disturbance by a parallel plant model and the reconstructed disturbance was then used for compensation of the reference setpoint value. Use of the parallel plant model was later generalized within the Internal Model Control concept (Morari and Zafiriou, 1989) that developed its own structure and tuning approaches to the PID control (Rivera et al, 1986; Skogestad, 2003) used frequently within the robust control.
All above controllers may be considered as different modifications of the

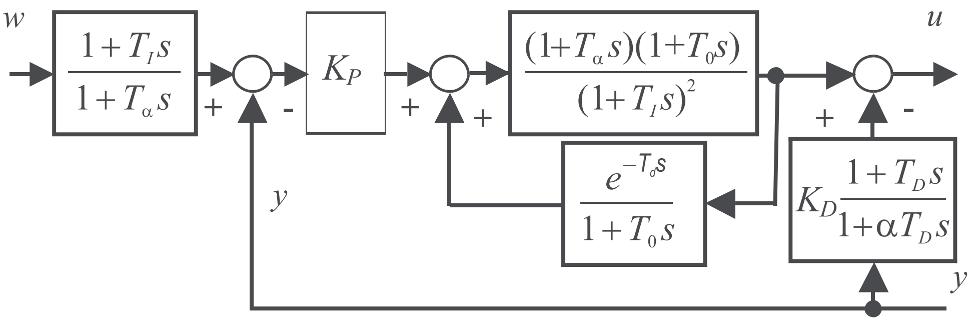

Figure 1.8: 2DOF MD-PID controller

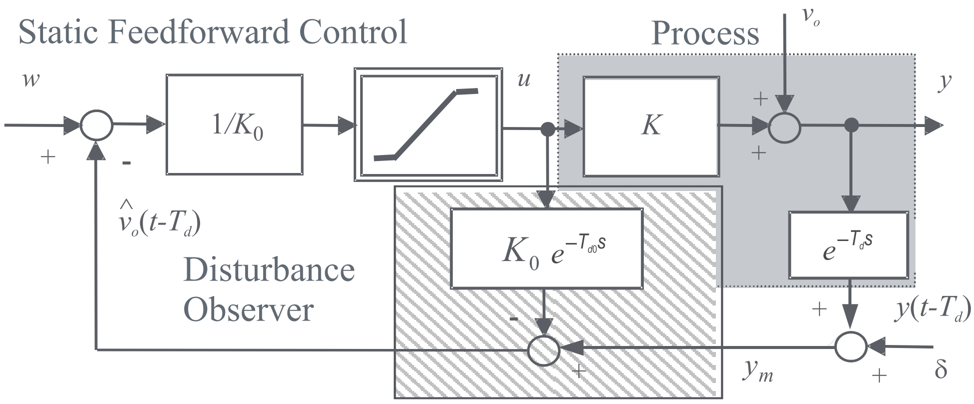

Figure 1.9: Controller for dead time compensation by Reswick (1956)

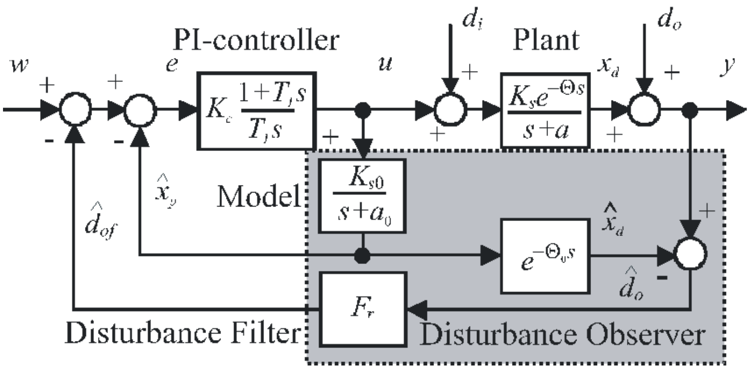

Figure 1.10: Filtered Smith predictor for dead time compensation; Smith (1957) proposed this scheme with $F_r(s) = 1$

IMC control derived for reconstruction and compensation of output disturbances that were dominantly used in process control. This area is typically dealing with stable processes, whereby the measured signals may be rather poor. The remarkable progress in mini- and microcomputers and power electronics technology in the 1980's made it also possible to improve the performance of motion control, what consequently lead to testing of traditional and novel theories of control appropriate for mechatronic systems. In this area with dominant influence of the input (load) disturbances of frequently unstable, or marginally stable plants, but the relatively high quality measured signals, much more frequently the so called Disturbance Observe based servo systems (Ohnishi , 1987; Ohnishi et al, 1987; Umeno and Hori, 1991), or the Disturbance Observer based PID control (Zhao, 2004) are used. This approach that is based on inversion of the model dynamics was also extended to systems with long dead time (Zhong and Mirkin, 2002; Zhong and Normey-Rico, 2002).

The new textbook on Robust Constrained PID control tries to compare both approaches based on reconstruction and compensation of input and output disturbances and the traditional approach to the PID control more systematically, what requires to adopt also some terminology changes. Due to the fact that also the IMC control actually uses DO for reconstructing output disturbances and both the IMC and DO based PID control are internally using plant models, where appropriate, the PID structures for reconstruction and compensation of input disturbances will be denoted more eloquently as the PID-IM (Inverse Model) controllers and the structures for reconstruction and compensation of output disturbances as PID-PM (Parallel Model) controllers. Of course, the question is, if this was the best choice that will enable a modular terminology development appropriate to cover also possible modifications with different mixed forms of solutions, but answers to this question will bring just the future development. With respect to this, author of this chapter will be thankfull for any comments regarding these proposals.

1.4 Performance of PID Control

Traditionally, PID control design may be carried out by using closed loop specifications in the time domain or in the frequency domain (Skogestad and Postlethwaite, 1996). In this book we will prefer the first ones, since their application in computer based design that would be based on exploiting information on the closed loop properties is extremely simple and

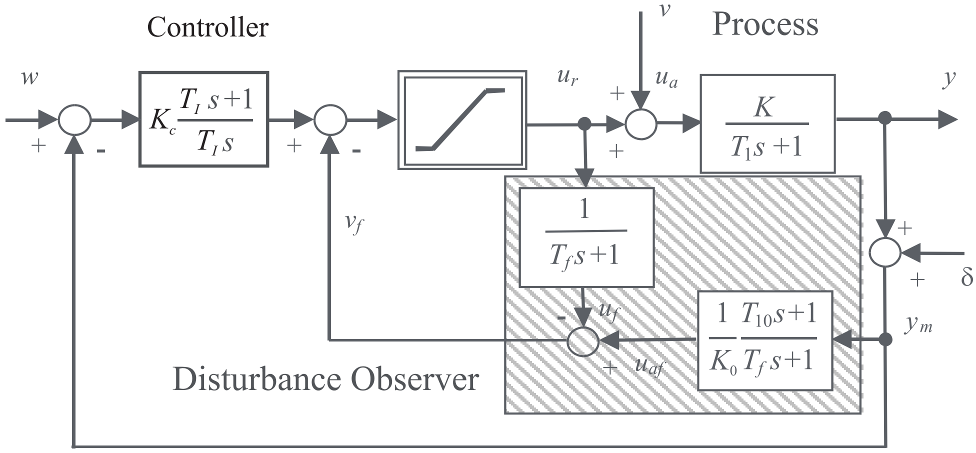

Figure 1.11: Disturbance Observer (DO) based servo-system proposed by (Umeno and Hori, 1991) and interpreted by (Zhao, 2004) as the DO-PI controller

straightforward. For characterizing the closed loop dynamics, we will use several qualitative and quantitative measures.

1.4.1 Settling Time t_s, IAE, TV, TV$_0$, TV$_1$ and TV$_2$

To characterize quality (speed) of control transients different performance indices are used as e.g. settling time, Integral of Absolute Error (IAE), Integral of Squared Error (ISE), or Total Variance (TV), whereby all these measures may be considered separately or in different logical combinations. Since we are always required to finish a control process in a limited time, it might seem that the basic performance index for process control should be defined as the *settling time*

$$y(t) - w = 0, \quad \forall t \geq t_s, \quad y_0 = y(0) \neq w \tag{1.8}$$

i.e. as time t_s required to reach by the output signal $y(t)$ starting from initial value $y_0 = y(0)$ a given setpoint value w. In general, however, the opposite is true. Here, we will exclusively deal with control problems that after a transient response to new reference state require maintaining system at its vicinity in steady state. In linear systems, transients to a constant setpoint value are theoretically infinitely long. So, finite settling time requires definition of certain neighborhood around it (Fig. 1.12). Such requirement also follows from the fact that all real control loops work with finite precision of measurement. To decide, when a transient finished by reaching steady state lying within defined neighborhood around reference value becomes yet more delicate problem in a noisy environment. Steady states can e.g. be indicated by fulfilling requirements put both on the plant

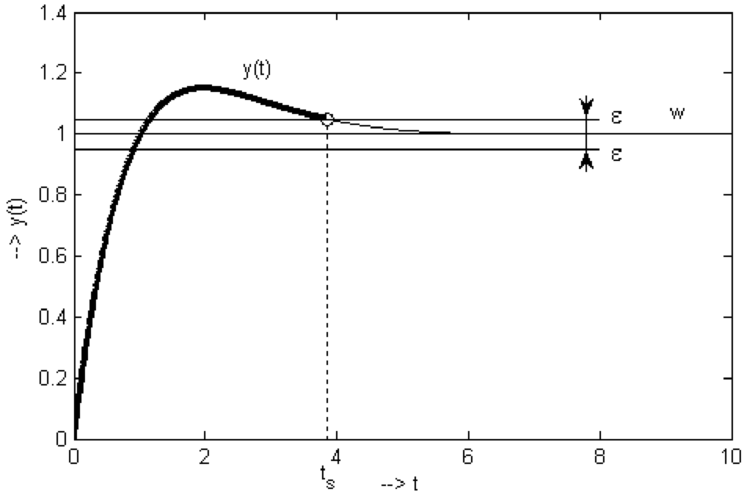

Figure 1.12: Definition of settling time t_s (1.9) based only on the plant output $y(t)$ with $\varepsilon = \varepsilon_y$

input and output

$$|y(t) - w| \leq \varepsilon_y \cap |u(t) - u_w| \leq \varepsilon_u, \quad \forall t \geq t_s \tag{1.9}$$

whereby the control signal value u_w corresponds to maintaining output at the setpoint value w and parameters $\varepsilon_y, \varepsilon_u$ may follow from particular technology (measurement noise & required control precision). Alternatively, they should be chosen in such a way to prevent a premature indication of steady state at flat extreme points of oscillatory transients. In general, plant output and plant input (controller output) may achieve steady states at different time moments, a nearly fixed output value may be achieved by oscillation at the input (dynamical steady state), there may exist steady states with nonzero steady state error, etc.
Since the settling time indication (1.9) depends on definition of several parameters, in order to characterize speed and duration of transients at the plant output in a simpler way, IAE (Integral of Absolute Error) or ISE (Integral of Squared Error) performance indices are frequently used defined as

$$\mathrm{IAE} = \int_0^\infty |[e(t) - e(\infty)]| \,\mathrm{d}t, \quad \mathrm{ISE} = \int_0^\infty [e(t) - e(\infty)]^2 \,\mathrm{d}t \tag{1.10a}$$

$$\mathrm{IAE} = \int_0^\infty |[y(t) - w]| \,\mathrm{d}t, \quad \mathrm{ISE} = \int_0^\infty [y(t) - w]^2 \,\mathrm{d}t \tag{1.10b}$$

where $e(\infty) = \lim_{t\to\infty} e(t)$
The first definition is usually preferred in situations, when some permanent error is allowed, but it should not lead to a permanent increase of the

integral values. The second definitions are appropriate for situations, where it is important to avoid permanent error.
With respect to problems with evaluating absolute value in analytical computations, i.e. due to the mathematical convenience, ISE is the criterion most frequently used by theoreticians for the analytical controller optimization. It is, however, also well known that such optimization underestimates small error values and leads to oscillatory transients. Therefore, with respect to practical requirements, in this book we are going to use dominantly IAE performance index, since. IAE is a good performance measure because the size and length of error is proportional to lost revenue (Shinskey, 1990). Because in optimizing controllers also minimal IAE values may correspond to transients with some overshooting, when aiming at monotonic transients, or transients without overshooting, it is not enough to look just for minimum of IAE, but one has to define also additional design constraints. The required output behavior can generally be achieved by different transients of the manipulated variable at the controller output. Therefore, it is useful to evaluate also Total Variance (TV), a criterion (Skogestad, 2003) introduced for characterizing "smoothness" and total "energy consumptuion" at the controller output. This was defined as

$$\mathrm{TV} = \int_0^\infty \left| \frac{du}{dt} \right| \mathrm{d}t \approx \sum_i |u_{i+1} - u_i| \tag{1.11}$$

Also this is mostly difficult to be evaluated analytically and therefore it is usually computed experimentally after appropriate discretization with sampling period as small as possible.

1.4.2 Basic Qualitative Shapes of Transient Responses

To describe qualitative properties of transients of PID control that may be composed from several exponentials, or periodic functions, we will introduce following definitions:

Definition 1.6 (Nonovershooting (NO) and Nonundershooting (NU) functions). Function of time $f(t)$ with initial value $f(0) = \lim_{t \to 0^-} f(t)$ different from its final value $f(\infty) = \lim_{t\to\infty} f(t)$ fulfilling conditions

$$\begin{aligned} [f(t) - f(\infty)] \operatorname{sign}\{f(0) - f(\infty)\} \geq 0 \quad \forall t \geq 0 \\ [f(t) - f(0)] \operatorname{sign}\{f(\infty) - f(0)\} \geq 0 \quad \forall t \geq 0 \end{aligned} \tag{1.12}$$

will be denoted as NonOvershooting (NO) and NonUndershooting (NU) function.

NO output property may follow from safety and technology requirements. It is important in many technologies, as e.g. in controlling machine tools, in traffic and flight control tasks, etc. In controlling systems with dead-time, nonovershooting properties may become different from the monotonic ones.

Definition 1.7 (Monotonic (MO) function). Function of time $f(t)$ with initial value $f(0) = \lim_{t\to 0^-} f(t)$ different from its final value $f(\infty) = \lim_{t\to\infty} f(t)$ and preserving direction of changes

$$[f(t_2) - f(t_1)]\,\mathrm{sign}\,\{f(\infty) - f(0)\} \geq 0 \quad \forall t_2 > t_1 \geq 0 \tag{1.13}$$

will be denoted as MOnotonic (MO) function.

Obviously, MO function is also NO and NU function, but not conversely. Monotonic functions typical for PID control may e.g. be given as $f(t) = 1 - \mathrm{e}^{-t/T_1}$; $y(0) = 0$; $y(\infty) = 1$, whereby $T_1 > 0$ is the time constant describing how fast the signal approaches new steady state value $y(\infty)$. For $t = T_1$ it should be at 63% of $y(\infty)$. By limiting $T_1 \to 0$ one gets from this exponential *step function* that so may represent *limit case of MO functions.*

MO transients at the controller output (plant input) and at the plant output may be motivated by energy savings in actuators, by minimizing their wear, generated noise and vibrations, by comfort of passengers in traffic control, or by precision increase in controlling systems with actuator hysteresis. MO controller output will also be expected to yield the lowest possible TV values.

Definition 1.8 (One-Pulse (1P) function). Function of time $f(t)$ that is continuous for $t > 0$ (with possible discontinuity at the origin) with initial value $f(0) = \lim_{t\to 0^-} f(t)$ and having with respect to the finite steady state value $f(\infty) = \lim_{t\to\infty} f(t)$ just single extreme point $f_m = f(t_m) \neq f(0)$ at $t_m \geq 0$, whereby it fulfills conditions

$$\begin{array}{l} [f(t_2) - f(t_1)]\,\mathrm{sign}\,\{f(t_m) - f(0)\} \geq 0 \\ [f(t_4) - f(t_3)]\,\mathrm{sign}\,\{f(\infty) - f(t_m)\} \geq 0 \end{array}, \ \text{for } 0 \leq t_1 < t_2 \leq t_m \leq t_3 < t_4 < \infty \tag{1.14}$$

will be denoted as One-Pulse (1P) function.

Obviously, 1P function may be defined as function with one extreme point that is MO before and behind this extreme point. By allowing discontinuity of $f(t)$ at the origin, e.g. for $f(t) = \mathrm{e}^{-t}\vec{1}(t)$ the extreme point may also move to origin from the right, when $t_m = 0^+$, whereby the interval before the extreme point shrinks to zero.

Examples of 1P functions may be represented by single exponential $f(t) = \mathrm{e}^{-t}\vec{1}(t)$ that has extreme point $f(0^+) = 1$ and discontinuity at the origin, or by difference of two exponentials $f(t) = \left(\mathrm{e}^{-t} - \mathrm{e}^{-2t}\right)\vec{1}(t)$ having extreme $f_m = 1/4$ at $t_m = \ln 2$

Definition 1.9 (Two-Pulse (2P) function). Function of time $f(t)$ that is continuous for $t > 0$ (with possible discontinuity at the origin) with initial value $f(0) = \lim_{t\to 0^-} f(t)$ that is having two extreme points $f_{m1} = f(t_{m1}) \neq f(0)$ and $f_{m2} = f(t_{m2})$ at $t_{m2} > t_{m1} > 0$ with respect to the finite steady state value $f(\infty) = \lim_{t\to\infty} f(t)$ and fulfilling conditions

$$\begin{aligned} [f(t_2) - f(t_1)]\,\mathrm{sign}\,\{f(t_{m1}) - f(0)\} \geq 0 \\ [f(t_4) - f(t_3)]\,\mathrm{sign}\,\{f(t_{m2}) - f(t_{m1})\} \geq 0 \\ [f(t_6) - f(t_5)]\,\mathrm{sign}\,\{f(\infty) - f(t_{m2})\} \geq 0 \end{aligned} \tag{1.15}$$

for $0 \leq t_1 < t_2 \leq t_{m1} \leq t_3 < t_4 \leq t_{m2} \leq t_5 < t_6 < \infty$

will be denoted as Two-Pulse (2P) function.

Obviously, 2P function may be defined as function with two extreme points that is MO on each interval not including one of them. By allowing discontinuity of $f(t)$ at the origin, the first extreme point may also move to origin from the right, when $t_{m1} = 0^+$, whereby the interval before this extreme point shrinks to zero. Example of such a function may again be given by difference of two exponentials $f(t) = \mathrm{e}^{-2t} - \mathrm{e}^{-t}$; $f(t) = 0$ with $f_{m1} = 1$ at $t_{m1} = 0^+$ and $f_{m2} = 1/8$ at $t_{m2} = \ln 4$.
By generalizing previous definitions to get a unique term for all above functions, we may come to notion of nP function. Within this text it will be mostly constraint to 0P, 1P and 2P functions.

Definition 1.10 (n-Pulse (nP) function). Function of time $f(t)$ that is continuous for $t > 0$ (with possible discontinuity at the origin) with initial value $f(0^-) = \lim_{t\to 0^-} f(t)$ that is having with respect to the finite final value $f(\infty) = \lim_{t\to\infty} f(t)$ n extreme points $f_{mi} = f(t_{mi})$, $i = 1, \ldots, n$ at $0 < t_{m1} < \cdots < t_{mn}$ and is MO on each interval not including one of these extreme points will be denoted as n-Pulse (nP) function. Again, by allowing discontinuity of $f(t)$ at the origin, the first extreme point may also move to zero from the right, when $t_{m1} = 0^+$, whereby the first MO interval before this extreme point shrinks to zero.

By introducing notion of nP function it is so possible to denote MO function as 0P one. Since by limiting values of nP function to any interval containing $f(\infty)$ one does not change number of extreme points, it can also be used in constrained control. After achieving saturation limits, by decreasing

duration of MO intervals among particular saturation pahses, nP functions may approach rectangular (relay) n-pulses of discontinuous MTC, but for $t > 0$ they always remain continuous in time.
To cover whole spectrum of transients typical for PID control we should yet complete the above list by definition of periodic functions interpreted as nP function with $n \to \infty$. Then, after specifying the damping ratio (as e.g. by Ziegler and Nichols (1942)) we could treat also oscillatory loop behavior. But, with respect to available space, within this text we will deal just with finite values of n.

1.4.3 Quantifying Qualitative Measures

By identifying properties like loop stability, NO, NU, MO, or nP shape of particular variable one typically get binary (true/false) information. On the other hand, performance indices like IAE or TV (1.8)–(1.11) give quantitative information about the loop behavior that enables refined evaluation of its quality. However, in control engineering it is frequently required to quantify also the above mentioned binary information, e.g. by expressing how far the system from stability, nonovershooting, or monotonicity border is. In the frequency domain there are broadly used robust design methods based on assigning stability degree, gain, phase and stability margin (see e.g. Anderson and Moore (1969); Datta et al (2000); Skogestad (2003); Skogestad and Postlethwaite (1996). Similarly, it is possible to introduce such quantitative measures for stability, nonovershooting, nonundershooting and monotonicity also in the time domain.
Quantitative measures for stability: In the time domain, stability or more precisely instability degree can be indicated in different ways – e.g. by requiring limited output value

$$|y(t)| < T_{\max} < \infty; \quad \forall t > 0 \tag{1.16}$$

by limiting possible settling time, IAE, ISE or TV values, maximal overshooting, by decreasing damping ratio, etc. When these measures increase over some predefined values chosen e.g. as a multiple of optimal value, or with respect to technological constraints, transients may be denoted as unstable. Despite this step seems to be mathematically vague, in fact it matches requirements of practice much closer than the usually used stability definition based on closed loop poles position – one can easily find example of stable system that is not usable in practice because of extremely high amplitudes of inner signals.
With respect to finite measurement precision and to quantization typical

for digital signals, it is more realistic to relate NO and NU signal properties not to a precise final value, but to an error band specified symmetrically (having width 2ε) around supposed final, or initial value. Then, for increasing signals over- and undershooting is signalized just after crossing this band. By introducing several levels ε it is possible to replace the binary (true/false) information by more detailed quantitative information telling e.g. that under measurement (evaluation) precision 2% of the maximal output value our system response may be considered as 0.02-NO, but this already does not hold for precision defined as 1% of the maximal output signal value. Similarly, for increasing, or decreasing signals it is possible to weaken strict monotonicity by introducing final evaluation precision into the monotonicity tests (Fig. 1.13).

Definition 1.11 (ε-NO and ε-NU functions). A continuous signal $f(t)$ with the initial value $f_0 = f(0)$ and with the final value $f_\infty = f(\infty)$ will be denoted as ε-nonovershooting, or ε-nonundershooting, when it fulfills conditions

$$\begin{array}{l} [f(t) - f(\infty)] \operatorname{sign}\{f(0) - f(\infty)\} \geq -\varepsilon \\ [f(t) - f(0)] \operatorname{sign}\{f(\infty) - f(0)\} \geq -\varepsilon \end{array} \quad \forall t \geq 0,\ \varepsilon > 0 \tag{1.17}$$

Definition 1.12 (ε-MO function). A continuous nearly MO signal $f(t)$ with the initial value $f_0 = f(0)$ and with the final value will be denoted as ε-monotonic when it fulfills condition

$$[f(t_2) - f(t_1)] \operatorname{sign}\{f(\infty) - f(0)\} \geq -\varepsilon;\ \forall t_2 > t_1 \geq 0;\ \varepsilon > 0 \tag{1.18}$$

To simplify program implementation, requirements (1.18) may be evaluated digitally by working with relatively small sampling period and by comparing just finite number of subsequent values $f(i)$ and $f(i+k)$, $k = 1, 2, \ldots, K$, whereby $K = T_h/T$, T being the sampling period, is chosen to cover at least one half-period T_h of possible high-frequency signal superimposed on the dominant monotonic signal

$$\begin{array}{c} [f(i+1) - f(i)] \operatorname{sign}\{f(\infty) - f(0)\} \geq -\varepsilon; \cap \ldots \\ \ldots \cap ([f(i+K) - f(i)] \operatorname{sign}\{f(\infty) - f(0)\} \geq -\varepsilon) \\ \varepsilon > 0, \quad K \geq 1, \quad i = 1, 2, \ldots, \infty \end{array} \tag{1.19}$$

Whereas (1.17)–(1.19) characterize amplitudes of superimposed high-frequency signals, deviations from strict monotonicity may also be characterized by limiting new integral measure that gives total contribution of high-frequency deviations (proportional not just to the amplitude, but also to the number of peaks) denoted as TV_0

$$\mathrm{TV}_0 = \sum_i |u_{i+1} - u_i| - |u(\infty) - u(0)| < \varepsilon_0 \tag{1.20}$$

TV_0 takes zero values just for strictly MO control signal transients.
In this way, it may be interesting to apply this criterion both to the plant output and to the plant input signals. Testing of amplitude deviations according to (1.19) may be reasonably simplified due to the following Lemma.

Lemma 1.13. *Constrained continuous signal $f(t)$ having initial value $f_0 = f(0)$ and final value $f_\infty = f(\infty)$ with local extreme points $f_{lei} = f(t_{lei})$ is ε-monotonic, if all subsequent local extreme points f_{lei} fulfill condition*

$$|y_{le,i+1} - y_{le,i}| \, sign\,(y_\infty - y_0) \geq -\varepsilon_y, \quad i = 1, 2, 3, \dots \tag{1.21}$$

Proof. Follows from the fact that the maximal signal increase in the direction opposite to $y_\infty - y_0$ in (1.18) will be constrained by two subsequent extreme points. interesting to apply this criterion both to the plant output and to the plant input signals.

Definition 1.14 (ε-nP function). Function of time $f(t)$ that is continuous for $t > 0$ (with possible discontinuity at $T = 0^+$) with the initial value $f(0^-) = \lim_{t\to 0^-} f(t)$, having for $t > 0$ n extreme points with respect to the finite final value $f(\infty) = \lim_{t\to\infty} f(t)$, whereby

$$|f_{mni} - f(\infty)| > \varepsilon; \quad f_{mni} = f(t_{mni}),\ i = 1, \dots, n \text{ at } 0 < t_{mn1} < \dots < t_{mnn} \tag{1.22a}$$

$$[f_{mni} - f(\infty)]\,[f_{mn,i+1} - f(\infty)] < 0,\ i = 1, \dots, n-1 \tag{1.22b}$$

that is ε-MO on each of $n+1$ intervals not including one of these extreme points will be denoted as the ε-nP function. By allowing discontinuity of $f(t)$ at $t = 0^+$, the first extreme point may also move to $t_{mn1} = 0^+$, whereby the first MO interval $(0, t_{tmn1})$ before this extreme point shrinks to zero.

Nearly nP-dynamics may also be specified by limiting TV_n values that take zero value exactly for signal consisting of $n+1$ monotonic intervals devided by n extremes. E.g. for signals with 1P dominant control it is possible to work with limited TV_1 criterion defined according to

$$\mathrm{TV}_1 = \sum_i |u_{i+1} - u_i| - |2u_m - u(\infty) - u(0)| < \varepsilon_1 \tag{1.23}$$

that takes zero values just for strictly 1P control signal, whereby it does not depend on possible control signal constraints. For control signals with superimposed higher harmonics it takes positive values.
Similarly, for systems with dominant 2P control function the acceptable contribution of higher harmonics may be limited by

$$\mathrm{TV}_2 = \sum_i |u_{i+1} - u_i| - |2u_{m1} - 2u_{m2} - u(\infty) - u(0)| < \varepsilon_2 \tag{1.24}$$

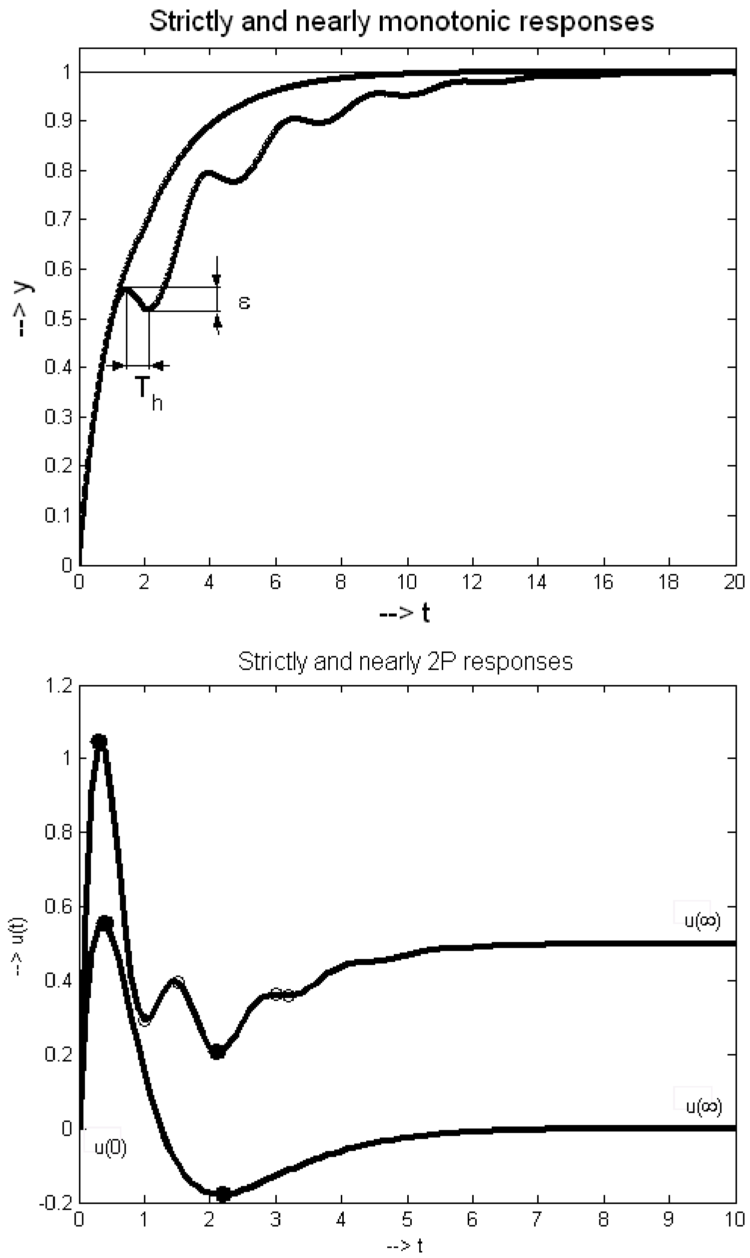

Figure 1.13: Above: Strictly monotonic signal satisfying (1.12) and "nearly monotonic" signal satisfying (1.18) with $K = T_h/T$, T being the sampling period, is chosen to cover at least one quarter-period T_h of possible high-frequency signal superimposed on the dominant monotonic signal $\varepsilon = \varepsilon_y$; Below: Nearly and strictly 2P responses; local extreme points denoted by "o" and significant extreme points denoted by "•"

For ideal 2P control functions it yields value $TV_2 = 0$

In applying weakened versions of ε-NO, ε-MO, or ε-nP properties it is, however, to remember that achieved information depends on ε – e.g. with acceptable overshooting 1% a transient may be classified as MO, but for acceptable overshooting 0.1% as 1P function. From one point of view it is quit normal that under final measurement precision one is not able to distinguish these two properties, when the error is below the system resolving power. On the other hand, it is clear that these weakened versions should be applied carefully with tolerances not exceeding acceptable measurement (evaluation) precision, otherwise one get unexpected and unusable results. Whereas in controlling stable plants it is possible to decrease the number of control pulses up to zero by keeping MO controller output, in controlling unstable plants the number of control pulses cannot decrease below the number of unstable poles.

NO specifications (not distinguishing between nonovershooting and monotonic control) exist also in the frequency domain (see e.g. Keel et al (2008)) but their application is extremely complicated, especially when speaking about dead time systems. Specific measure for deviations from monotonicity was also introduced by Åström and Hägglund (2004). Here, we have preferred new measures for deviations from NO, MO and nP function properties that may not only be tested numerically, by evaluating simulated or experimentally measured transients corresponding to the setpoint and disturbance step responses, but they represent a modular system and are also appropriate for constrained control. These specifications may be hierarchically organized into trees, whereby the closed loop stability will be considered as the root property, NO, NU, MO and nP as child properties. Graphically represented in the plane of loop parameters, together with quantitative measures, such properties will be giving *performance portrait* of particular control loop.

1.4.4 Performance Portrait (PP)

The closed loop PP represents information about the closed loop performance corresponding to setpoint and disturbance step responses expressed over a grid of (possibly normalized normalized) loop parameters. For a loop represented by a D-dimensional parameter vector $P = \{p_1, p_2, \ldots, p_k, p_{k+1}, p_{k+d}\}$; $D = k + d$, whereby some part of parameters p_i; $i = 1, \ldots, k$ is a priori given, some parameters will be fixed during the loop analysis and $p_i \in [p_{imin}, p_{imax}]$; $i = k_1, \ldots, k + d$ may vary over some (known) intervals. So, the performance portrait will be considered in the space with the total dimension D, whereby the variable parameters forming subspace with the dimension d will take levels $p_{i,j} = p_{imin} + (p_{imax} - p_{imin})j/n_i$; $j = 1, 2, \ldots, n_i > 1$.

By containing information about required loop properties PP may be used both for optimally localizing a nominal operating point by appropriate controller tuning, or for optimally localizing an uncertainty set of all possible operating points corresponding to specified intervals of variable loop parameters. When e.g. working with the plant model (1.1) the loop parameters are K_s, a_0, a_1, T_0, T_d. Many control method are based on inversion of the plant model, whereby model parameters could be denoted as $K_{s0}, a_{00}, a_{10}, T_{00}, T_{d0}$. Inversion will require at least first order filter with a time constant T_f. Specification of the setpoint response will require determination of at least one time constant T_w. It means that in total there are 12 parameters that determine the resulting dynamics. If e.g. two of them, say $K_s \in [K_{s,min}, K_{s,max}]$, $T_d \in [T_{d,min}, T_{d,max}]$ are variable, the task of the control design will be to choose appropriate model parameters $K_{s0}, a_{00}, a_{10}, T_{00}, T_{d0}$ and free design parameters T_s, T_w in such a way that over all points over the uncertainty set corresponding to $K_s \in [K_{s,min}, K_{s,max}]$, $T_d \in [T_{d,min}, T_{d,max}]$ chosen according to $p_{i,j} = p_{imin} + (p_{imax} - p_{imin})j/n_i$; $j = 1, 2, \ldots, n_i > 1$ required performance measures will be achieved. PP required for such a design may be generated by simulation, or by real time experiments. Although its generation may be connected with numerical problems, especially those related to the nature of grid computations, when one has to balance precision of achieved results (quantization level in considered grid) with the total number of evaluated points and the corresponding computation time, it gives very promising results especially when dealing with dead time systems.

1.5 Dynamical Classes (DC) of Control

By introducing qualitative measures for transient responses, we are now able to categorize all PID controllers that are able to yield MO step responses at the plant output according to the shape of their manipulated (control) variable. If this has properties of nP functions, we will include the corresponding control into the dynamical class of control with index n, shortly DCn.

Today, also people without a background in optimal control understand that if they wish to move with their car monotonically from one point to another they have at least once accelerate (it means to increase kinetic energy of the car) and then to brake (decrease the energy). Or, if they wish to charge a container, they must open the input valve for some interval of time. So, control processes are by its nature related with energy accumulation, or dissipation and the transients are expected to be the fastest one if they are related with maximal values of the manipulated variable. This fact was reflected by the Feldbaum's theorem (Feldbaum, 1965) published firstly in 1949.

Theorem 1.15 (Feldbaum's Theorem). *For the MTC of the n-th order system from one constant reference output value to another one there are required n-intervals of optimal control, when the control signal step-by-step changes from one limit value to the opposite one.*

Despite the fact that later works (by Bushaw (1958); Pontryagin et al (1964), etc.) showed that in controlling oscillatory systems and initial states sufficiently far from the required ones the total number of intervals can also be higher (see e.g. Athans and Falb (1966)), by restricting our treatment to monotonic output transients from a steady state to another one, Feldbaum's theorem still represents one of the corner stones of optimal control. But when examining majority of existing textbook on PID control, about n-interval of optimal control you are going to find practically nothing - one of few positive exception is the already mentioned book by Glattfelder and Schaufelberger (2003).
It is true that in practice it is just rarely permitted to apply control dealing exclusively with limit control values and with their instantaneous changes. The "rectangular" pulses with sharp rims excite in controlled systems theoretically an infinitely broad frequency spectrum of higher harmonics. Excitation of higher harmonics could cause ineligible reactions. Furthermore, we are mostly not able to perform such control perfectly, and such a control is usually not acceptable with respect to the technological constraints. An admissible rate of the control signal changes or an admissible acceleration use to be constrained by construction, or have to be respected by control. So, if the physical substance of the Feldbaum's theorem has to be respected, then in modified "softer" versions, when the control pulses will not be rectangular, but continuous in time – i.e. somehow "rounded". Besides of the amplitude constraints, one has to respect also rate constraints, or even constraints on higher-order control signal derivatives. Under such constraints it may happen that some interval does not take the limit value, or even some of them fully melt away from the control responses. To cover also such "softer" control responses and to distinguish them from the rectangular train of pulses of the MTC it is then better to speak about *dy-*

namical classes of control and about corresponding *fundamental* controllers establishing bridges between smooth linear PID control and nonlinear discontinuous MTC.
The aim of this chapter is to explore dynamics of PID control and make it compatible also with the MTC. In relay MTC of simple plants, output responses corresponding to transition from one steady state to a new reference steady state are typically monotonic. The already mentioned exception of systems with complex roots with larger initial deviation from final state is not relevant for such a problem. The rectangular shape of control signal in relay MTC is, however, possible just in a limit case of control

- without constraints on the rate of the control signal and/or on its higher derivatives,
- with negligible nonmodelled dynamics,
- for negligible fluctuations of plant parameters and
- for negligible measurement noise (full information about state).

To respect all these additional factors control signal must become "softer". Thereby some its pulses may not hit the constraints, or even some pulses may fully disappear. So, by stressing importance of the above mentioned limitations, acceptable closed loop dynamics may naturally tend to lower number of control pulses, in the limit case of stable systems to single interval with MO controller output.

Example 1.16 (Smooth control of an n-tuple integrator). When considering stable single integrator $\dot{y} = u_1$ (Fig. 1.14) with output y changing monotonically from an initial value $y_0 = y(0)$ to a final value $y_\infty = y(\infty) > y(0)$, y will be increasing (not decreasing) if its derivative is a positive (non negative) function of time, i.e. $\dot{y}(t) > 0$, or $\dot{y}(t) \geq 0$, $t \in (0, \infty)$. With respect to the plant equation $\dot{y} = u_1$, $t \in (0, \infty)$ it also means that for $t \in (0, \infty)$ the control $u_1(t)$ must take positive (non negative) values and in the initial and final steady states satisfy conditions $u_1(0^-) = \dot{y}(0) = 0$ and $u_1(\infty) = \dot{y}(\infty) = 0$ - signal $u_1(t)$ continuous for $t \geq 0$ and satisfying these conditions must take a maximum $u_{m1} - u(t_{m1}) > 0$; $\dot{u}(t_{m1}) = 0$ for some $t_{m1} \in (0, \infty)$. Under constrained control, when the control signal saturates, the maximum value may also be achieved over an interval $t \in [t_{max1}, t_{max2}]$. It is also obvious that in order to achieve as fast as possible output increase, the maximum u_{m1} should be as large as possible and, in order to keep MO output increase, $u_1(t)$ must remain positive even

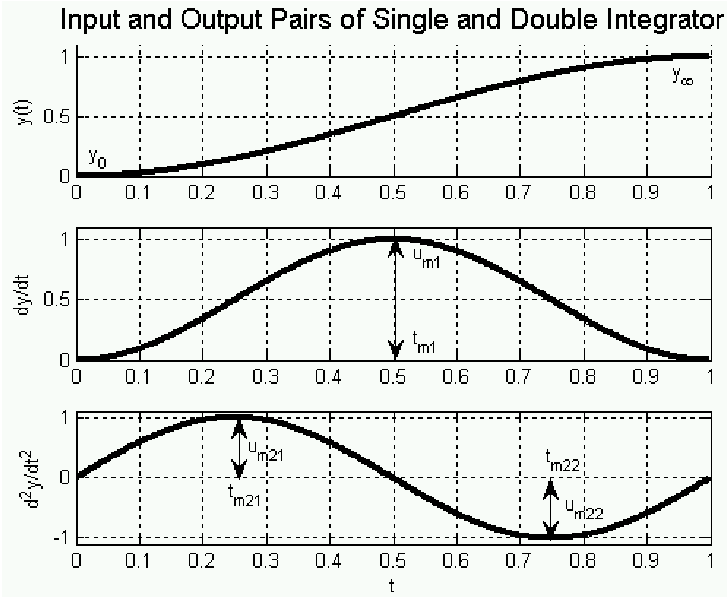

Figure 1.14: MO output y satisfying (1.18) for $\varepsilon = \varepsilon_y = 0$ (above) with the corresponding 1P input signal of single integrator $u_1(t) = \dot{y}(t)$ (middle), or with the corresponding 2P input signal of the double integrator $u_1(t) = \ddot{y}(t)$ (below)

in the case when it has several local extreme points $u_{1e}(t_{ei})$; $i = 1, 2 \ldots$ corresponding to $\dot{u}_1(t_{ei}) = 0$.

The simplest control, however, corresponds to situation with $u_1(t)$ having just a single local extreme that separates the overall control into two monotonic intervals: the first one monotonically increasing from $u(0) = 0$ up to $u_{m1} = u(t_{m1})$ and then the second one monotonically decreasing from $u_{m1} = u(t_{m1})$ up to $u(\infty) = 0$.

Similarly, we could treat also a decrease of the setpoint value. So, we may conclude that in a general case the ideal control guaranteeing MO output transition between two steady state values of single integrator will be characterized by smooth continuous 1P $u_1(t)$ satisfying given initial and final conditions and having one extreme point and being monotonic before and after this extreme point. By accepting possible control discontinuity at $t = 0^+$, when the extreme point moves to $t_{m1} = 0^+$, the first MO interval may shrink to zero. However, the MO output increase finishing by reaching steady state cannot be achieved by simpler 0P (e.g. step) control signal.

In order to control the double integrator, one has to put additional integrator in front of the previous one and to consider that for achieving a MO increase of $\dot{y}(t)$ (the earlier input, now output of the added integrator) for $t \in (0, t_{m1})$, the new continuous input $u_2(t)$ must be described by a function having one maximum $u_{m21} > 0$ at an interior point $t_{21} \in (0, t_{m1})$ that

divides the whole interval $(0, t_{m1})$ into two MO subintervals $(0, t_{21})$ and (t_{21}, t_{m1}).
During the earlier second phase of control with $t \in (t_{m1}, \infty)$, in order to achieve a monotonic decrease of $\dot{y}(t)$, input of the new integrator $u_2(t)$ must firstly decrease to its minimal value $u_{m22} < 0$ at some $t_{m22} \in (t_{m1}, \infty)$ and then monotonically increase to its final value $u(\infty) = 0$. So, instead of the originally two control intervals, now one has to consider three monotonic control intervals. When accepting control discontinuity at $t = 0^+$ the number of intervals may drop to two. But by requiring MO output increase finishing by reaching new steady state control of the double integrator cannot be achieved by simpler input, e.g. by a 0P, or by a 1P signal.

Similar conclusions may also be derived for controlling unstable systems – the number of required pulses cannot be lower than the number of unstable poles. For these systems we get similar conclusions like for the relay MTC. For stable plant poles poles their influence on the resulting closed loop dynamics may vary. If a plant has only stable poles and its open-loop response is monotonic, then it is always possible to find a controller guaranteeing monotonic closed loop setpoint response at the plant output by a monotonic signal at the plant input. By requiring shorter transients, one extreme point in the setpoint response may become visible, or right two extreme points, etc. A lot will depend on the plant dynamics, on the chosen controller and on required speed of processes. Once you decide to use controller producing at least one extreme point in the control signal, by speeding up the response you may expect problems with the control signal saturation. In order to solve all associated problems, we need to introduce some internal classification of all possible control tasks. This will be achieved by introducing dynamical classes of control.

Definition 1.17 (Dynamical classes (DC) of control). With n being nonnegative integer, under *Dynamical Class* n (shortly DCn) of PID control we understand all control tasks and their solutions with MO plant output and nP plant input.

In characterizing shape of the control signal by nP function n corresponds to the non-negative integer used in denoting number of possible extreme points or intervals with saturated control signal values that also corresponds to the number of constrained pulses occurring under MTC. Such control signals have to bring the plant output monotonically from one steady state to another one. With respect to Definition 1.1 and the Feldbaum's Theorem it is possible to conclude that all tasks and dynamical

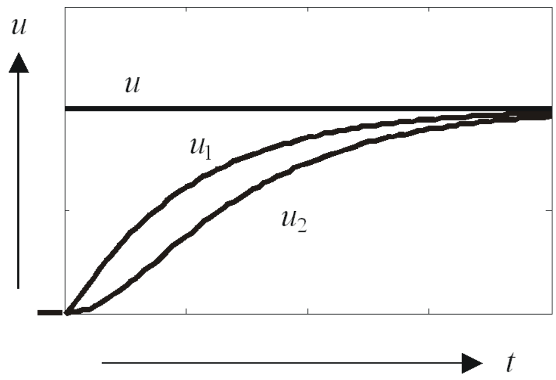

Figure 1.15: DC0: Control signal reaction to a setpoint step; u – without rate constraints, u_1 – with a rate constraint, u_2 – with constrained 2nd derivative of the control signal

processes of the PID control correspond to DC0, DC1 and DC2. What does it mean?

1.5.1 Dynamical Class 0 (DC0)

In this dynamical class, after a step change of reference variable both the manipulated variable (controller output) and the plant output change monotonically, from one steady state to another one.

Definition 1.18 (Step response dynamics of DC0). MO control signal both at the controller and plant output initiated by a setpoint step change characterize step response dynamics of DC0.

Examples of such control signals at controller output are in Fig. 1.15. Limit case of such monotonic transients at the plant input, or output is the step function.
Processes of DC0 can be met in situations, where the dynamics of transients in plant may be neglected, i.e. it is not connected with a reasonable energy, or mass accumulation. Such processes are e.g. typical in controlling flows by valves. After constraining rate of control signal changes, transition to a new control signal value can be exponential one. After constraining also amplitude of the 2nd control signal derivative, the control response takes form of S-function (Fig. 1.15). Since for properly dimensioned actuators and admissible inputs the control constraints will never be active, these control loops are traditionally well treated within the framework of the linear control theory.
It is yet to note that validity of the NO condition (1.12) does not automatically mean that the control transient must be stable. Therefore, condition (1.12) should yet be combined with some measure indicating system

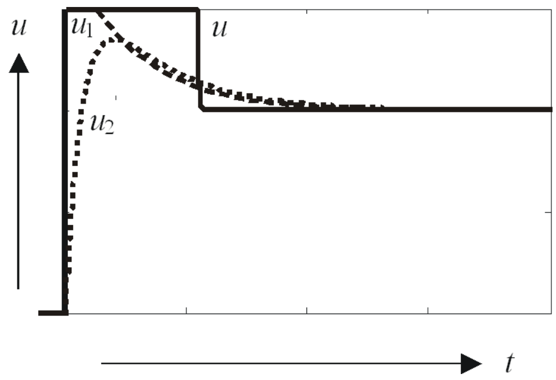

Figure 1.16: DC1: Control signal reaction to a setpoint step change; u – time optimal (without rate constraints), u_1 – with rate constraints for the transient from the limit to the steady state value, u_2 – as u_1, with an additional limit on the control signal increase.

stability. Simultaneous fulfillment of monotonicity (1.13) with constrained values at the plant output and input usually fully guarantee also the parent property – BIBO system stability.

1.5.2 Dynamical Class 1 (DC1)

In DC1, for the initial phase of control response initiated by a setpoint step it is typical accumulation of energy in the controlled process. This is associated with a gradual increase (decrease) of the controlled variable that runs most rapidly under impact of the limit control signal value. Control signal may be qualified as one-pulse function.
E.g. by charging a container with liquid, in the first phase of control the input valve should be fully opened, whereas the output value (liquid level) monotonically increases and only in the vicinity of the required level the input flow starts to decrease to a steady state value what will stabilize required output value. Similar transients can frequently be met in speed control in mechatronic systems, in temperature, pressure and concentration control, etc.

Definition 1.19 (Step response dynamics of DC1). Dynamics with MO output response and 1P control signal reaction corresponding to a setpoint step change (involving one extreme point, or one control interval with control signal at one of the control signal constraints, Fig. 1.16) will be classified as dynamics of DC1.

From requirement of single extreme point of control signal (one interval at the limit control value) it follows that the transition from initial control

signal value to its extreme point and transition from this extreme point to the steady state value u_∞ will be monotonic.
Rectangular pulse of MTC with infinitely short transient from limit control signal value to the steady state value represent limit situation not fully achievable in practice. After limiting rate of changes during the control signal decrease to the steady state (response u_1), the span of the limit control action decreases, but the total length of transient to the new steady state increases. When constraining also the control signal increase (response u_2), the control signal does not catch to reach the limit value, since the necessary control decrease to the steady state has to start yet before it – the length of transient grows further. Whereas the single interval of control is still visible, by constraining rate of the control signal changes the control signal reaction slowly approaches monotonic shape typical for DC0.
With respect to one possible interval with constrained controller output, for dealing with DC1 it is usually not enough to remain within the linear control. Typical solutions for this dynamical class are frequently achieved with different aw - controllers.

1.5.3 Dynamical Class 2 (DC2)

In DC2 output changes are associated with accumulation and recurrence or dissipation of energy required for achieving state and output changes and stabilization at a new steady state.

Definition 1.20 (Step response dynamics of DC2). Dynamics corresponding to MO output response to a setpoint step change with 2P control signal reaction involving two extreme points, or two control intervals (Fig. 1.17) with control value subsequently constrained to the upper and lower limit value (or conversely), will be classified as dynamics of DC2.

Within the DC2 the control signal reaction to a setpoint step bounded to monotonic output response can already involve two extreme points. After these two intervals (two extreme points) control signal is monotonically tending to a new steady state value u_∞ .
According to the Theorem 1.15 the MTC is typical with two (rectangular) control pulses approaching both limit control values (Fig. 1.17). After introducing rate constraints for both the switching from one limit value to the opposite one and for transient to the steady state value u_∞, the 2^{nd} control interval is typically rounded, or even disappears. Such response u_2 is typically converging to the next lower DC.

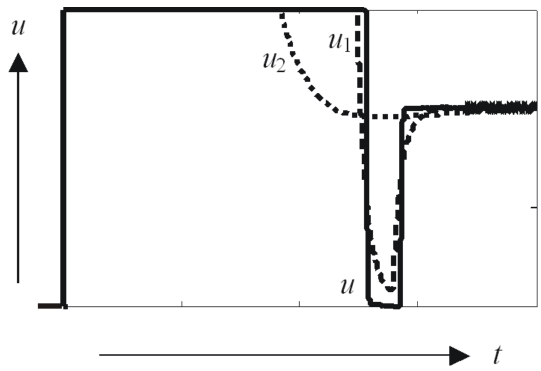

Figure 1.17: DC2: Control signal reaction to a setpoint step change; u – time optimal (without rate constraints), u_1 – by limiting rate of changes in transient from one control limit to the opposite one and in transient to the steady state, u_2 – as u_1 but with stronger constraints.

In the case when the rate constraints allow the control signal to attack both the upper and lower control limit, also the majority of aw approaches fail (Rönnbäck, 1996) (improved solutions are e.g. given by Hippe (2006)). The windup phenomenon is not only connected with the integral (I) action, but also with the controlled process, when it is denoted as the plant windup Glattfelder and Schaufelberger (2003). In the literature simple and reliable solution for this dynamical class that could enable an arbitrary dynamics shaping ranging from the fully linear one up to the on-off MTC are still missing. For all that the needs on such solutions are very high: Let's mention just the automotive industry. Here, the historically known cascaded linear structures are not able to fulfill sufficiently the existing expectations. Although this task is practically solved (see e.g. Huba (2003, 2006)), the new solutions are not yet widely known.

1.6 Fundamental and “ad hoc” Solutions

Under fundamental controllers we understand solutions that for the nominal loop dynamics offer continuum of transient responses parameterized by the closed loop poles (or equivalent parameters as time constants of bandwidths) and enable to achieve any speed of control ranging from linear pole assignment control up to the relay MTC. Under “ad hoc” controllers we will understand solutions offering single (not adjustable) closed loop dynamics, or dynamics adjustable just in a limited range.
E.g. the relay MTC may be denoted as a typical “ad hoc” solution, since it offers unique closed loop dynamics that cannot be simply slowed down.

1.6.1 Setpoint Response

Since all solutions of DC0 will always remain linear, the corresponding fundamental solutions may be derived by the linear pole assignment control. How it is possible to characterize their substance?
Let us consider specification of the closed loop dynamics by two n-tuples of poles[1] α_1 and α_2 satisfying

$$-\infty < \alpha_2 < \alpha_1 < 0 \tag{1.25}$$

The setpoint response $\bar{y}(\alpha_i, t) = y(\alpha_i, t)/w(t)$ representing output reaction to the setpoint step $w(t) = w\vec{1}(t)$; $w = \text{const}$ is starting for $\alpha_i; i = 1, 2$ in a steady state with zero initial condition $\bar{y}(\alpha_i, 0) = 0$.

Definition 1.21 (Fundamental controller of DC0 – setpoint response). Controllers offering for a setpoint step and poles (1.25) output responses satisfying Ineqs.

$$1 > \bar{y}(\alpha_2, t) > \bar{y}(\alpha_1, t) > 0; \;\; \forall t > 0 \tag{1.26}$$

and asymptotic properties

$$\lim_{t\to\infty} \bar{y}(\alpha_i, t) = 1; \quad i = 1 \text{ or } 2 \tag{1.27}$$

will be denoted as *fundamental* one.

Fundamental solution simply means that by shifting closed loop poles to the left the corresponding outputs converge to the reference value faster, but monotonically, i.e. without overshooting, or undershooting, or without changing somehow their shape.
Similar effect in increasing speed of control we would like to achieve in constrained systems treated in DC1 and DC2. Here, the step response of the MTC representing the not really achievable limit dynamics will be denoted as $\bar{y}_{topt}(t)$. Let us suppose that the required state may be achieved by monotonic output transient, i.e. the Feldbaum's theorem holds. Expectations on the fundamental controller may then be expressed by following definition.

Definition 1.22 (Fundamental controllers of DC1 and DC2 – setpoint response). Controller yielding for the nominal dynamics $S(s)$ and for the closed loop poles (1.25) MO setpoint step responses of the output variable and fulfilling Ineqs.

$$1 > \bar{y}_{topt}(t) = \bar{y}(-\infty, t) \geq \bar{y}(\alpha_2, t) \geq \bar{y}(\alpha_1, t) > 0; \;\; \forall t > 0 \tag{1.28}$$

[1]Poles need not to be n-tuple, but in both vectors there should be kept fixed ratio of corresponding entries to the representative value α_i, $i = 1, 2$

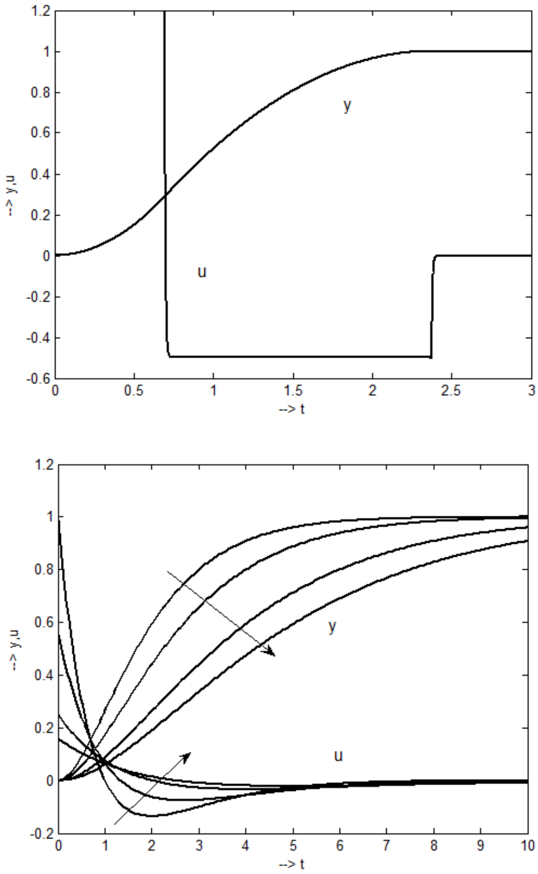

Figure 1.18: Output and control signal transients of double integrator plant: left - for the relay MTC with control signal constrained by $U_{max} = 1.2$ and $U_{min} = -0.5$; right – for linear pole assignment control with double real pole values $-1, -0.75, -0.5$ and -0.4 arrows indicate pole shifting to zero (i.e. slowing down transients

and asymptotic requirement

$$\lim_{t \to \infty} \bar{y}(\alpha_i, t) = 1; \quad i = 1 \text{ or } 2 \tag{1.29}$$

will be denoted as *fundamental* one.

Requirement (1.28) means that the closed loop dynamics may be specified by the closed loop poles (or other appropriate parameters, as e.g. closed loop time constants, closed loop bandwidth, etc.), whereas it is ranging arbitrarily from fully linear dynamics of pole assignment control up to the relay MTC one (Fig. 1.17). Both these situations are considered as limit cases of the generalized approach.

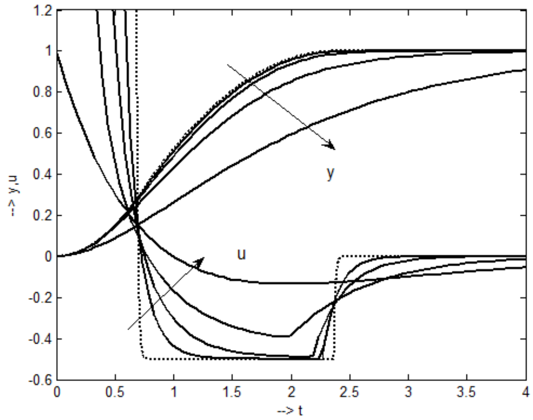

Figure 1.19: Output and control signal transients of double integrator plant by constrained pole assignment control $U_{max} = 1.2$ and $U_{min} = -0.5$; for double real pole values $-8, -4, -2, -1$ – arrows indicate pole shifting to zero (i.e. slowing down transients); dotted – MTC transients

1.6.2 Disturbance Response

Notion of the dynamical classes can be applied both to the setpoint step responses as well as to the disturbance responses. The main difference is connected with the fact that whereas in the case of a step change of the reference signal the controller is instantaneously able to react, after a disturbance step it take some time up to moment when the disturbance observer sufficiently reconstructs the new disturbance value. Due to this reconstruction time, controller is able to stop control error increase appearing during this phase of disturbance response just with some delay. Just then controller starts to remove the already existing deviation. From this moment, output and control signal behavior can be analyzed in the same way as after a setpoint step. So, when speaking about disturbance response of DC0, instead of MO output considered at the setpoint response we are actually dealing with 1P output that can be characterized as MO just from the turnover point.

In the following we will deal just with controllers offering disturbance output response that after initial deviation monotonically tends to the required reference value.

In evaluating disturbance step responses one can test the same properties as for the setpoint response, but (besides of the already mentioned delay in disturbance compensation) it is to note that:

- By a prefilter in the reference signal it is possible to slow down dynamics of the setpoint step responses and by the disturbance observer filter to modify the disturbance response – in this way it is possible to tune both responses at least partially separately – we are speaking about two-degree-of-freedom controllers.

- In general, areas of parameters guaranteeing NO, MO, or nP disturbance responses are different from those corresponding to setpoint step responses.

Let the disturbance response $\bar{y}(\alpha_i, t) = y(\alpha_i, t)/d(t)$ represents output reaction to the disturbance step $d(t) = d\vec{1}(t)$; $d = \text{const}$ starting for $\alpha_i; i = 1, 2$ in a steady state with zero initial condition $\bar{y}(\alpha_i, 0) = 0$. Again, we would like to deal with controllers that by pushing the closed loop poles to minus infinity enable to increase speed of removal of the control error caused by disturbance step and thereby to decrease its amplitudes up to zero.

Definition 1.23 (Fundamental controller– disturbance response). Controllers offering for a disturbance step and poles (1.25) output responses satisfying to Ineqs.

$$0 \leq |\bar{y}(\alpha_2, t)| \leq |\bar{y}(\alpha_1, t)|; \ \forall t > 0 \tag{1.30}$$

and asymptotic properties

$$\lim_{t\to\infty} \bar{y}(\alpha_i, t) = 0; \quad i = 1 \text{ or } 2 \tag{1.31}$$

$$\lim_{\alpha_i\to\infty} \bar{y}(\alpha_i, t) = 0; \quad \forall t > 0; \quad i = 1 \text{ or } 2 \tag{1.32}$$

will be denoted as *fundamental* ones.

Since the delay in the disturbance reconstruction and compensation may be significantly long and due to this also the control error occurring at the moment of output turnover, it is again possible to consider different DC associated with its removal.

1.6.3 Internal and Zero Dynamics

In systems with relative order of considered output r less than the total system order n there always exist states that are not directly controllable by input. They represent the so called internal dynamics. To achieve some simplicity, in nonlinear control this internal dynamics is usually characterized by simpler expressible zero dynamics.

Definition 1.24 (Relative order of the system output). As the relative degree of the plant output we denote integer r, telling, how many times it is required to differentiate output to get in the resulting formula control signal (plant input).

Definition 1.25 (Zero dynamics). Zero dynamics of a system with relative degree $r < n$ describes dynamics associated with maintaining output and its first r derivatives at zero. For linear system given by its transfer function with highest power in numerator m and highest power in denominator n the relative degree is given as the pole-zero excess

$$r = n - m \tag{1.33}$$

From Definition 1.25 it follows that the zero dynamics is interesting just for systems with the relative degree less than the system's degree. In designing PID controllers, when dealing with dominant dynamics up to the 2^{nd} order (i.e $n \leq 2$) the only situation with zero dynamics corresponds to systems with the relative degree $r = 1$ and $r = 2$. In a special case of systems with real poles it corresponds to situation when the total plant dynamics may be decomposed into two parallel first order plants

$$F(s) = \frac{K_1}{1 + T_1 s} + \frac{K_2}{1 + T_2 s} = K \frac{1 + T_0 s}{(1 + T_1 s)(1 + T_2 s)} \tag{1.34}$$

$$K = K_1 + K_2, \quad T_0 = \frac{K_1 T_2 + K_2 T_1}{K_1 + K_2} \tag{1.35}$$

In this case, zero dynamics is characterized by the time constant T_0 of the numerator of the transfer function. Number $-1/T_0$ is thereby denoted as the plant *zero*. Fundamental solutions that would enable an arbitrarily close tracking of the reference variable may be found just for systems with stable zero dynamics, when $T_0 > 0$. From the point of view of the MTC this situation corresponds to the so called singular problem (Athans and Falb, 1966), when during the 2^{nd} period of control, the manipulated variable does not go to saturation limit, but takes values denoted as zeroing input (Isidori, 1995) that varies with the time constant T_0. So, Feldbaum's Theorem is valid just for systems with the relative degree equal to the full degree. For $r < n$ the total number of control intervals remains to be given by n, but just r from them may run with the limit control values.
For systems with unstable zero dynamics ($T_0 < 0$) fundamental solutions do not exists and it is possible to design just solutions preserving this unstable zero dynamics leading usually to some output undershooting.

1.7 Dead Time Systems

Another theoretically challenging and due to this being backward segment of the control theory is represented by the time-delayed systems. For this area, two early historical solutions are known: the Disturbance-Response

Feedback by Reswick (1956) for dead-time compensation and the Smith Predictor (Smith, 1957). Practical experiences referred by many papers show that both structures have strong limitations: they are highly sensitive to parameters fluctuations and they do not enable an arbitrarily close approximation of optimal solutions. Controllers with dead time compensation are frequently denoted as the predictive ones (see e.g. predictive PI-controller in Åström and Hägglund (1995)), but sometimes also as the PI - dead-time controllers (Shinskey, 1996, 2000). In the time of analogue pneumatic and electronic controllers the main reason for rare use of the corresponding structures was given by the problems of dead time modeling. So, for many decades' traditional PID controllers without dead time compensation substituted optimal solutions for the dead time compensation. These approaches did not guarantee strictly optimal results and so they have reasonably contributed to the inflation of different "optimal" controller tuning. They further survive due to the conservativeness of practice despite the fact that the new digital controllers enable an easy dead time modeling and compensation.

1.7.1 Delayed Fundamental Controllers

One important question is if it is possible to achieve for the time delayed systems the same dynamics as for the delay-free systems.
For the setpoint response answer to this question depends on the dead time position within the closed loop. If it is situated in the feedback loop, controllers with dead time compensation enable to achieve at the output the same dynamics as in the delay free systems. For known initial conditions the delay-free controller-plant connection enables an immediate action.
Feedback controller compensating dead time present somewhere in the loop, will be, of course, more complicated and the closed loop behavior will be much more sensitive to any model imperfection. But, theoretically, for in advance known input signals it is possible to achieve at the plant output any speed of control transients.
However, when dealing with response to unknown disturbances, or when dealing with step response and dead time is located within the feedforward path, at the output any result of control actions can appear just after the time delay. Therefore, in such situations, the best achievable behavior will be delayed by this dead time and it has to be respected also by the corresponding definitions of fundamental controllers that will be denoted as *delayed fundamental controllers*. The difference will especially be visible in the disturbance response, where it is no more possible to achieve

requirement (1.32).

1.7.2 Fundamental Controllers – a New Concept?

Many of the known approaches to the controller design do not fulfill the requirements on the fundamental solutions, since they:

- are linear and so they do not enable to arbitrarily speed up transients to approach in the limit under consideration of constraints the MTC transient responses, or
- do not involve free design parameters at all.

E.g. Klán and Gorez (2000) (as many others) tried to find optimal PI controller tuning for stable 1st order systems with relatively long dead time T_d. The problem, however, is that structure of the PI controller was generically derived for the 1st order plant dynamics without the dead time. Short dead time values can be allowed by limiting choice of the applicable controller parameters (closed loop poles). This, however, violates requirements put on the fundamental solutions. For the 1st order plant with long dead time the fundamental controller will already have more complicated structure than simple PI controller – involving some features of the Smith predictor. So, the above mentioned solution cannot be treated within the group of fundamental solutions for the first order plants with long dead-time, just as a special (ad hoc) suboptimal solution.
The above example represents typical feature of majority of existing solutions. The PI controller represents an easy to use, but not a fundamental solution. In the time of analogue pneumatic and electronic controllers the main reason for rare use of the optimal dead time structures was given by the problems of the dead time implementation. So, for many decades' traditional PID controllers without dead time compensation substituted them. These approaches do not guarantee strictly fundamental properties and so they have reasonably contributed to the inflation of different "optimal" controller tuning. They further survive due to the conservativeness of practice despite the fact that the new digital controllers enable an easy dead time modeling and compensation. Of course, it has no sense to fight against their use, but it should be shown what they are able to offer. In such a way, all the ambiguity of solutions reported e.g. by O'Dwyer (2000, 2006) can be reasonably reduced.

1.8 Table of Fundamental PID Controllers

As it was already mentioned above, Glattfelder and Schaufelberger (2003) tried to design the pole assignment PI controller in such a way that its control signal step reaction would converge to one pulse of the MTC. This point shows other important discrepancy of the PI control theory. When comparing their design criterion with that one introduced by Klán and Gorez who required the optimal PI control signal step reaction to have a stepwise character (see e.g. paper by Klán and Gorez (2000); or the discussion by Strmčnik and Vrančič (2000) we see a clear contradiction. Who is right in this conflict? The response may surprise many people — both requirements are right!

Simply, there exist two dynamical classes of PI control. Whereas the traditional linear PI control having the control signal response required by Klan and Gorez corresponds to DC0, controllers using anti-windup circuitry and trying to approach the MTC response characteristic by one saturated pulse of control represent already solution of DC1. It has no sense to ask, which one is better — each has its unique properties that cover specific group of applications!

Generalizing this way of arguing, it is then possible to define three dynamic classes of the PID control (Tab. 1.1).

Introduction of dynamical classes of control together with introduction of fundamental solution enable transparent practically motivated classification of the existing controller structures and tuning rules.

Up to now, works on PID control usually did not pay attention to the dynamical classes. So it can e.g. happen that whereas Vítečková et al (2000) proposed for the plant

$$S_2(s) = \frac{K_s}{s(T_1 s + 1)} \tag{1.36}$$

PD controller tuning that corresponds to the DC1, for the plant

$$S_2(s) = \frac{K_s}{(T_1 s + 1)(T_2 s + 1)} \tag{1.37}$$

they already gave PID controller tuning that corresponds to the DC0. Because solutions corresponding to a particular DC need not be unique, it is to expect that a rigorous classification of the existing solutions according to the dynamical classes represents a complex and long term problem. Classification of the existing solutions is complicated also by the fact that the practically attractive properties may lie on the border of two dynamical classes.

Table 1.1: Table of the fundamental PID controllers

		Dominant dynamics							
Dynamic class	I-action	K	Ke^{-T_d}	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}$	$\left[\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}\right]e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	PD	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

FF – static feedforward control is involved also in all feedback controllers
Pr – abbreviation for predictive (dead time) controllers with compensation of the dominant dead time

Rows of the newly introduced table of fundamental controllers correspond to different DCs (Tab. 1.1). These are naturally given by the relative degree of the output defined by the dominant dynamics located in the forward path of the control loop. The type of the controller is then given by the dominant closed loop dynamics including also the feedback dynamics.

The new table of controllers may seem to be much richer than the traditional basis of PID control consisting of the I, P, PI, P-P, P-PI, PD and PID controllers. Besides of the basic dynamics, traditional controllers were classified according to the implementation form (series, parallel, interactive, noninteractive, with setpoint weighting, with different anti-windup structures, etc.).

Similar features are to find also in the new approach. Generic schemes of the constrained PID control are derived by the state space approach and extended by DOB based I action. These are shown to have equivalent schemes that are more or less similar to the traditional structures used in different modifications in practice. Particular controllers are represented by structures – i.e. they are more complex than simple transfer functions. Such a development is not surprising – e.g. the generalization to controllers with two-degree-of-freedom controllers started already in 1960s.

All traditional linear structures are involved in the DC0, while the higher DCs are already essentially nonlinear.

Note that the PI controllers and their predictive versions for dead time compensation are included in two dynamical classes. Each solution is, however, different! The traditional linear PI controller represents optimal solution from DC0. This has either to be combined with a prefilter (usually used in older analogue electronic solutions), or implemented as the I-P controller with error acting on I only $b = 0$. It guarantees dynamics minimizing the actuator wear.

Up to now, the solutions really optimal for the DC1 were approximated by linear controllers equipped by some anti-windup circuitry. However, while the structures used for ages in the series implementation of PI controllers for constrained systems (Åström and Hägglund, 1995; Glattfelder and Schaufelberger, 2003; Kothare et al, 1994) represent substantial part of the newly derived ones – the question is if the missed parts of the new optimal solutions are not simply result of a vague controller description, or of the intentional know-how protection?

The new solutions fully explain needs on setpoint weighting, or prefilters used equivalently in older analogue electronic controllers, needs on structure variations (switching between a linear and a nonlinear PD-controller, etc.

Let us remind that the table brings two structures of the PI and PD controllers and even 3 different structures of the PID ones. Because it is no problem to show that all of them are important for practice, then it is also clear, why each attempt to replace one fundamental solution by an "optimal" retuning of some other structure lead finally to the already mentioned inflation of optimal tunings: for each set of initial states, input signals and process parameters the optimization gave some results and libraries are crowded by all such results.

While the fundamental PD controller of DC1 is essentially near to the linear PD controller compensating usually the largest loop time constant (it has linear control algorithm extended by saturation), the new PD controllers of DC2 are already fully nonlinear. For each special loop dynamics (the double integrator, single integrator+time constant, two different or equal time constraints, oscillatory dynamics, etc.) it is possible to derive a special controller. Despite the possibility to derive for each plant new controller, a unique importance has the solution derived for the double integrator that can be used universally. Again, it is not something completely new. Feldbaum (1965) cites patent of Russian engineers from 1935 based on improving dynamics of the rolling mills positional control by quadratic velocity feedback that is typical for the time optimal control of the double integrator. Later, similar idea was used in the industrial controller Speedomax produced by Northrup.

Later we will deal with the fundamental solutions that enable by choosing the closed loop poles to modify the closed loop dynamics from the fully

linear one up to the on-off dynamics of the MTC. The latter corresponds to pushing poles up to $-\infty$. All controllers are extended by the I-action based on reconstruction and compensation of acting disturbances. Since it would be too demanding to explore in details all possible solutions, this book concentrates on solutions based on 1- and 2-parameter models.
After choosing for the identified dominant loop dynamics a fundamental controller, one has to determine its tuning. When is a controller tuning reliable? Everything depends on the loop properties. If the time constants and gain of the identified dominant dynamic seem to be constant, besides of the possibly fixed nominal dynamics reliable system approximation has to take into account also the nonmodelled (perturbation dynamics). In general it should considers possible plant-model mismatch that determines borders of the closed loop poles choice used for the controller tuning and also other parasitic aspects as e.g. the measurement noise.

1.9 Generic and Intentionally Decreased DC

Introduction of dynamical classes of control into the controller design brings structure for classification of available approaches, methods and solutions. Why such a system may be useful, it may be obvious from the following theorem.

Theorem 1.26. *For a given plant, the generic dynamical class of control is given by the output relative degree. However, for plants with stable subsystems it can be intentionally decreased up to the number of remaining unstable or marginally stable poles.*

Each dynamical class of control is related to some control properties. Since from the above theorem it follows that e.g. for stable 2^{nd} order systems it is possible to design controllers from DC2, DC1 and DC0, without having in mind specific properties of each solution comparing of resulting solutions may be very questionable.
To illustrate related problems, let us start with inspecting possible loop configurations with dynamics of the 1^{st} and 2^{nd} order. It is to remember that whereas identifying some process from the measured input-output behavior (by evaluating step response, relay experiment, measurement at the stability border, etc.) without additional information it is not possible to decide about the actual dynamics distribution within the loop. But, for each particular distribution another controller should be chosen.
For the 1^{st} order loop dynamics (Fig. 1.20) we have to decide upon 2 solutions, for the 2^{nd} order one (Fig. 1.21) upon 3 (or even 4, since we have

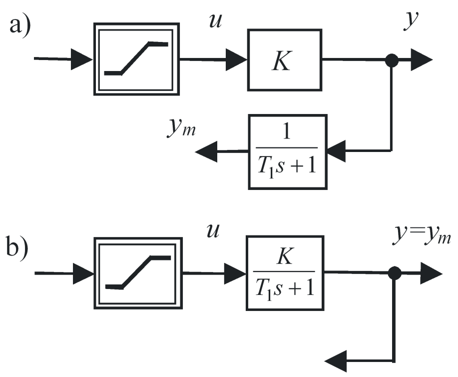

Figure 1.20: Control loops with the 1st order dominant dynamics and with relative degrees a) 0 and b) 1.

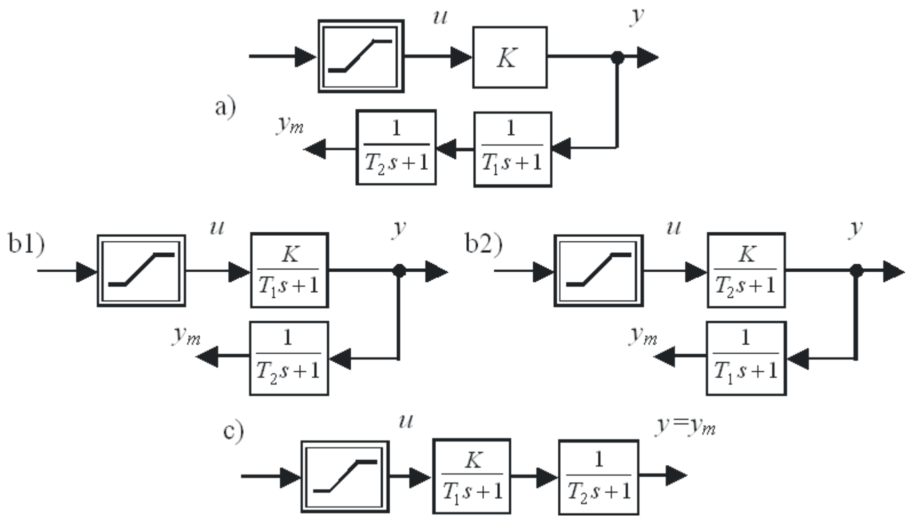

Figure 1.21: Control loops with the 2nd order dominant dynamics and the relative degrees: a) 0; b) 1 and c) 2.

to distinguish among the configurations b1 and b2). When remembering, how many authors tried to propose universal autotuner based on evaluating the input-output behavior, here you can see one of the reasons, why no of them can be generally accepted. There are too many degrees of freedom: the optimal order of the approximation, distribution of the dynamical term within the loop and choice of the dynamical class of control that can be based on the relative degree of the actual output, or intentionally decreased. Besides of the natural allocation of the dynamics within the control loop the design has to consider also other aspects as the availability of different signals, the measurement and quantization noise level of particular measurements, nonmodelled dynamics, possible fluctuations of system parameters, etc. It is e.g. to show that for the same measured output the sensitivity to measurement noise is increasing by increasing the dynamical class of used controllers. So, in a noisy environment of industrial control

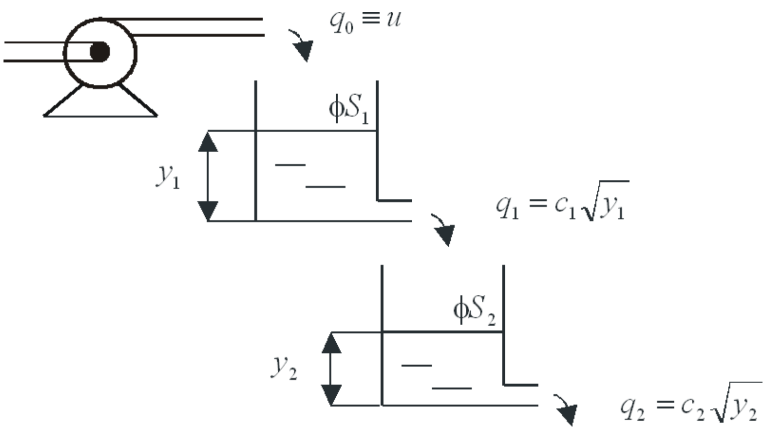

Figure 1.22: Two tank system with a "noninertial" pump

with oftenly put additional requirements on minimizing wear of actuators one has intentionally to choose solution corresponding to the lowest DC. Similar conclusions can also follow in controlling processes with variable dead time, gain and other parameters. But still there exist many situations, where a decrease of the dynamical class is not permitted – e.g. in controlling unstable systems.

Example 1.27 (Two-tank hydraulic system). For a hydraulic system with two containers and a pump having a dynamics negligible with respect to the dynamics of containers characterize generic dynamical classes of control associated with the output (Fig. 1.20) defined by:

a) $y = q_0$,

b) $y = q_1$ (or $y = y_1$),

c) $y = q_2$ (or $y = y_2$).

Solution: When it is required to control the input flow q_0 of the "memoryless" (noninertial) pump, it is possible to speak about process of DC0. This follows from the assumption of the noninertial pump, when it is possible to react to step setpoint changes by step flow changes. Of course, due to the inertia of the pump and of the liquid contained in pipeline the real processes will be smoother (e.g. exponential as in Fig. 1.15), but these transients can fully be neglected with respect to the time of filling the containers.

This holds without respect to the question, which information about the pump flow is used for establishing feedback. Depending on available sensors the pump can be controlled by measuring the controlled flow by a sensor located immediately at its output, or it is controlled by using flow observer based on measuring levels y_1 and/or y_2 (or the flows q_1 and q_2). In the 2^{nd}

case it may be necessary to respect also the length of the inlet pipeline that brings dead-time into the control loop. Of course, for each situation the final controller (including also the relevant reconstruction) will be different.

When it is required to control the flow q_1, or the level y_1 of the first tank, each larger setpoint step change may already require to let the pump to work for some time in one of the limit regimes: either fully switched on (Fig. 1.16), when the level has to be increase, or fully switched off, when it has to be decreased. A “linear” control has sense just in the vicinity of the reference flow (level). The length of transient from a limit regime to a new steady state can sometimes be fully neglected with respect to the time required for filling the container. By its nature, this process can be classified as from the DC1. As above, the final control algorithms will depend on, which process variables are measured and they will differ from those proposed for the above problem.

When it is required to control the second tank output flow q_2, or its level y_2, after each larger setpoint change upward it will be firstly required to run the pump fully switched on for some time. In such a way the level in the first tank will increase most rapidly what results in the fastest possible filling of the 2$^{\text{nd}}$ tank. However, yet before reaching the required level in the 2$^{\text{nd}}$ tank, the pump has to be switched off to decrease the first tank level y_1 up to the value corresponding to the required steady state (Fig. 1.17). “Linear”‘ control process may appear just in the vicinity of the required output flow q_2 (level y_2) and its duration can usually be neglected with respect to the duration of the nonlinear transient. By its nature, the process can be assigned into the DC2. The control algorithms will depend on, which process variables are measured and differ from those proposed for both above problems.

Since the two-tank system can be shown to be stable, according to Theorem 1.26 the output flow q_2 (or level y_2) may be set to the reference level also by control algorithm of DC0 that would simply set the input flow $q_0 = q_2$. It can be easily shown (e.g. by simulation) that the corresponding transient would be much longer than the transient from DC2. For the same task, the solution of DC1 might be based on bringing the flow q_1 to the value corresponding to the desired output, i.e. by $q_1 = q_2$. Again, the overall transient would take more time than by using controller from DC2, but it would be faster than controller from DC0.

1.10 Summary

1. The existing classification of PID controllers into ISA, series and parallel ones reflects spontaneous development of the technology that shows still to be not completely finished. In early period, many details of developed solutions were considered as proprietary information and the internal structures were frequently kept secret instead of being published in literature. Much useful information was also scattered in the literature and finally forgotten, so that it is not sure that today we know fully to argue all existing forms of controllers and to transfer their essential features to the newer technology solutions. Some effort for a more detailed description was already spent in early period of PID control development motivated by requirements of a reliable analytical tuning.
2. With respect to real needs of practice it is obvious that the traditional modules of PID control do not cover all its requirements: there is lack on solutions for higher dynamical classes (DC) of constrained control (unstable systems), including also time-delayed systems.
3. Problems are also caused by the fact that control community has still not accepted notion of dynamical classes of the PID control and requirements that must fulfill fundamental controllers to be included into the PID basis. Despite to the fact that many authors are already respecting these requirements intuitively, without moving forward in this point, it is not possible to develop consistent theory that would cover all requirements ranging from quasi minimum-time control up to the fully linear "smooth"transients.
4. Solutions of the DC0 are fully compatible with the traditional linear solutions that in transition from a steady state to another one yield monotonic output and control signal responses.
5. Within the DC1 containing already one phase of energy accumulation, the fundamental solutions are up to now typically being replaced by traditional linear controllers extended by anti-windup (aw) circuitry.
6. Within the DC2 containing energy accumulation and dissipation phases the fundamental solutions are usually being replaced by less effective and just locally applicable cascaded structures, by the sliding mode control, or by new development in model predictive control.
7. By index of the dynamical class we denote a non-negative integer denoting number of possible intervals with the limit control signal values

(or extreme points) that can occur under MTC with monotonic output.

8. Dynamical classes of control (control processes, controllers) are physically closely related to the energy accumulation/dissipation taking part in controlled plants, when they denote relevant number of energy accumulation phase and, mathematically, to the relative degree of the specified outputs.

9. Definition of dynamical classes of control enables to classify existing design methods and approaches, to increase reliability of the control design, to explain several existing gaps between theory and practice and so finally to improve the acceptance of the control theory by practice.

10. Historically, the notion of dynamical classes of control is closely related to the Feldbaum's theorem about n-intervals of the relay MTC. However, ideal rectangular pulses of such control are considered just as limit case of the constrained pole assignment control (CPAC) with poles shifted to minus infinity. Generating of such rectangular pulses would require an infinitely broad frequency band of the control loop. Putting additional constraints on the rate of control signal changes (or even on its higher derivatives), or on the loop frequency band, respectively, CPAC gives smoother control and some its intervals may shrink to smoother signals with extreme points, or even they fully disappear.

11. Generic DC index related to the specified output relative degree cannot be determined by measuring input-output characteristics, if the measured output is different from the specified one. So, for a rigorous decision in defining optimal controller we usually need also additional information about the distribution of dynamical elements within the control loop (system structure): ideal universal autotuner based fully on input-output measurement is not possible.

12. In the case of relatively high measurement (quantization) noise, or perturbation dynamics it may be useful to intentionally decrease dynamical class of control against the generic one and so to decrease the systems sensitivity, actuators wear, etc.

13. For covering all typical situations in controlling systems with transfer functions up to the 2$^{\text{nd}}$ order with dead time we have introduced table of fundamental controllers containing besides of static feedforward control (contained also in all feedback structures) 18 different feedback controllers.

14. The proposed table includes all traditional (linear) PID controllers within the DC0. Besides of this it systematically covers many solutions used in practice that are up to now staying outside of mostly “linear” theory of the PID control. Despite it is much richer than the basis of traditional PID control, it still does not cover all possible situations with the 2^{nd} order plants with dead time – some rarely appearing situations were up to now omitted due to the sake of simplicity.

15. The existing inflation of different PID tuning rules and anti-windup structures results from attempts to replace one fundamental solution by another one by retuning its parameters. By this process based mostly on different optimization procedures one can get a local optimum corresponding to a particular choice of the initial conditions, input signals and system parameters, but never a globally valid solution.

16. There are many arguments supporting acceptance of the proposed table and solutions contained. As at any change of the settled-down theory, against acts the inertia of human thinking. This cannot be eliminated by any arguments or facts: this is also result of the system dynamics.

1.11 Questions and Exercises

1. How could you define PID control?

2. Could you formulate an alternative definition?

3. What does characterize index of a dynamical class?

4. How it is related to the Felbaum’s theorem about n intervals of time optimal control?

5. Which criteria must fulfill a controller to be considered as the fundamental one?

6. What was the technological reason for lag of theory of the time delayed systems and by which technology generation it was eliminated?

7. Identify some reasons for lags in development of theory of constrained PID control.

8. Name some factors contributing to the inflations of optimal tuning rules for PID control.

9. Does the well-known method by Ziegler and Nichols optimize the controller tuning for the set point step of for the disturbance step responses?
10. Characterize processes of the dynamical classes 0, 1 and 2!
11. Sketch the table of fundamental PID controllers!

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Chapter 2

Basic Fundamental Controllers of DC0

The Dynamical Class 0 (DC0) contains all known linear PID control structures enabling to achieve monotonic plant output course after a setpoint step by monotonic control signal at the controller output. Ideally, control signal reaction to such a setpoint step may be arbitrarily fast and in a limit case to approache step function. Plants that enable to achieve such transients may be considered as generalization of a memoryless plant with additional stable dynamics. Considered structures will be derived from the static feedforward open-loop control extended by reconsruction and compensation of acting disturbances by measuring the plant and controller output signals. For this purpose, fundamental solutions will be proposed based on parallel plant model (PID-PM, or IMC like PID structures) and inverse models of the (invertible) dominant loop dynamics (PID-IM, or DO based PID structures). In a correctly tuned loop and under effect of admissible input signals the control signal constraint will never be active: neither in steady states nor during monotonic transient responses among them. So, in DC0 the control loop may be fully treated by means of linear control theory. A typical feature and important advantage of transients in DC0 is the lowest wear and energy consumption of the actuators.

2.1 I, I_0 and FI_0 Controllers

Relation of the input and output variable of the memoryless plant (2.1) with constrained control signal u_r, with input disturbance v_i and output disturbance v_o is described as

$$y = K(u_r + v_i) + v_o \tag{2.1}$$

In order to consider also asymptotic behavior, we will deal just with piecewise constant loop inputs.

2.1.1 Output Disturbance Reconstruction

By measuring output y that corresponds to control signal u (Fig. 2.1a) for a plant gain estimate K_0 it is possible to reconstruct the output disturbance value $\hat{v}_o$ by means of $\hat{v}_o = y - K_0 u_r$. This value can then be used

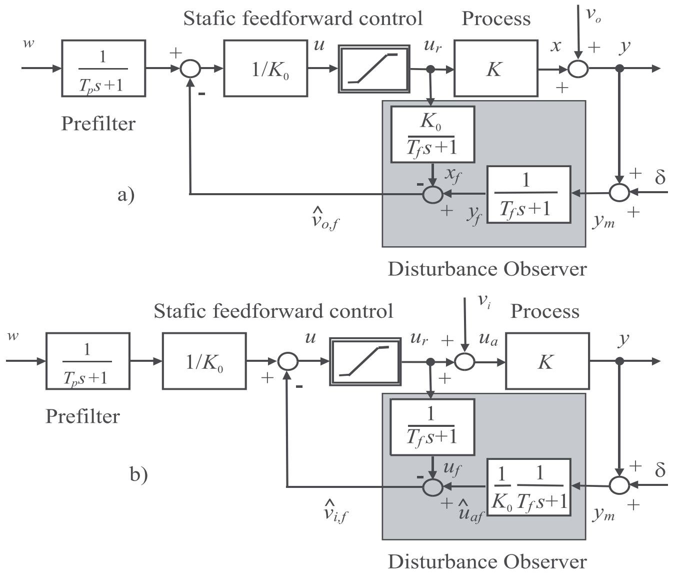

Figure 2.1: FI_0 controllers: static feedforward control extended 1) by reconstruction and compensation of (a) output and (b) input disturbances of a memoryless plant and 2) by a prefilter; in nominal case $K_0 = K$

for the distrubance compensation by a counteractive signal added to the reference value. In order to avoid algebraic loop, to achieve required noise filtration and robustness, and or to limit rate of control signal changes after a disturbance step, it is required to work with filtered reconstructed signal

$$\hat{v}_{of} = \frac{1}{1 + T_f s}\left[y - K_0 u_r\right] \tag{2.2}$$

To limit rate of the control & output changes after a reference step, a prefilter with time constant T_p can be used.

2.1.2 Input Disturbance Reconstruction

An input disturbance v_i may be reconstructed as difference between the reconstructed plant input u_a and the controller output u as $v_i = y/K_0 - u_r$. Again, it will be required to work with filtered reconstructed disturbance

$$\hat{v}_{if} = \frac{1}{1 + T_f s}\left[y/K_0 - u_r\right] \tag{2.3}$$

To limit rate of the control & output changes after a reference step, also here (Fig. 2.1b) a prefilter with the time constant T_p may be used. For admissible input signals the saturation can be omitted (it will never be active, i.e. $u_r = u$) and the structures in Fig. 2.1 be replaced by more frequently used structure (Fig. 2.2) with integrating controller

$$R(s) = \frac{K_I}{s}; \quad K_I = \frac{1}{K_0 T_f} \tag{2.4}$$

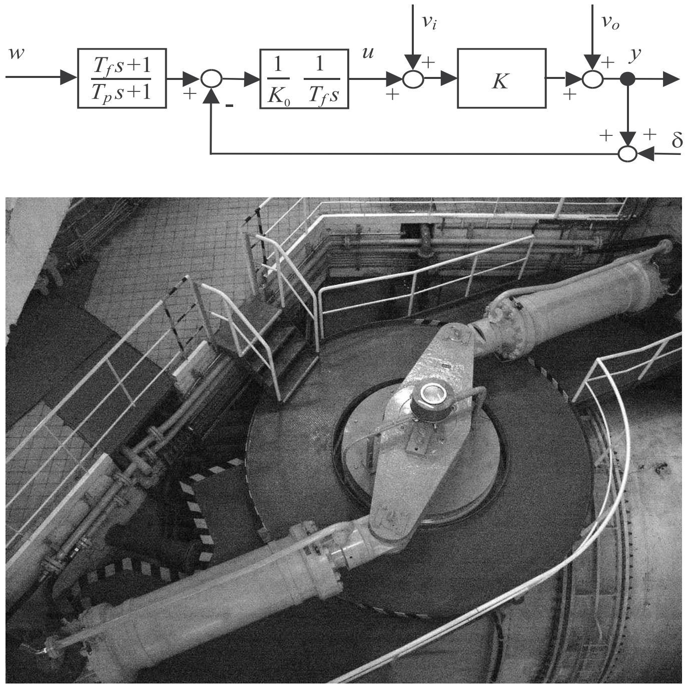

Figure 2.2: Closed loop with I_0 controller equivalent to the structures from Fig. 2.1 (above) and example of hydraulic actuators with I character (below).

Instead of the filter time constant T_f it may be simpler to work with the reconstruction filter bandwidth

$$\Omega_f = \frac{1}{T_f} = K_0 K_I \tag{2.5}$$

Equivalent loop prefilter is defined as

$$T_e(s) = \frac{1 + T_f s}{1 + T_p s} \tag{2.6}$$

By its omitting, i.e. by setting $T_p = T_f$ in the generic scheme in Fig. 2.1 with

$$T_p(s) = 1/(1 + T_f s) \tag{2.7}$$

one gets a continuous control response after a setpoint step. For keeping the step character of feedforward control after a reference signal step, the equivalent loop should include ideal prefilter with as small as possible value T_p (ideally $T_p \to 0$). This equivalent structure of FI_0 controllers may be usefull not just due to its simplicity, but also in situations when the controller and actuator are physically not separable. Such a situation is typical e.g. in using hydraulic and electrical drives in roles of actuator. These genericly have integral character - for a constant nonzero input their output is linearly increasing.

Since an output disturbance can be fully replaced by equivalent input disturbance and this is more fequent in practice, in the sequel we will mostly limit our treatment to the case of input disturbances whereby the index "i" may be omitted.

Definition 2.1 ($\mathbf{I_0}$ and $\mathbf{FI_0}$ controllers). Under I_0 controller we will understand static feedforward control extended by DO based reconstruction and compensation of input, or output disturbances according to Fig. 2.1. When extended by the prefilter with the time constant T_p to the FI_0 controllers, they may also be represented by the equivalent structure according to Fig. 2.2.

Definition 2.2 (I controller). Under I controller we will understand FI_0 controller with the prefilter time constant $T_p = T_f$ that may also be represented by the equivalent structure according to Fig. 2.2 in which the input filter dissapears.

In order to get acceptable filtration of the measurement noise and also to achieve desired robustenss against non-modelled loop delays and plant-model mismatch the DO time constant T_f cannot be set arbitrarily small. In a closed loop tuned for monotonic transient responses and under effect of admisssible input signals the control saturation will never be active and therefore it can be omitted from Fig. 2.1. That means the loop can be fully treated by linear methods. It is also not to forget that the first order filter used in FI_0-controllers does not represent the only available solution. Usefull properties as e.g. improved robustness and noise filtering can be achieved by using higher order DO filters.

2.1.3 Fundamental Properties of I_0 and FI_0 Controllers

When the gain K_0 used for the controller tuning is not equal to real plant gain K, the transfer functions corresponding to reference/disturbance signal responses become

$$\begin{aligned}
I_0 :\ F_{wI0}(s) &= \frac{Y(s)}{W(s)} = \frac{K(T_f s + 1)}{K_0 T_f s + K} = \frac{s/\Omega_f + 1}{s\kappa/\Omega_f + 1}\ ; \quad \kappa = \frac{K_0}{K} \\
I :\ F_{wI}(s) &= \frac{Y(s)}{W(s)} = \frac{K}{K_0 T_f s + K} = \frac{1}{s\kappa/\Omega_f + 1}\ ; \quad \Omega_f = \frac{1}{T_f} \\
F_{v,o}(s) &= \frac{Y(s)}{V_o(s)} = \frac{s K_0 T_f}{K_0 T_f s + K} = \frac{s\kappa/\Omega_f}{s\kappa/\Omega_f + 1}\ ; \quad F_{v,i}(s) = K F_{v,o}(s)
\end{aligned} \tag{2.8}$$

So, both responses depend on the reconstruction filter bandwidth Ω_f and on the ratio of the estimated and real plant gains $\kappa = K_0/K$. For $\kappa > 0$, the closed loop in Fig. 2.2 wil be (theoretically) stable for any positive value of Ω_f, (for any negative value of the closed loop pole $\alpha_f = -\Omega_f/\kappa$). By increasing $\Omega_f \to \infty$, $F_w(s) \to 1$, i.e. the exponential closed loop setpoint step responses $1-\exp(-\Omega_f t/\kappa)$ are approaching unit step, what according to Def. 1.21 means that for the setpoint response this solution represents fundamental controller. Similarly, by increasing $\Omega_f \to \infty$, $F_{vi}(s) \to 0$, i.e. the disturbance closed loop step responses $K\exp(-\Omega_f t/\kappa)$ are converging to zero (more precisely, to the Dirac pulse $K\delta(t)$), what according to requirements of Def. 1.21 means that for the disturbance response this solution again represents fundamental controller.

2.1.4 Nonmodelled Dynamics Approximated by Dead-time – Analytical Treatment

In controlling ideal memoryless plant the controller gain (2.4) may increase to infinitely large values (DO filter time constant T_f may converge to zero). The only condition is that $\kappa > 0$, i.e. the estimated plant gain K_0 has the same sign as the real gain K. However, in controling real plants, no control loop is strictly memoryless one. By increasing reconstruction bandwith $\Omega_f = 1/T_f \to \infty$, the speed of transients increases. Since the concept of memoryless plants covers just situation, when the transients are sufficiently slow and negligable, at some value of Ω_f this concept will become inappropriate to real loop behavior. As a result, overshooting and oscillations of loop variables occur.

Next, we are going to determine borders for validity of the concept of memoryless plant and to propose measures to enable its reliable use. In doing so, we will start with analyzing influence of the simplest loop dynamics. So, let us consider loop with I controller (Fig. 2.2) for $T_p = T_f$), with a memoryless plant and dead-time T_d, when the relation between the control signal u and the measured output $y_m = y_1$ (Fig. 2.3) is given as

$$F_{yd}(s) = \frac{Y_m(s)}{U(s)} = K\mathrm{e}^{-T_d s} \tag{2.9}$$

$$\begin{aligned} F_{w0}(s) &= \frac{Y_0(s)}{W(s)} = \frac{K_I K}{s + \mathrm{e}^{-T_d s} K_I K} = \frac{\mathrm{e}^{T_d s}\Omega_f/\kappa}{s\mathrm{e}^{T_d s} + \Omega_f/\kappa} = \frac{B_0(s)}{A(s)}\,; \quad \kappa = \frac{K_0}{K} \\ F_{w1}(s) &= \frac{Y_1(s)}{W(s)} = \frac{K_I K\mathrm{e}^{-T_d s}}{s + \mathrm{e}^{-T_d s} K_I K} = \frac{\Omega_f/\kappa}{s\mathrm{e}^{T_d s} + \Omega_f/\kappa} = \frac{B_1(s)}{A(s)}\,; \quad \Omega_f = \frac{1}{T_f} \end{aligned} \tag{2.10}$$

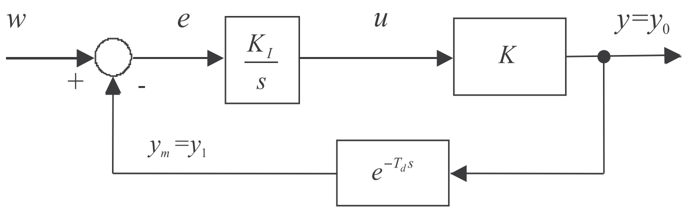

Figure 2.3: Loop with I controller, memoryless plant and dead time

After introducing new complex variable

$$p = T_d s \tag{2.11}$$

these equation may be fully expressed in a normalized form

$$\begin{aligned} F_{w0}(p) &= \frac{Y_0(p)}{W(p)} = \frac{\mathrm{e}^p \Omega/\kappa}{p\mathrm{e}^p + \Omega/\kappa} = \frac{B_0(p)}{A(p)}; \;\; \Omega = \frac{T_d}{T_f} \\ F_{w1}(p) &= \frac{Y_1(p)}{W(p)} = \frac{\Omega/\kappa}{p\mathrm{e}^{T_d p} + \Omega/\kappa} = \frac{B_1(p)}{A(p)}; \;\; \kappa = \frac{K_0}{K} \end{aligned} \tag{2.12}$$

It is to see that the setpoint response fully depends on the parameter

$$q = \Omega/\kappa \tag{2.13}$$

For some tasks, it may be more appropriate to introduce instead of the parameter q its reciprocal value

$$\tau = \kappa/\Omega \tag{2.14}$$

In the case of nominal tuning ($\kappa = 1$) it denotes ratio of the filter time constant to the dead time $\tau = T_f/T_d$. One of the first method for analytical controller tuning (Oldenbourg and Sartorius, 1944, 1951) was based on derivation of conditions of the double real dominant pole.

Theorem 2.3 (I controller gain corresponding for T_d to the Double Real Dominant Pole (DRDP)). *Tuning of the I controller in the loop in Fig. 2.3 that should guarantee the fastest possible monotonic transients may be derived by using conditions for the double real dominant pole p_0 of the closed loop characteristic equation $A(p) = 0$ by satisfying conditions*

$$A(p_0) = 0; \;\; \dot{A}(p_0) = 0 \tag{2.15}$$

as

$$\begin{aligned} q_{opt} = \Omega/\kappa &= \exp(-1) \\ \Omega_f T_d \exp(1) &= \kappa \\ T_f &= T_d \exp(1)/\kappa \end{aligned} \tag{2.16}$$

In the plane of loop parameters (κ, Ω) equation represents a line corresponding to the fastest possible transients without overshooting. For $\Omega \exp(1) > \kappa$ the transients already have overshoots.
It is once more to remind that strictly MO transients can really be achieved for prefilter with $T_p \geq T_f$ in Fig. 2.1. Solution equivalent to $T_p = T_f$ is equivalent to omitting prefilter in the scheme in Fig. 2.2.

Definition 2.4 (Optimal DO bandwith, optimal DO time constant, optimal I controller gain for loop with T_d). As optimal DO time constant T_f, optimal DO bandwith $\Omega_f = 1/T_f$, optimal normalized DO bandwith $\Omega = T_d/T_f$ and optimal integral gain $K_I = 1/(K_0 T_f)$ of the dead-time system in Fig. 2.3 will be denoted those corresponding to the double real dominant pole (DRDP) given as

$$\begin{aligned} \Omega = \kappa/exp(1)\,;\ \ \Omega_f = \kappa/\left(exp(1)T_d\right)\,;\ \ T_f = 1/\Omega_f \\ K_I = 1/\left(K_0 T_d \exp(1)\right) \end{aligned} \tag{2.17}$$

Theorem 2.5 (Critical I controller gains). *Sustained closed loop oscillation with period $P_u = 2\pi/\omega$ orresponds to the root $s = j\omega$ of the characteristic equation $A(s) = 0$. Critical tuning and the corresponding period of oscillations P_u determined by substituting $s = j\omega$ (Neimark , 1973) into $A(s) = se^{T_d s} + \Omega_f/\kappa$ are*

$$\begin{aligned} \omega = 0 \ \Rightarrow \ P_u \to infty\,;\ \Omega_f/\kappa = 0 \\ \omega = 2\pi/T_d \ \Rightarrow \ P_u = 4T_d\,;\ \Omega_f/\kappa = \pi/\left(2T_d\right) \end{aligned} \tag{2.18}$$

The upper critical DO bandwith and the corresponding critical DO time constant are then given as

$$\Omega_{crit} = \kappa\pi/2\,;\ \Omega_{f,crit} = \kappa\pi/\left(2T_d\right)\ ;\ T_{f,crit} = 2T_d/\left(\kappa\pi\right) \tag{2.19}$$

2.1.5 Nonmodelled Dynamics Approximated by Dead-time – Treatment by Performance Portrait

Alhough it might seem at the first glance that the analytically derived border of MO responses based on the DRDP gives reliable results, a detailed computer based analysis based on the Performance Portrait shows that the experimentally determined area of MO responses is slightly larger than the analytically derived one. In this alternative approach the loop behavior is mapped and analyzed over a grid of loop parameters in the plane (κ, Ω). From these data it is then possible to visualize the loop Performance Portrait in Fig. 2.4, or to derive parameters corresponding to a tolerable overshooting shown in Tab. 2.1. It is interesting to note that all

Table 2.1: IAE0 and IAE1 values and the corresponding controller tuning corresponding for $\kappa = 1$ to the outputs y_0 and y_1 under ϵ_y NO&MO setpoint step responses of the loop with I controller and nonmodelled dynamics approximated by the dead time T_d

$100\varepsilon_y$ [%]	10	5	4.04	2	1	0.1	0.01	0.001	0	**0**
$\tau = T_f / T_d$	1.724	1.951	2.0	2.162	2.268	2.481	2.571	2.625	2.703	**2.718**
$\Omega = T_d / T_f = 1/\tau$	0.580	0.515	0.5	0.465	0.441	0.403	0.389	0.381	0.37	**0.368**
IAE_0 / T_d	1.105	1.147	1.17	1.240	1.314	1.486	1.571	1.625	1.703	**1.718**
IAE_1 / T_d	2.105	2.147	2.17	2.240	2.314	2.486	2.571	2. 625	2.703	**2.718**

values $\tau = \tau(\epsilon_y)$, including e.g. the simple tuning $\tau = 2$ proposed by Skogestad (2003) that corresponds to $100\epsilon_y = 4.04\%$, which are for $\epsilon_y \to 0$ converging to the value

$$\tau \to 2.703 \ldots \tag{2.20}$$

are smaller than

$$\tau_{opt} = exp(1) = 2.718 \ldots \tag{2.21}$$

corresponding to (2.14) and (2.17).

Of course, one could deal with the question, if the discrepancy in results is due to the limited precision of numerical computations, or it is expressing influence of infinitely many poles neglected in the double real dominant pole method. Although in this case differences may be observed just on the third decimal position, in general, the experimental qualitative & quantitative computer based analysis of step responses may give much deeper insight into closed loop properties than the analytical analysis of infinitely many closed loop poles of such dead time system. Simultaneously it e.g. shows on numerical issues that may be important not just for simulations, but also for the real time control. However, for vast majority of engineering tasks the identified differences in results do not play a primary role and we could conclude that in the case of the I-controller the analytically derived conclusions a coinciding with the results achieved by using the Performance Portrait. This method gives, however a more detailed information, what will become yet more important in dealing with more complex control tasks.

2.1.6 Nonmodelled Dynamics Approximated by Time Constant – Analytical Treatment

Another elementary possibility for approximating the nonmodelled loop dynamics is represented by single time constant (accmulative delay) corre-

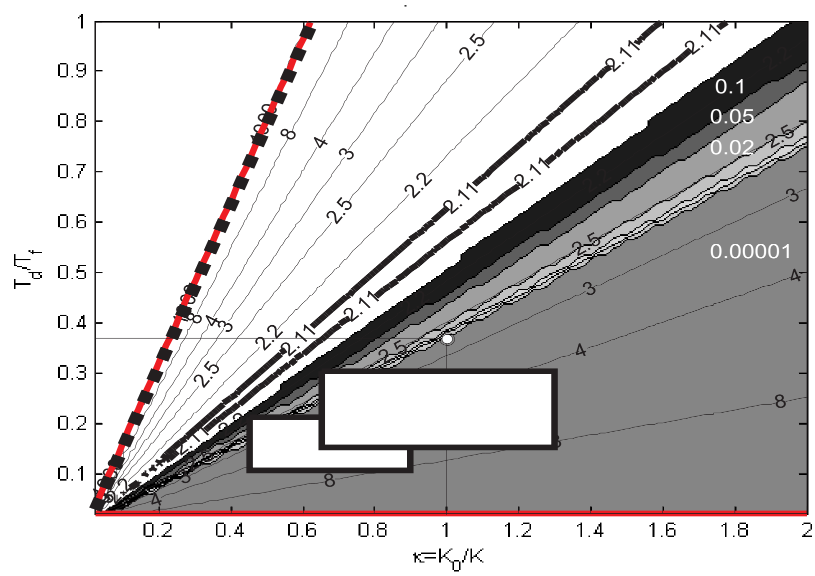

Figure 2.4: Performance portrait of the loop with I controller and dead time from Fig. 2.3 including level contours corresponding to IAE values of the output y_1; areas of NO&MO output step responses identified for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$; a boundary point of the strictly NO&MO area is given by (2.17); no one of given ϵ_y areas does reach up to the area of optimal IAE1 values outlined by bold curces; the stability border (2.19) is given by bold dotted line; examples of uncertainty boxes are explained in following chapters

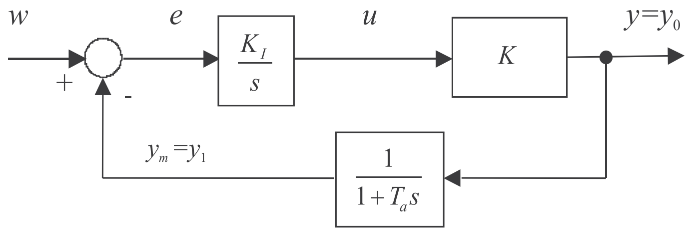

Figure 2.5: Loop with I controller, memoryless plant and non-modelled dynamics approximated by time constant

sponding to the plant transfer function

$$F_{ya}(s) = \frac{Y_m(s)}{U(s)} = \frac{K}{1 + T_a s} \tag{2.22}$$

For passive compensation of the nonmodelled dynamics (I controller tuning), the location of this delay within the control loop is not important and in order to cover both possible loop configurations we will consider transfer functions corresponding to two possible loop outputs in Fig. 2.2 (i.e. with $T_p = T_f$ in the generic schemes in Fig 2.1). For the new complex variable

$$p = T_a s \tag{2.23}$$

they again depend on the parameters $q = \Omega/\kappa$, or $\tau = \kappa/\Omega$

$$\begin{aligned} F_{w0}(p) &= \frac{Y_0(p)}{W(p)} = \frac{(1+p)\,\Omega/\kappa}{p\,(1+p) + \Omega/\kappa} = \frac{B_0(p)}{A(p)}; \quad \Omega = \frac{T_a}{T_f} \\ F_{w1}(p) &= \frac{Y_1(p)}{W(p)} = \frac{\Omega/\kappa}{p\,(1+p) + \Omega/\kappa} = \frac{B_1(p)}{A(p)}; \quad \kappa = \frac{K_0}{K} \end{aligned} \tag{2.24}$$

Theorem 2.6 (I controller gain corresponding for T_a to the Double Real Dominant Pole (DRDP)). *Tuning of the I controller in the loop in Fig. 2.5 that should guarantee the fastest possible monotonic transients may be derived by using conditions for the double real dominant pole p_0 of the closed loop characteristic equation $A(p) = 0$ by satisfying conditions*

$$A(p_0) = 0; \quad \dot{A}(p_0) = 0 \tag{2.25}$$

as

$$q_{opt} = \Omega_{opt}/\kappa = 1/4; \quad T_f = 4T_a/\kappa \tag{2.26}$$

In the plane of loop parameters (κ, Ω) equation (2.26) represents a line corresponding to the fastest possible transients without overshooting. For $\Omega \exp(1) > \kappa$ the transients should already have overshooting.

Definition 2.7 (Optimal DO bandwith, optimal DO time constant, optimal I controller gain for T_a). As optimal DO time constant T_f, optimal DO bandwith $\Omega_f = 1/T_f$, optimal normalized DO bandwith $\Omega = T_d/T_f$ and optimal integral gain $K_I = 1/(K_0 T_f)$ of the system with time constant in Fig. 2.5 will be denoted those corresponding to the double real dominant pole (DRDP) given as

$$\Omega = \kappa/4\,; \;\; \Omega_f = \kappa/(4T_a)\;; \;\; T_f = 1/\Omega_f\,; \;\; K_I = 1/(4K_0 T_a) \tag{2.27}$$

Theorem 2.8 (Critical I controller gains for T_a). *Sustained closed loop oscillation with period $P_u = 2\pi/\omega$ corresponds to the root $s = j\omega$ of the characteristic equation $A(s) = 0$. In difference to the loop with dead time, for $\kappa > 0$ this loop remains stable for any $T_f > 0$, when the critical tuning and the corresponding period of oscillations P_u determined by substituting $s = j\omega$ into $A(s) = s(T_a s + 1) + \Omega_f/\kappa$ are*

$$\begin{aligned} \omega = 0 \;\; &\Rightarrow \;\; P_u \rightarrow infty\,; \;\; \Omega_f/\kappa = 0\,; \;\; K_{I,min} = 0 \\ \omega \rightarrow \infty \;\; &\Rightarrow \;\; P_u \rightarrow 0\,; \;\; \Omega_f/\kappa \rightarrow \infty\,; \;\; K_{I,max} \rightarrow \infty \end{aligned} \tag{2.28}$$

2.1.7 Nonmodelled Dynamics Approximated by Time Constant – Treatment by Performance Portrait

Comparison of results achieved for dead time Tab. 2.1 and time constant Tab. 2.2 shows that in the case of the time constant T_a increased values of tolerated overshooting lead to reasonably faster IAE decrease than in the case of the dead time T_d (Fig. 2.6). For 10% tolerated overshooting the IAE values corresponding to T_a and T_d are roughly equal. That has an important consequence on plant identification: by using the step responses based e.g. on measuring the average residence time. For systems with tolerable overshooting it has no sense to distinguish the type of nonmodelled dynamics. And conversely, it may be important in aiming to achieve the fastest possible MO responses with low admissible overshooting and low deviations from monotonicity.

2.1.8 Tuning Based on Maximal Sensitivity $M_s = 1.4$

Today, controllers are frequently tuned with the aim to guarantee chosen maximal sensitivity to modeling errors. This can be expressed as the maximal value of the sensitivity function defined as $S(s) = 1/(1 + L(s))$, whereby $L(s) = R(s)F(s)$ is the open loop transfer function with $R(s)$ being the transfer function of the controller and $F(s)$ being the plant transfer function. The maximal sensitivity is then given as

$$M_s = \max\{S(j\omega),\; S(s) = 1/(1 + L(s))\}; \quad L(s) = R(s)F(s) \tag{2.29}$$

Table 2.2: IAE0 and IAE1 values and the corresponding controller tuning corresponding for $\kappa = 1$ to the outputs y_0 and y_1 under ϵ_y NO&MO setpoint step responses of the loop with I controller and nonmodelled dynamics approximated by the time constant T_a

$100\varepsilon_y$ [%]	10	5	2	1	0.1	0.01	0.001	0	**0**
$y_0 : \tau = T_f / T_a$	1.754	2.169	2.604	2.857	3.367	3.597	3.731	3.968	**4**
$\Omega = T_a / T_f = 1/\tau$	0.570	0.461	0.384	0.350	0.297	0.278	0.268	0.252	**0.25**
IAE_0 / T_a	1.348	1.510	1.760	1.941	2.376	2.598	2.732	2.968	**3**
$y_1 : \tau = T_f / T_a$	1.398	1.908	2.433	2.724	3.311	3.571	3.717	3.968	**4**
$\Omega = T_a / T_f = 1/\tau$	0.715	0.524	0.411	0.367	0.302	0.280	0.269	0.252	**0.25**
IAE_1 / T_a	1.921	2.221	2.581	2.806	3.321	3.573	3.717	3.968	**4**

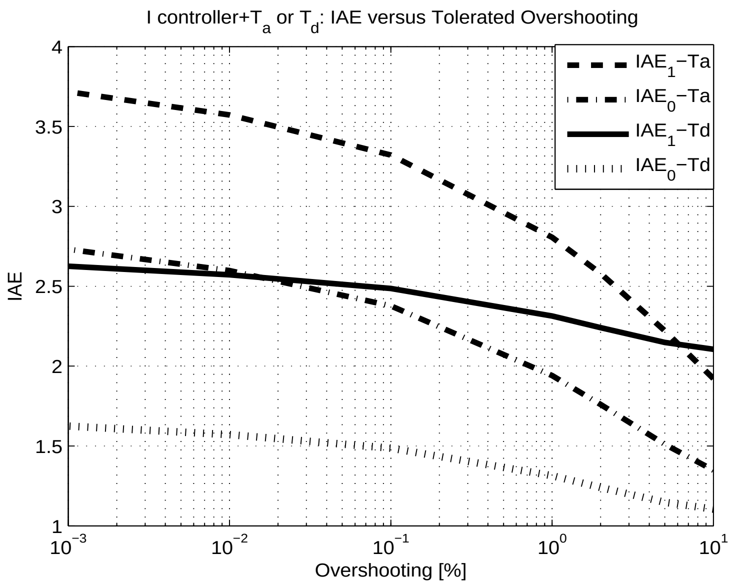

Figure 2.6: IAE1 and IAE0 values versus tolerated overshooting for the time constantT_a and dead time T_d according to Tab. 2.1 and Tab. 2.2

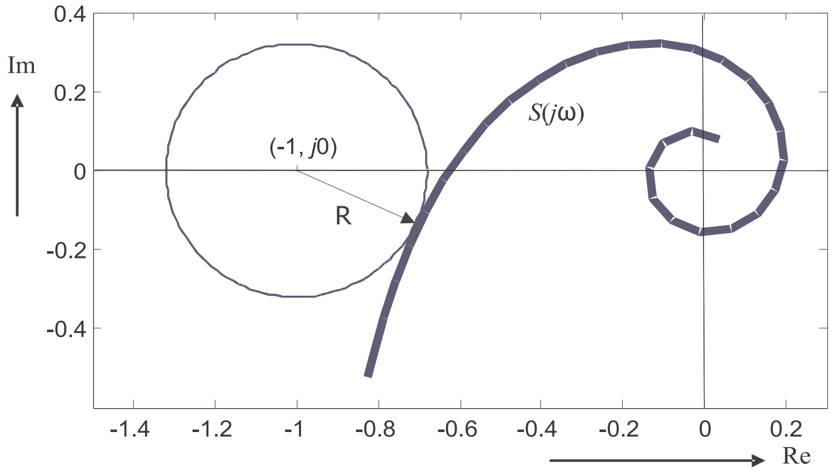

Figure 2.7: Maximal sensitivity $M_s = 1/R$ is defined as reciprocal value of the maximal radius R of the circle with centre in critical point $(-1, 0j)$ that is touging Nyquist curve $L(j\omega)$

Thereby M_s represents inverse value of the shortest distance R of the critical point $(-1, j0)$ from the Nyquist curve of the open loop transfer function $L(s)$. This may be determined by making circle with centre in the critical point that tougches Nyquist curve (Fig. 2.7). Typical M_s values (Åström and Hägglund, 1995; Åström et al, 1998) appropriate for control lie in the range $1.2 - 2.0$. Lower M_s values give slower, but less oscillatory transient responses. Why exactly these values? Do they represent universal constants defined by the nature? In order to get some interpretation we may compare the maximal sensitivity method with results achieved by the double real dominant pole.

By evaluating maximal sensitivity corresponding in the nominal case to tuning (2.17) one gets $M_s = 1.3936 \approx 1.4$. The result does not depend on the particular dead time value T_d. It shows that the DRDP and the maximal sensitivity approaches are somehow related and explains possible motivation for Aström and coworksers to prefer exactly the value $M_s = 1.4$. However, calculating the maximal sensitivity corresponding to tuning (2.27) gives already different value $M_s = 1.155$. It means that the tuning corresponding to DRDP does not introduced universally valid *optimal* M_s value. Simultaneously, question arrises, which delay is more appropriate for approximations dealing with non-nmodelled loop dynamics: dead time (2.9) or time constant (2.22)? Experimental results show that in dealing with real loops tuning (3.18) gives mostly too conservative controller values (as it is also illustrated by Fig. 2.6) that gives argument to work with T_d. For approximation (2.22) we might use also other alternative: to derive new controller tuning corresponding to value $M_s = 1.4$

what gives

$$T_f = 1.5T_a\,;\;\; T_p = 2.1T_f \tag{2.30}$$

Thereby, the prefilter time constant value has to be increased from $T_p = T_f$ more than two times to achieve nearly monotonic step response of y_1. Fig. 2.8 shows that such transients are much faster as for the DRDP tuning. By further increase of T_p it would also be possible to remove slight overshooting in the transients of the output y_0 and so also of the control signal. It is, however, to note that in the disturbance response the overshooting remains. So, approach based on loop shaping and dealing with the maximum sensitivity and the maximum complementary sensitivity (Skogestad and Postlethwaite, 1996; Skogestad, 2003) is indeed possible, but not primarily oriented to respect nonovershooting and monotonicity conditions at the plant and controller outputs. Furthermore, the expected dynamics is guaranteed just around the nominal operating point. Of course, it is expected that the less aggresive tuning with lower M_s values will allow higher degree of the plamt-model mismatch, but there are no easy ways of quantifying these expectations. Similar comments may also be used for other design methods based on the frequency response, as e.g. the Disturbance Rejection Magnitude Optimum method (DRMO, Vrančič et al (2004)) that gives

$$T_f = 2T_a; \quad T_p = 1.35T_a \tag{2.31}$$

These method may be advantageous in working with plant models achieved by identification based on the frequency response. Else, more direct approach of the Performance Portrait method that uses direct technological parameters as tolerated overshooting, or tolerated deviations from monotonicity will be preferred.

2.1.9 Short Summary of Nominal I_0-Controller Tuning

Previous analysis showed that already in designing simple I controller there exist several degrees of freedom: by choice of the prefilter T_p it is to decide about character of the control signal dynamics after setpoint steps – should it have a step character, or a softer, exponential one? That is: are we going to use controller according to Fig. 2.1 with equivalent structure in Fig. 2.2 with two unknown parameters T_p and T_f, or simplified solution according to Fig. 2.1 with prefilter $T_p = T_f$ that is equivalent to Fig. 2.2 without prefilter? This process of deciding about complexity of the solution could be continued by considering higher order DO filters in the generic scheme that might be interesting both from the point of view of noise filtering as well as from the closed loop robustness point of view.

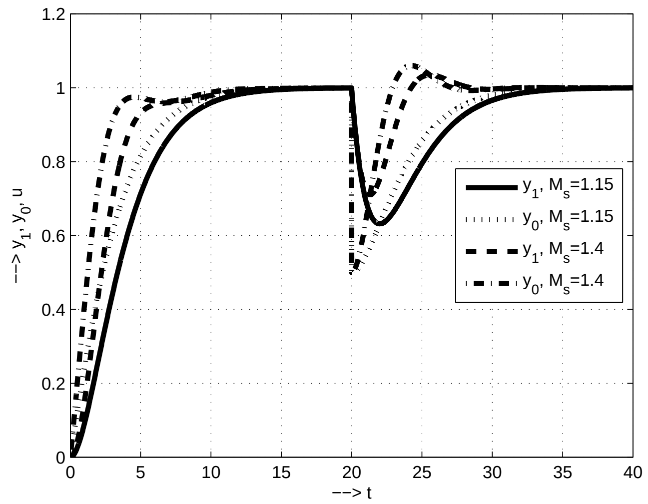

Figure 2.8: Loop with the FI_0 controller and plant (2.22) with $T_p = T_f$ and T_f corresponding to (2.27) that yields $M_s = 1.15$ and with the tuning (2.30) corresponding to $M_s = 1.4$

It was also shown that the concept of memoryless plant used in deriving controller structure has to be refined by appropriate approximation of the non-modelled dynamics. This plays an important role in controller tuning enabling achieving the fastest possible ϵ_y-MO&NO transient responses. In this step we have analysed basic properties achieved by approximations of the nonmodelled dynamics by the dead time and by single time constant. Of course, in practice also their combination, or higher order approximations may be proposed, as e.g. the approximation by $1/(1 + T_a s)^2$, used by Glattfelder and Schaufelberger (2003). But, when allowing tolerated overshooting around 5% of the setpoint step, influence of both types of approximations of the nonmodelled dynamics is approximately equal and more important question becomes, which approximations are easier to be achieved. A more rigorous approximation of the nonmodelled dynamics has sense just for a high precission control.

In specifying the loop dynamics we have concentrated our effort on conditions of achieving ϵ_y-NO&MO transients that showed to correspond to identical conditions in this case. This may be important both in the technological context, both in decreasing acturator wear and in the constrained control design. At the controller output, MO transient from one admissible steady state to another one will never excite control saturation and so it is possible to omit its effect from the control loop analysis.

Aproximation of the loop dynamics may be based on measuring setpoint step responses, by evaluating system response at the stability border (when it is possible and allowed to bring system to oscillations by appropriate controller tuning), by relay experiment, etc.

For loop with memoryless plant and dead (2.9) gave the I controller tuning based on the DRDP already one of the first control text books by Oldenbourg and Sartorius (1944, 1951) that indicates importance of the solution for practice. Since the controller is derived for the simplest plant model it may be universally used for controlling broad spectrum of stable plants, what e.g. inspired Datta et al (2000) to speak about "magic of integral control". I controllers are appropriate also for systems with long (and possible variable) dead times. Rugh and Shamma (2000) e.g. presents there use in combustion engines that are typical by long delay between engine fuelling and exhaust emissions. But, above mentioned tuning rules are still fixed just to a nominal point and should futher be extended to more general situation with plant parameters varying over broader intervals.

2.1.10 Robust Controller Tuning and Characteristics

In practical applications, loop parameters are mostly known with some degree of uncertainty. Plant properties may vary in time (time variable plants), due to operating point changes (nonlinear plants), or they may be simply identified with a limited precission. How to tune the controller, when it is required to guarantee some performance, whereas the loop parameters K, T_a or T_d are not known exactly, but they are given just with interval uncertainty as

$$
\begin{aligned}
K \in \langle K_{min}, K_{max} \rangle \ ; \ c_K = K_{max}/K_{min} \geq 1 \\
T_d \in \langle T_{d,min}, T_{d,max} \rangle \ ; \ c_d = T_{d,max}/T_{d,min} \geq 1 \\
T_a \in \langle T_{a,min}, T_{a,max} \rangle \ ; \ c_a = T_{a,max}/T_{a,min} \geq 1
\end{aligned}
\tag{2.32}
$$

When interpreting such a situation by means of Fig. 2.4 it is to note that changes of the plant gain K influence possible values of $\kappa = K_0/K$. For a chosen value K_0 it is possible to find limit values

$$
\kappa_{min} = K_0/K_{max} \ ; \ \kappa_{max} = K_0/K_{min} \tag{2.33}
$$

that within the parameter plane (κ, Ω), determine range of horizontal movement of the working point. For a constant and exactly know value of $\Omega = T_d/T_f$, or $\Omega = T_a/T_f$ the uncertainty set is reduced to a horizontal uncertainty line segment (ULS) with vertices corresponding to (2.33).

Similarly, for a chosen DO badwidth $\Omega_f = 1/T_f$ it is possible to find limit values of $\Omega = T_d/T_f$, or $\Omega = T_a/T_f$ as

$$\begin{aligned} \Omega_{min} &= T_{d,min}/T_f\,;\ \Omega_{max} = T_{d,max}/T_f \\ \Omega_{min} &= T_{a,min}/T_f\,;\ \Omega_{max} = T_{a,max}/T_f \end{aligned} \tag{2.34}$$

If the only uncertainty is related to the nonmodelled dynamics, whereby the plant gaing is exactly known, the uncertainty set will be given by vertical ULS with the horizontal position $\kappa = K_0/K$ and vertices (2.34).
By combining extreme values of two independent parameters (2.32) one gets uncertainty box (UB) with vertices corresponding to (2.33) and (2.34) as

$$UB = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \kappa_{\min}, \Omega_{\max} & \kappa_{\max}, \Omega_{\max} \\ \kappa_{\min}, \Omega_{\min} & \kappa_{\max}, \Omega_{\min} \end{bmatrix} \tag{2.35}$$

To guarantee required property (ϵ_y-NO&MO control with specified tolerance ϵ_y) for all possible situations, it is then required that the whole UB lies in parameter area guaranteeing given property. With respect to the shape of the border of NO & MO control in Fig. 2.4 it is obvious that the critical role will be played by the upper left vertex

$$B_{11} = (\kappa_{min}, \Omega_{max}) \tag{2.36}$$

whereby $\Omega_{max} = T_{d,max}/T_f$, or $\Omega_{max} = T_{a,max}/T_f$. In the analytical design this can be placed at one of the line borders (2.17), or (2.27). That means to fulfill for all possible working points requirements

$$K_0 T_f \geq exp(1) K_{max} T_{d,max}\,;\ K_0 T_f \geq 4 K_{max} T_{a,max} \tag{2.37}$$

Due to the radial shape of the performance portrait, by shifting ULS or UB along chosen lines to any position the closed loop properties do not vary. However, by increasing ratio of the upper and the lower limit value in (2.32) the mean value of IAE index over the uncertainty set will increase. As it is evident from Fig. 2.9, the rate of increase depends on the type of the nonmodelled dynamics and on the tolerated overshooting.
For T_a and strictly MO tuning both IAE values increase due to the ucertainty much more rapidly than for T_d, but for the 10% tolerated overshooting the increase is in both cases practically equivalent and much less intensive than in the MO case. From this point of view we come to a surprising result: in the case of the I controller it is easier to control loops with dead time than loops with equivalent time constant value. Since by increasing the uncertainty coefficients c_a, or c_d the maximal values in (2.32) and so also the required T_f values in (2.37) increase linearly, also the IAE values in 2.9 increase linearly. It is also to note that in the case of a time

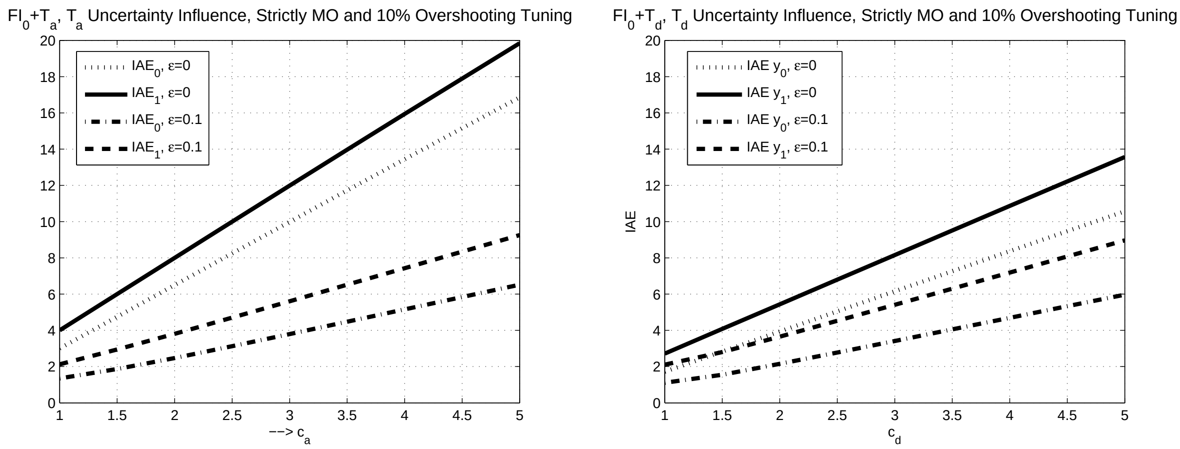

Figure 2.9: T_a uncertainty influence on average IAE values of outputs y_0 and y_1 for strictly MO tuning ($K_I = 0.25/(KT_{a,max})$) and tuning with 10% tolerable overshooting of y_0 with $K_I = 0.57/(KT_{a,max})$ (left) and equivalent uncertainty influence for T_d with strictly MO tuning ($K_I = 0.37/(KT_{d,max})$) and tuning with 10% tolerable overshooting with $K_I = 0.58/(KT_{d,max})$

constant tuning corresponding to certain overshooting of the output y_0 differs from that corresponding to the output y_1. From this point ov view it is to expect that the step disturbances entering to the closed loop at different points will in the case of considering tolerable overshooting require special attention.

Example 2.9. In this illustrative example we will show robust design and the corresponding robust performance achievable by using the simplest possible I controller and then compare these results with the much more complex Filtered Smith Predictor (FSP) according to Normey-Rico and Camacho (2007); Example 6.1. The uncertain plant to be controlled is

$$\begin{aligned} F\left(s\right) = & \frac{K_p \mathrm{e}^{-Ls}}{\left(1+s\right)\left(1+0.5s\right)\left(1+0.25s\right)\left(1+0.125s\right)} \\ & K_p \in \langle 0.8,\, 1.2\rangle \;;\; L \in \langle 9,\, 12\rangle \end{aligned} \tag{2.38}$$

The FSP controller using primary PI-controller

$$C\left(s\right) = K_c \frac{1+T_I s}{T_I s} \tag{2.39}$$

was tuned using standard robust approach in the frequency domain based on a nominal plant and norm bounded multiplicative uncertainty. As the nominal model an approximation of the original plant by the FOPDT one with

$$F_{apr}\left(s\right) = \frac{K_n \mathrm{e}^{-L_n s}}{1+T_n s}\;;\; K_n = 1\,;\; L_n = 10.5 \tag{2.40}$$

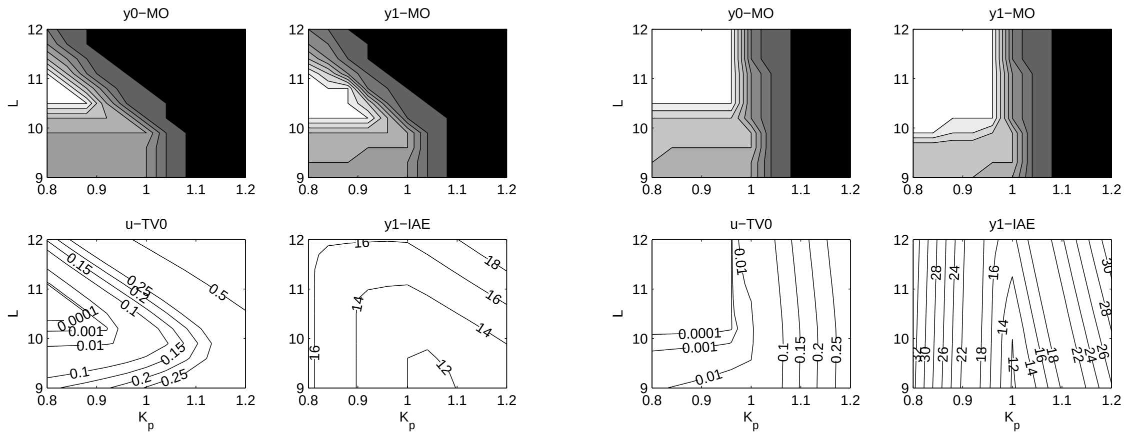

Figure 2.10: PP of the plant (2.38) with the FSP controller based on (2.39)-(2.40) with $T_f = L_n/2$ (left) and $T_f = 10L_n$ (right); ϵ_y-MO areas identified for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$ with white denoting the best performance

was used. Robust stability was proven for $K_c = 1\,;\ T_I = T_n$ and $T_f = L_n/2$. But, this method is not able to guarantee higher requirements on MO transients, expressed e.g. by the amplitude related deviations, or TV_0 values as it is evident from the PP in Fig. 2.10 left. So, it does not enable to design controller for more advanced applications. From Fig. 2.10 right it is to see that even the 20 times larger filter time constant does not reasonably improve the considered loop performance for larger plant gains: the controller needs to be fully retuned, possibly by the PP method.
Tuning of the I-controller will be based on the average residence time interpreted as dead time T_d

$$A_0 = KT_d\,;\ A_0 = \int_0^\infty \left[y\left(\infty\right) - y\left(t\right)\right] dt \tag{2.41}$$

by the step responses (Åström and Hägglund, 1995), or by a general input signal according to Ingimundarson (2000), when for the maximal dead time L one gets for (2.38)

$$T_{d,max} = L_{max} + 1 + 0.5 + 0.25 + 0.125 = 13.875 \tag{2.42}$$

When choosing $K_0 = 1$, the filter time constant may be determined according to (2.37) as $T_f = \tau(\epsilon_y)T_{d,max}K_{max}$, whereby the values for 2% and 5% were taken from Tab. 2.1. From the PP in Fig. 2.11 it is obvious that for the output y_1 the deviations achieved in the critical corner exactly match the expectations, so that no corrections are necessary. Since the MO conditions are nearly matched also by the output y_0, it means that any output corresponding to some distribution of dynamical terms in (2.38) among the

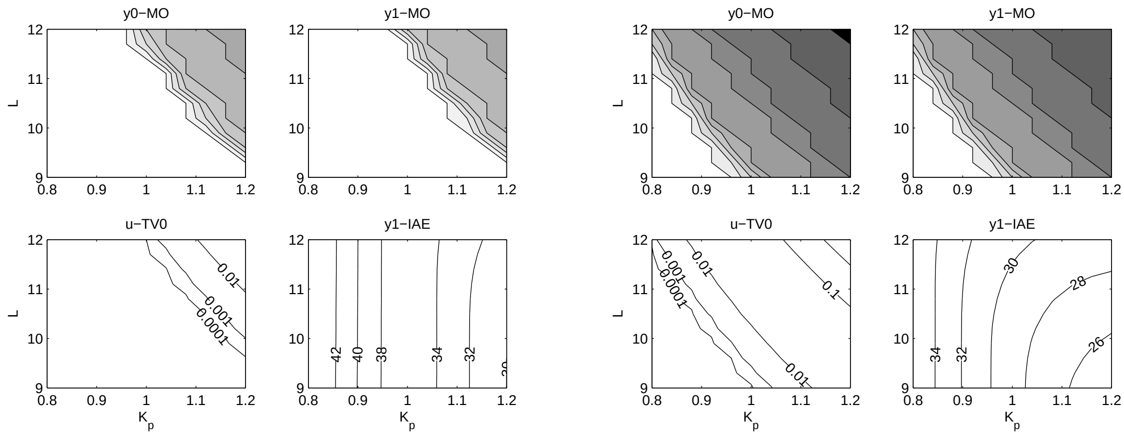

Figure 2.11: PP of the I controller with T_f tuned to guarantee lower than 2% ($\epsilon_y = 0.02$, left) and lower than 10% deviations (right) from MO. MO areas correspond to $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$ with white showing the best performance

feedback and the feedforward path would match the required specification. Explanation for this (may be surprising result), when the extremely simple model gives precise results, may be taken from the same source as the above example (Normey-Rico and Camacho (2007), pp. 174): "when the dead-time is dominant, the contribution of the open loop poles to the closed loop response will be small thus their elimination will contribute with a small increment in the speed of the transients". Model used for tuning of the I controller fully respects this statement - whereas the model (2.40) used by authors of this statement not. The PPs in Fig. 2.10 and Fig. 2.11 fully confirm also another statement of above authors (pp. 145) "the effect of dead-time error is not symmetric", just the method they have used does not allow dealing effectively with this problem.

Simple I controller yields indeed higher IAE values than the much more complex FSP. However, up to now there exists no method for reliable tuning of FSP with respect to higher performance requirements. Having this fact in mind, several authors developed interactive tools to fight with this problem by the 'trial and error' method. Using the PP method the FSP may be redesigned to respect also this problem in a direct way.

2.2 PI$_0$ Controllers

In order to get MO transients, in the case of increasing time constant (accumulative delay) it is frequently not enough to compensate its influence just by restricting the closed loop bandwidth. Besides of slower transients this way of compensation brings also increased influence of disturbances. In many situations such impact is not acceptable and there is arising demand on active compensation of the time delay that would avoid these negative phenomena. Active compensation of dominant loop time constant leads to new control structures denoted here as PI, PI_0 and FPI_0 controllers. In deriving their structure the time constant T_a aproximating originally nonmodelled dynamics will now be denoted as the dominant loop time constant T_1. We will start by extending structure in Fig. 2.1 (similarly as in Fig. 2.5) by such a time constant denoted now as T_1.

2.2.1 Different Types of PI_0 and FPI_0 Controllers

Definition 2.10 (Active compensation of stable time constant - inversion of dynamics). Within the DC0, under "active compensation" of the loop time constant T_1 ("acumulative delay") it will be understood reconstruction of the estimate $\hat{y}$ of the actual loop output $y = y_0$ from the measured delayed output $y_m = y_1$ that can be expressed as

$$\begin{aligned} \hat{Y}(s) &= (1 + T_1 s)\, Y_m(s) \\ \hat{y}(t) &= y_m(t) + T_1 \mathrm{d}y_m(t)/\mathrm{d}t \end{aligned} \tag{2.43}$$

Such a reconstruction of the input signal of a dynamical system from measured values of its output is called as inversion of its dynamics.

Active compensation of single time constant, i.e. inversion of its dynamics, is based on using inverse plant transfer function (inverse model). Such inversion may, however, be carried out just for stable systems. Output of an unstable system located in the feedback of Fig. 2.12 would be for a constant output increasing exponentially to infinite magnitudes. Under finite precission, such a signal cannnot be processed by real equipment. Besides of this, with respect to physical feasibility of inversion and to avoid algebraic loops, reconstruction of actual output will require additional filtration.

Active compensation of the time delay incorporated into reconstruction and compensation of disturbances leads to control structure in Fig. 2.12. In a loop with admissible inputs and monotonic transients of control signal the control saturation will never be active and so it can be omitted and the loop may be represented by the well known structure with the linear PI-controller $R(s)$ and the equivalent prefilter $T_e(s)$

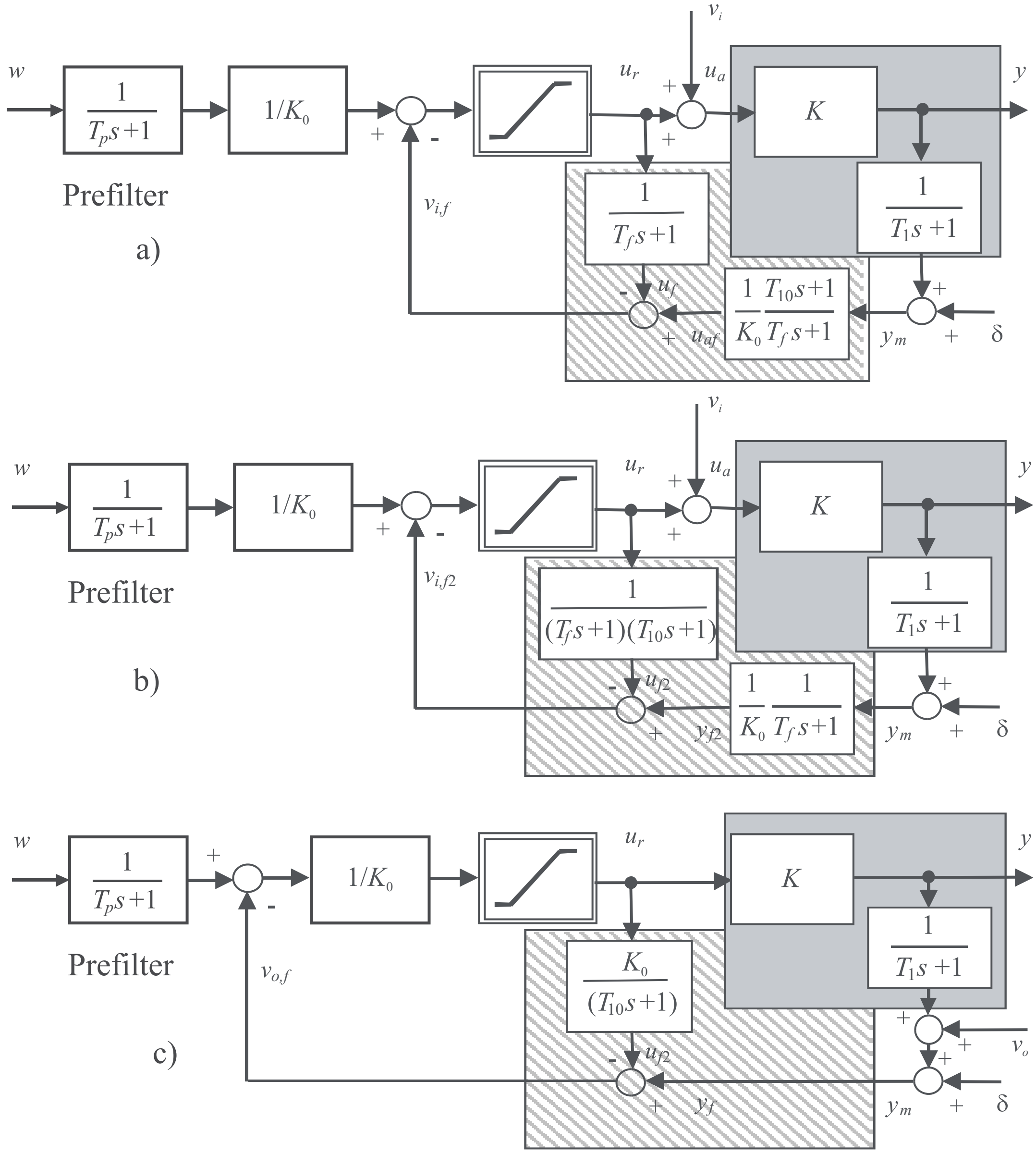

Figure 2.12: a) Fundamental PI_0-IM controller designed as static feedforward control extended by input disturbance reconstruction and compensation using Inverse plant Model (cancelling time constant T_1); b) Modification of the I_0 controller for input disturbance reconstruction with balanced DO including loop time constant estimate in the channel from the controller output and c) Parallel plant Model of the PI_0-PM controller (PI_0-IMC controller); all structures may be extended by prefilter

$$
\begin{aligned}
R(s) &= K_c \frac{1+T_I s}{T_I s}\,;\; K_c = \frac{T_{10}}{K_0 T_f}\,;\; T_I = T_{10} \\
T_e(s) &= \frac{1+T_{f1}s}{(1+T_{p1}s)\,(1+T_{p2}s)}\,;\; T_{f1} = T_f\,;\; T_{p1} = T_{10}\,;\; T_{p2} = T_p
\end{aligned}
\tag{2.44}
$$

After cancelling T_f in the prefilter numerator (2.44) by choosing $T_{p1} = T_{10} = T_f$ and $T_{p2} = T_p > 0$, step changes of control signal produced by a setpoint step will change to smoother exponential changes and the structure corresponds to the Two Degreee of Freedom (2DOF) PI controller that is also equivalent to the PI controller with error acting on I action (integral part) only, while the P-action has as input negative measured output. When furthermore $\mathrm{T}_{p2} = T_p = 0$ the structure reduces to the traditional PI controller without the equivalent prefilter.

$$
U(s) = -K_p Y_m(s) + E(s)/\left(K_0 T_f\right)\,;\; E(s) = W(s) - Y_m(s) \tag{2.45}
$$

When starting by defining the traditional PI controller parameters K_p and T_I, then according to (2.44) it is possible to calculate the equivalent prefilter parameters of the structure in Fig. 2.12 and to denote them as

$$
T_{p1} = T_I\,;\; T_{p2} = 0\,;\; T_{f1} = T_I/\left(K_p K_0\right) \tag{2.46}
$$

Because of pole zero cancellation in the prefilter, controller (2.45) with the error acting on I only that gives smoother (exponential) control error decrease, may be simpler described by the prefilter parameters

$$
T_{p1} = T_I\,;\; T_{p2} = 0\,;\; T_{f1} = 0 \tag{2.47}
$$

Definition 2.11 (Fundamental PI_0-IM, FPI_0-IM and PI controllers based on Inverse Model of the dominant loop dynamics). The static feedforward control extended by the reconstruction and compensation of input disturbances and compensation of single loop time constant using inverse model of the dominant loop dynamics will be denoted as the PI_0-IM controller here. It is a fundamental solution fulfilling conditions of Def. 1.21. When extended by a prefilter with the time constant $T_p > 0$ it will be denoted as the FPI_0-IM controller. For $T_{f1} = T_{p1} = T_f = T_{10}$, when also the prefilter (2.44) of the equivalent loop in Fig. 2.13 reduces to the prefilter with single time constant $T_{p2} = T_p$, one gets the structure of the 2 degree of freedom (2DOF) PI controller. When also $T_p = 0$ the structure reduces to the traditional PI controller. The closed loop transfer functions of the FPI_0-IM and 2DOF PI controller with the plant-model parameter

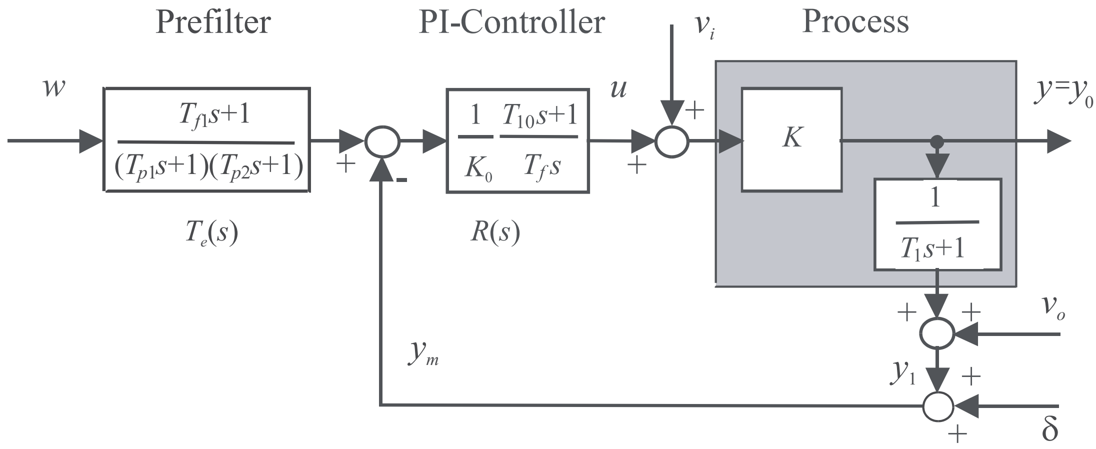

Figure 2.13: Equivalent scheme of the PI_0-IM controller from Fig. 2.12; tuning $T_{f1} = T_{p1} = T_f$ gives the 2DOF PI controller with the simplest prefilter $T_e = 1/(1 + T_{10}s)$

dismatch are given as

$$
\begin{aligned}
F_{w0}(s) &= \frac{Y_0(s)}{W(s)} = \frac{K(1+T_f s)(1+T_1 s)}{(1+T_p s)\left[K_0T_fT_1s^2 + (K_0T_f + KT_{10})s + K\right]} \\
F_{w1}(s) &= \frac{Y_1(s)}{W(s)} = \frac{K(1+T_f s)}{(1+T_p s)\left[K_0T_fT_1s^2 + (K_0T_f + KT_{10})s + K\right]} \\
F_{w0p}(s) &= \frac{K(1+T_1 s)}{K_0T_fT_1s^2 + (K_0T_f + KT_{10})s + K}\,;\ T_{f1} = T_{p1} = T_{10} \\
F_{w1p}(s) &= \frac{K}{K_0T_fT_1s^2 + (K_0T_f + KT_{10})s + K}\,;\ T_{f1} = T_{p1} = T_{10}
\end{aligned}
\tag{2.48}
$$

It is to note that the equivalent scheme according to Fig. 2.13 is now allways realizable, i.e. also for the FPI_0-IM controller with $T_p = 0$. Since for $K_0/K > 0$, $T_f > 0, T_{10} > 0$ and $T_1 > 0$ all denominator coefficients are positive, system remains robustly stable for any such a tuning.

Besides of use of the inverse dynamics, an equal balancing of both reconstruction channels may also be achieved by alternative solutions with the time constant T_1 included in the DO path from the controller output in Fig. 2.12b. Instead of this it is more frequently used solution with reconstruction and compensation of the output disturbance using the parallel plant model in Fig. 2.12c.

Definition 2.12 (PI_0-PM controller with Paralel plant Model typical for the IMC structures). In loops with the time constant T_1 the input disturbance reconstruction used in the I_0 controller can be improved by inserting identified time constant T_10 into the DO reconstruction branch leading from the controller output (Fig. 2.12b) to balance equally both reconstruction channnels. When designed for reconstruction and compensation of the output disturbance (Fig. 2.12c), the resulting PI$_0$-PM controller will become identical with the IMC control structure that is known by low noise sensitivity and good robustenss. Transfer functions describing setpoint responses may be achieved from (2.48) by susbstituting T_{10} instead of T_f. Similarly, also the responses to input disturbances cannot be arbitrarily speeded up and are determined by the time constant T_{10}. This structure does not fullfill requirements on the fundamental solutions from Def. 1.21 and therefore it will not be further analyzed here.

2.2.2 PI$_0$-IM: Analytical Versus Numerical Robust Tuning

Analytical approach to controller tuning may be based on the position and character of the closed loop poles. In such a case, character of the transient responses is determined by roots of the characteristic polynomial corresponding to different loop parameters. Closed loop poles corresponding to (2.48) are

$$s_{1,2} = -\frac{K_0 T_f + K T_{10}}{2 K_0 T_f T_1} \pm \frac{\sqrt{(K_0 T_f + K T_{10})^2 - 4 K K_0 T_f T_1}}{2 K_0 T_f T_1} \tag{2.49}$$

Transients are expected to change qualitatively when the discriminant in (2.49) changes its sign, i.e. when

$$(K_0 T_f + K T_{10})^2 - 4 K K_0 T_f T_1 = 0 \tag{2.50}$$

By denoting

$$\kappa = K_0/K > 0\,;\ \tau_f = T_f/T_{10} > 0\,;\ \tau_1 = T_1/T_{10} > 0\,;\ \tau_p = T_p/T_{10} > 0 \tag{2.51}$$

the last equation may also be rewritten as

$$\tau_1 = \frac{(\kappa \tau_f + 1)^2}{4 \kappa \tau_f} \tag{2.52}$$

The closed loop performance portrait showing dependance of the shape of transient responses on the loop parameters is appropriate to be identified for dimensionless parameters (2.51). It can be derived from (2.48) by introducing new complex variable

$$p = T_{10} s \tag{2.53}$$

when

$$
\begin{aligned}
F_{w0}(p) = \frac{Y_0(p)}{W(p)} &= \frac{(1+\tau_f p)\,(1+\tau_1 p)}{(1+\tau_p p)\left[\kappa\tau_f\tau_1 p^2 + (\kappa\tau_f+1)\,p+1\right]} \\
F_{w0p}(p) &= \frac{(1+\tau_1 p)}{\kappa\tau_f\tau_1 p^2 + (\kappa\tau_f+1)\,p+1}\,;\ \tau_p = \tau_f
\end{aligned}
\tag{2.54}
$$

New complex variable p means also new scale in the time domain, whereby, for the same input, the time transients corresponding to p in (2.54) will be times faster than transients corresponding to s in (2.48). It means that all properties related to time (as e.g. IAE or ISE performance indices) identified in the dimensionless variables have to be multiplied by this factor. Note (Fig. 2.14) that for the output y_0 (input of the time constant) of the PI_0 controller achieved for $T_p = 0$ the NO areas are no more identical with those corresponding to MO output, as in the case of the I controller and that to larger values of $\tau_f = T_f/T_{10}$ correspond enlarged areas of NO and MO control.

From the closed loop transfer functions of the PI_0 controller achieved for $T_p = 0$ from (2.48), or (2.54) it may be deduced that for $K > K_0 \Rightarrow \kappa < 1$ due to the 2nd order polynomials in numerator and denominator giving $F_{w0}(\infty) > 1$ the setpoint step responses of $y_0$incline to overshooting. This will restrict choice of appropriate tuning for achieving NO and MO output to $K_0 \geq K_{max}$. Only here gives the aperiodicity border (2.50), (2.52) some usefull information. Output monotonicity of the setpoint responses of y_0 could be be improved by canceling one of the numerator time constants by prefilter. Since the plant time constant may vary in time, the simplest solution is to use prefilter with tuning $T_p = T_f$, or the equivalent controller without prefilter (i.e. with $T_{p1} = T_{f1}$ in (2.44)). For the output y_1 the areas of NO and MO control coincides. The aperiodicity border (2.50), (2.52) gives for $\kappa < 1$ some usefull information just for small values of τ_f. All these subtle nuances shows that impact of the robust analytical tuning based on the closed loop pole is very restricted by its nature. The u-TV values with $\mathrm{TV}_{min} = 1$ correspond to unit step of the setpoint signal and $K = 1$. In order to eliminate dependance on these parameters, it is more appropriate to work with TV_0 criterion.

2.2.3 PI_0-IM: Impact of the Parameter Mismatch on Setpoint Steps

In tuning the controller one will usually ask: "which tuning will guarantee chosen qualitative shape of transients and simultaneously give minimal IAE values for the whole possible extent of parameter changes?"

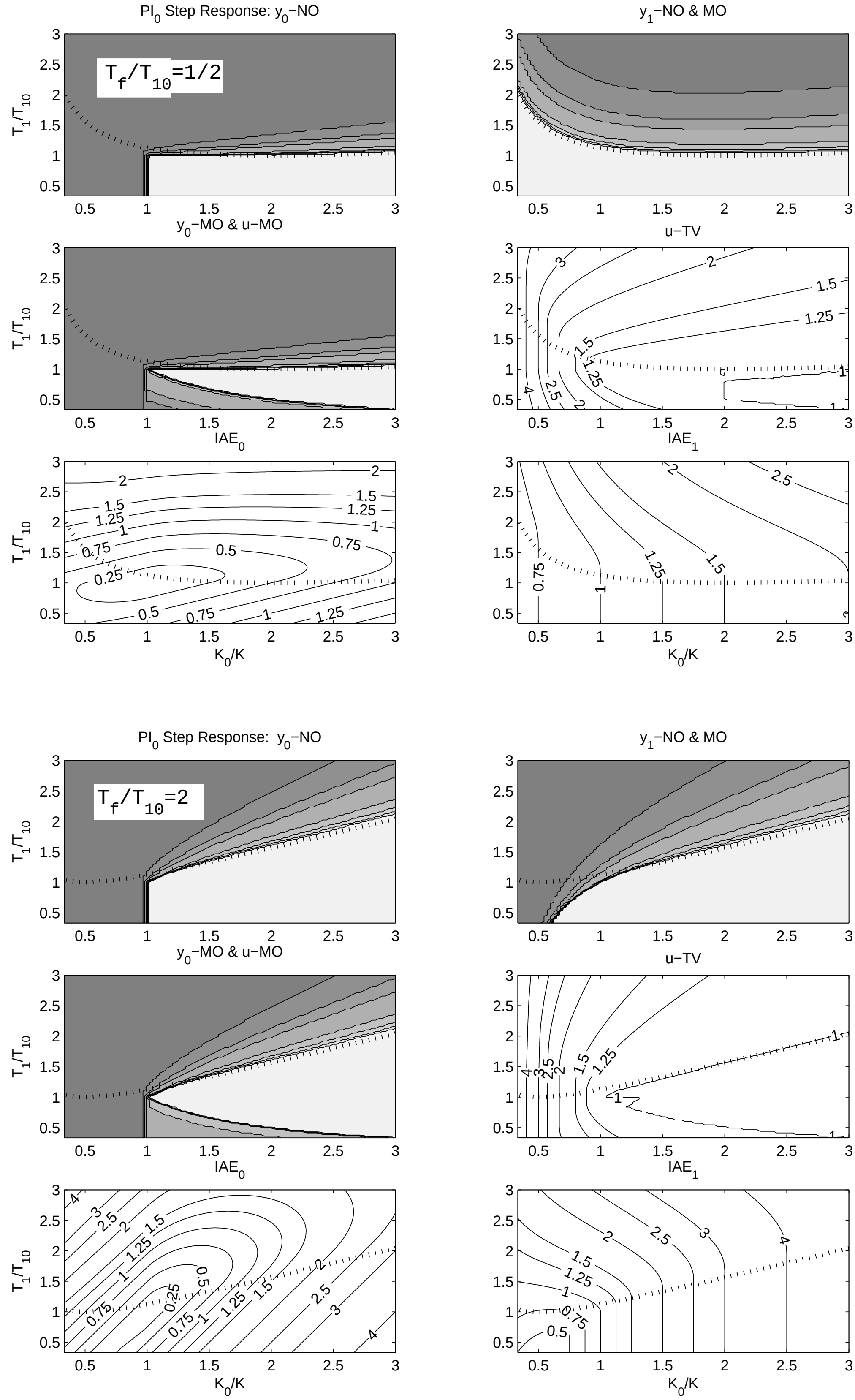

Figure 2.14: Performance portrait of the PI_0-IM controller for setpoint response with two values of $\tau_f = 1/2$ and $\tau_f = 2$ and different measurement precissions $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$ with white showing the best performance; dotted border of complex poles (2.50), (2.52)

In answering this question, let us formulate the control task in robust design of the PI_0-IM controller more precisely. For the plant uncertainty given as

$$\begin{aligned} K &\in \langle K_{min}, K_{max}\rangle \ ; \ c_K = K_{max}/K_{min} \geq 1 \\ T_1 &\in \langle T_{1,min}, T_{1,max}\rangle \ ; \ c_T = T_{1,max}/T_{1,min} \geq 1 \end{aligned} \tag{2.55}$$

that in the plane of normalized parameters (κ, τ_1) yields uncertainty boxes of all possible operating points

$$UB = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \kappa_{min}, \tau_{1,max} & \kappa_{max}, \tau_{1,max} \\ \kappa_{min}, \tau_{1,min} & \kappa_{max}, \tau_{1,min} \end{bmatrix} \tag{2.56}$$

or in the case with single uncertain parameter corresponding to the uncertainty line segment (ULS) the task is to find controller tuning guaranteeing fastest possible transients (with minimal average IAE value). In fulfilling this task it is firstly required to identify the loop performance portrait corresponding to dimensionless variables (e.g.by fixing plant values $K = 1, T_1 = 1$ and by mapping system behavior for interesting range of plant values K_0 and T_{10} (for intervals larger than given by (2.55) and for some range of values $\tau_f = T_f/T_{10}$. All interesting results achieved by computer simulation will then be stored within the 3D space of dimensionless parameters (2.51).

Examples of sweeping parameter area corresponding e.g. to ϵ-MO output y_0 and looking for appropriate UB lying completely in it are in Fig. 2.15. By using uncertainty information represented by (2.55) it is necessary to sweep over all possible values of T_f for UB (2.56) or ULS defined by ratios of extreme values of uncertain parameters c_K, or c_T and lying in the required performance area. During this step, from identified values of particular UB (2.55) one has to recalculate the task from fixed controller tuning K_0, T_{10} and variable plant parameters K, T_1 to fixed limit loop values (2.55) and variable controller tuning corresponding to the optimal position of UB according to

$$K_0 = \kappa_{min}^{opt} K_{max} \ ; \ T_{10} = T_{1,min}/\tau_{min}^{opt} \ ; \ IAE_{mean} = T_{10} IAE_{mean}^{opt} \tag{2.57}$$

Finally, the identified optimal tuning has to be verified by simulation to guarantee required degree of output monotonicity and overshooting. Due to truncation errors, results fulfilling given condition may be shifted by one quantization step that may be important especially when working with lower number of points in the parameter grid. In such a case, finer controller tuning could reasonably improve the resulting control performance. The calculation may be accelerated by generating new performance portrait just for a limited range of unknown parameters. Minimal IAE values usually

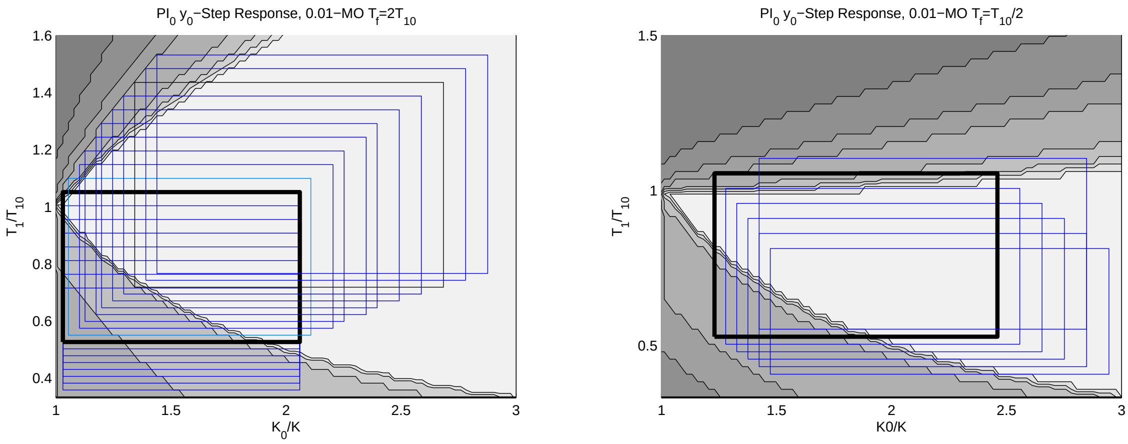

Figure 2.15: Uncertainty boxes corresponding to interval plant parameters (3.43) for $\tau_f = 2$ (left) and $\tau_f = 1/2$ (right) and tolerated overshooting 1%; optimal UB (bold) $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$ with white showing the best performance;

correspond to UB shifted as much as possible to the values with $\kappa \geq 1$, i.e. to $K_0 \geq K_{max}$.

We may use this fact in simplifying visualization problems in 3D, when by supposing $K_0 = K_{max}$ it is possible to decrease the number of uncertain parameters to one and to work in 2D parameter space. Also here we may expect that for the output y_0 of the PI$_0$ controller the transients may be monotonic for $\tau_1 \leq 1$ and $\tau_f > 0$. Without considering nonmodelled dynamics, the transient responses may be arbitrarily speeded up by decreasing the DO filter time constant and the IAE value over the ULS may be made to be arbitrarily small. From this point of view, use of more complex PI$_0$-IM controller seems to be much more advantageous than the use of I$_0$ controller with uncertainty characteristics in Fig. 2.9. However, even in the situations with negligable uncertainty of the dominant dynamics parameters, in real loops the process of speeding up transient responses by decreasing the DO filter time constant will be limited by the every time present nonmodelled dynamics. The DO filter time constant must remain larger that the largest time constant or dead time approximating the nonmodelled dynamics.

Example 2.13 (Tuning of the PI$_0$ Controller for Limited TV$_0$ Values). The task is to tune robustly PI$_0$ controller to guarantee for setpoint step response limited values of TV$_0$<TV$_{0,max}$ $= 0.1$ for the plant uncertainty limits

$$T_1 \in \langle 1, 1.5 \rangle \ ; \ K \in \langle 10, 20 \rangle \tag{2.58}$$

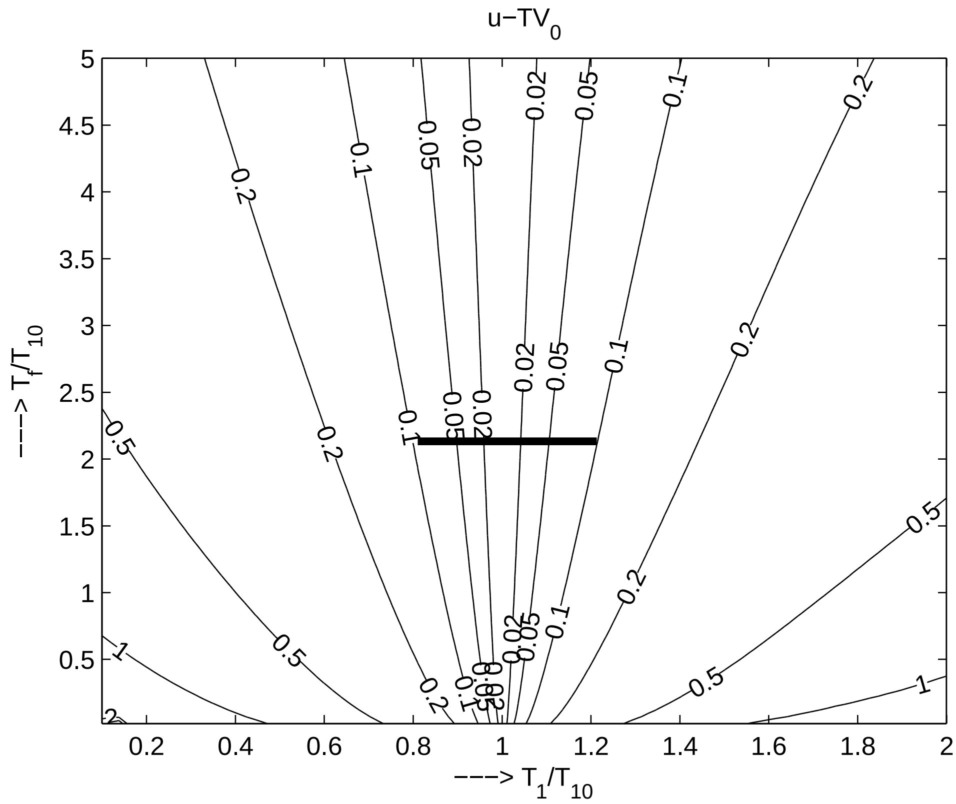

Figure 2.16: PI_0: Performance portrait for the TV_0 values of the setpoint step responses with the ULS corresponding for the minimal possible value τ_f to the limit T_1 values (2.58) and to $\mathrm{TV}_0 < 0.1$

Performance Portrait (Fig. 2.16) was generated over 100x100 points for $T_{10} = 1, K = K_0 = 1$, $\tau_1 \in \langle 0.01, 2\rangle$ and $\tau_f \in \langle 0.01, 5\rangle$. In order to keep the disturbance response as fast as possible, controller will be tuned by using the smallest possible T_f value enabling to achieve the required performance. Localization of the corresponding ULS in the PP is shown by bold line segment. The tuing parameters are determined according to (2.57). By sweeping the PP one gets

$$\begin{aligned}\tau_f = 2.1327\,;\ \tau_{1,min}^{opt} = 0.8101\,;\ IAE_{1,mean}^{opt} = 1.0267 \\ T_{10} = T_{1,min}/\tau_{1,min}^{opt} = 1.2344\,;\ T_f = \tau_f T_{10} = 2.6327 \\ IAE_{1,mean} = T_{10} IAE_{1,mean}^{opt}\end{aligned} \tag{2.59}$$

From the setpoint step responses corresonding to limit values (2.58) and satisfying $\mathrm{TV}_0 < 0.1$ it is to see that the amplitude deviations from monotonicity and nonovershooting may be approximatelly expressed as

$$\epsilon_y \approx TV_{0,max}/2 \tag{2.60}$$

Such a relation holds, however, just in situations, when the transients show one pulse superimposed on monotonic (in the limit case step) variables.

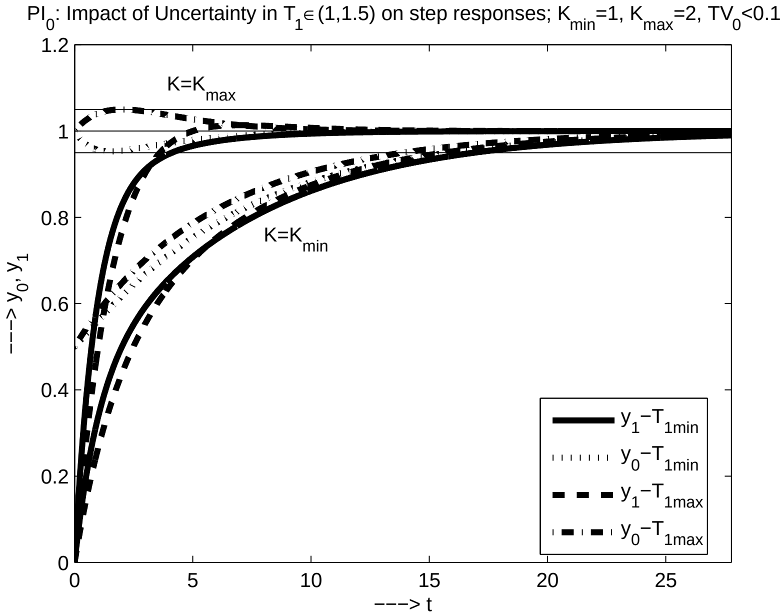

Figure 2.17: PI_0: Setpoint step responses corresonding to limit values (2.58) and satisfying $\mathrm{TV}_0 < 0.1$ show amplitude deviations from monotonicity $\epsilon_y \approx TV_{0,max}/2$

2.2.4 PI$_0$-IM: Impact of Parameter Mismatch for Disturbance Step

Up to now, all our attention was concentrated on the setpoint response, but already there we tried to find such a controller tuning that would fulfill the performance requirements with the minimal T_f values that are expected to give the fastest possible disturbance reconstruction and compensation. Now we will focuss our attention also to the disturbance response. For an input disturbance v_i the disturbance responses are defined by transfer functions

$$\begin{aligned} F_{vi0}(s) = \frac{Y_0(s)}{V_i(s)} = \frac{sKK_0T_f\,(1+T_1s)}{\left[K_0T_fT_1s^2 + (K_0T_f + KT_{10})\,s + K\right]} \\ F_{vi1}(s) = \frac{Y_1(s)}{V_i(s)} = \frac{1}{(1+T_1s)}F_{vi0}(s) \end{aligned} \tag{2.61}$$

As it is obvious from these transfer functions that yield $F_{vi0}(0) = 0$ and $F_{vi1}(0) = 0$, a piecewise constant input disturbances will cause no permanent error. From $F_{vij}(0) = 0\,; j = 0, 1$ it is obvious that in steady states influence of admissible piecewise constant input disturbances disturbances is completely eliminated. For generating performance portrait it is again advantageous to introduce normalized loop parameters (2.51) and (2.53) that yield

$$
\begin{aligned}
F_{vi0}(p) = \frac{Y_0(p)}{V_i(p)} &= \frac{pK_0\tau_f\,(1+\tau_1 p)}{\left[\kappa\tau_f\tau_1 p^2 + (\kappa\tau_f+1)\,p + 1\right]} \\
F_{vi1}(p) = \frac{Y_1(p)}{V_i(p)} &= \frac{1}{(1+\tau_1 p)} F_{vi0}(p)
\end{aligned}
\tag{2.62}
$$

As it is obvious from these transfer functions, the performance analysis may not be fully realized in the normalized variables and the disturbance response will also depend on the tuning parameter K_0. So, localization of UB in space of normalized parameters will be based on sweeping the parameter portrait for position corresponding to minimal IAE value that has to respect not only the time scale (2.53), but also scaling imposed by K_0.

It is to remember that NO, or MO areas of controller parameters identified by the computer based analysis for a disturbance step are different from equivalent areas corresponding to setpoint step step. When it is required to keep some property for setpoint as well as for disturbance response, the uncertainty box corresponding to possible loop values must lie in intersection of corresponding areas.

2.2.5 Influence of the Nonmodelled Dynamics

Results of the previous analysis show that controller tuning is dominantly influenced by robustness issues. This holds also in situations, when the parameter changes are relatively negligible and plant is supposed to have time invariant dynamics.

The first intuitive expectation migt be that by decreasing plant uncertainty and by canceling the dominant time constants by inverse dynamics, the remaining loop dynamics can be arbitrarily speeded up (as for fundamental solutions) by deceasing $T_f \to 0$. This is, however, not true in practice. In such situations a reliable PI_0 controller tuning would require to determine not only the dominant loop time constant T_1 but also some parameter approximating the nonmodelled dynamics. In the simplest case it is again possible to approximate the nonmodelled dynamics by a time constant T_a (accumulative delay), or by a transport delays T_d. Such loop approximations would be based on models as

$$
F_{yd}(s) = \frac{Y_m(s)}{U(s)} = \frac{K\mathrm{e}^{-T_d s}}{1+T_1 s}
\tag{2.63}
$$

or

$$
F_{yd}(s) = \frac{Y_m(s)}{U(s)} = \frac{K}{(1+T_1 s)\,(1+T_a s)}
\tag{2.64}
$$

In the nominal case with $T_{10} = T_1$, the first estimate of appropriate T_f values for which the nonmodelled dynamics might be important could be based on Tab. 2.1 and Tab. 2.2 derived for the I_0 controller. Such approach was already mentioned by textbooks (Huba, 2003, 2006). Using approximation of the nonmodelled dynamics by dead time and $K_0 = K, T_{10} = T_1$ one gets e.g values recommended by Vítečková et al (2000), or many other results summarized by O'Dwyer (2000).
To get more detailed picture of the resulting loop dynamics for $T_{10} \neq= T_1$, the computer based analysis can be used again. For both models 2.63 and 2.64 introduction of additional parameter for the nonmodell dynamics leads to increase of the dimension of the solved problem. It means that already when using possibility for simplification by choosing $K_0 = K_{max}$ it is necessary to work in a 3D space. Therefore, it is always advantageous to check the possibility to simplify the problem e.g. by choosing $T_{10} = T_{1,max}$ and to solve the problem in 2D space of parameters (τ_d, τ_f), ; $\tau_d = T_d/T_{10}$.

2.2.6 Effect of Measurement and Quantization Noise

For a possible measurement noise δ the responses of both possible outputs are defined by

$$\begin{aligned} F_{\delta 0}(s) = \frac{Y_0(s)}{\delta(s)} &= \frac{K\,(1+T_{10}s)\,(1+T_1 s)}{[K_0 T_f T_1 s^2 + (K_0 T_f + K T_{10})\, s + K]} \\ F_{\delta 1}(s) = \frac{Y_1(s)}{\delta(s)} &= \frac{1}{(1+T_1 s)} F_{\delta 0}(s) \end{aligned} \tag{2.65}$$

In the robustness analysis in previous sections we came to conclusion that from the robustness point of view it is better to use the more complex PI_0 controller than the simpler I_0 controller. However, in tuning the fundamental PI_0-IM controller given by Fig. 3.12a it is important to remember that a measurement noise step by $\Delta\delta$ produces in the control signal kick with amplitude

$$\Delta u = \lim_{s\to\infty} s \frac{1+T_{10}s}{K_0 T_f s} \frac{\Delta\delta}{s} = \frac{T_{10}}{K_0 T_f} \Delta\delta \tag{2.66}$$

When for a given value $\Delta\delta$ one chooses the filter time constant T_f too small, noise amplification and due to this the corresponding "kick" of the manipulated variable Δu may increase over acceptable values. So, the filter time constant (or the equivalent gain of the P action (2.44)) should also consider acteptable levels of such control signal kicks defined by the maximal amplitudes of the measurement noise. Whereas for the I_0 controller the integral character of controller is guaranteeing homogenous filtration over all frequencies, for the PI_0 controller filtration properties dominate just for frequencies over the DO bandwidth $\Omega_f = 1/T_f$. So, the measurement noise and required filtration properties represent the key aspects in deciding if to use I_0 or PI_0 control.

2.2.7 Conclusions PI_0

Space available for this contribution has not enabled to go into detailed comparing of all possible structures of the PI_0 controllers, of possible DO filters and prefitlers. But, in interpreting results from the computer based analysis of robust controller tuning it is important to note several points:

1. For the setpoint step responses of output y_0 (input of the time constant) NO areas are different from MO ones. For the disturbance responses of output y_0 and for both responses of output y_1 NO and MO areas are identical.
2. NO and MO areas corresponding to output y_0 are different from those corresponding to the output y_1. It is to remember that just the tasks with output y_0 with the relative degree zero genericly fall into DC0.
3. Transients from DC0 may also be designed for the output y_1 (see Theorem 1.15), but there faster dynamics may be achieved by solutions of DC1, treated e.g in Huba (2011).
4. Setpoint step responses are more sensitive to the plant-model mismatch than the disturbance responses. This sensitivity typical for the setpoint step responses of the output y_0 may be reasonable decreased by using prefilter with $T_p = T_f$.
5. For NO and MO step responses of output y_0 it is important to work with $K_0 \geq K$ and $T_{10} \geq T_1$.
6. By limiting the admissible TV_0 values in tuning the PI_0 controller it is simultaneously possible to limit the amplitude deviations from nonovershooting and monotonicity to approximatelly $\epsilon_y \approx TV_{0,max}$

2.2.8 Performance Portrait of the FPI_0 Controller for $T_p = T_f$

After extending the PI_0 controller by prefilter with the time constant $T_p = T_f$ to the FPI_0 (2.44) it is possible to reasonably enlarge areas of NO and MO step responses without increasing number of tuned parameters. The performance portrait in Fig. 2.18 shows that with the tuning $T_{10} =$

$T_{1,max}$ and $K_0 = K_{max}$it is possible to achieve outputs u, y_0 and y_1 with zero TV_0 values for practically arbitrary T_f values. This, however, holds just for systems with negligable nonmodelled dynamics. Therefore, in real applications this controller could be reliably tuned after approximating the nonmodelled dynamics and by increasing number of normalized parameters by using the PP generated in 3D. A simplified approach could e.g use the analyzes of the I_0 tuning to choose T_f and according to the PP in 2D to set

$$T_{10} = T_{1,max}\ ;\ \ K_0 = K_{max} \tag{2.67}$$

2.3 Predictive I_0 and Filtered Predictive I_0 Controllers (PrI_0 and FPrI_0)

Tuning of the closed loop systems involving dead-time still represents a challenging domain of control research. Thereby, importance of dead-time systems that are being used to describe transport of mass, energy and information and to approximate accumulation of time lags in a chain of low order systems is permanently increasing, to mention just different new applications arising in the field of remote control via computer networks and telecommunication links. As it was shown in many contributions (see e.g. Normey-Rico and Camacho (2007)), an increase of the dead-time values with respect to the dominant plant time constant leads in the loops with PID controllers without active dead time compensation to rapid performance deterioration. Consider a stepwise constant reference signal $w(t)$ and an uncertain plant with dominant dead-time

$$\begin{aligned} F(s) &= K\mathrm{e}^{-T_d s} \\ K \in \langle K_{min}, K_{max}\rangle\ ;\ c_K &= K_{max}/K_{min} \geq 1 \\ T_d \in \langle T_{d,min}, T_{d,max}\rangle\ ;\ c_d &= T_{d,max}/T_{d,min} \geq 1 \end{aligned} \tag{2.68}$$

The task is to design robust controller that would guarantee step responses of the output and control variable with tolerable deviation from monotonicity defined e.g. by specifying the amplitude deviations ϵ_y, or ϵ_u, or by specifying the integral measures for deviations u-TV_0 or y-TV_0.
The plant model (2.68) may be simply identified by evaluating the average residence time (2.41) by the step responses Åström and Hägglund (1995), or by a general input signal according to Ingimundarson (2000).
In the simplest case, based on estimate of the plant gain K_0, to set output of the considered plant to the reference value w the static feedforward control $1/K_0$ maight be used. For the plant with an output disturbance

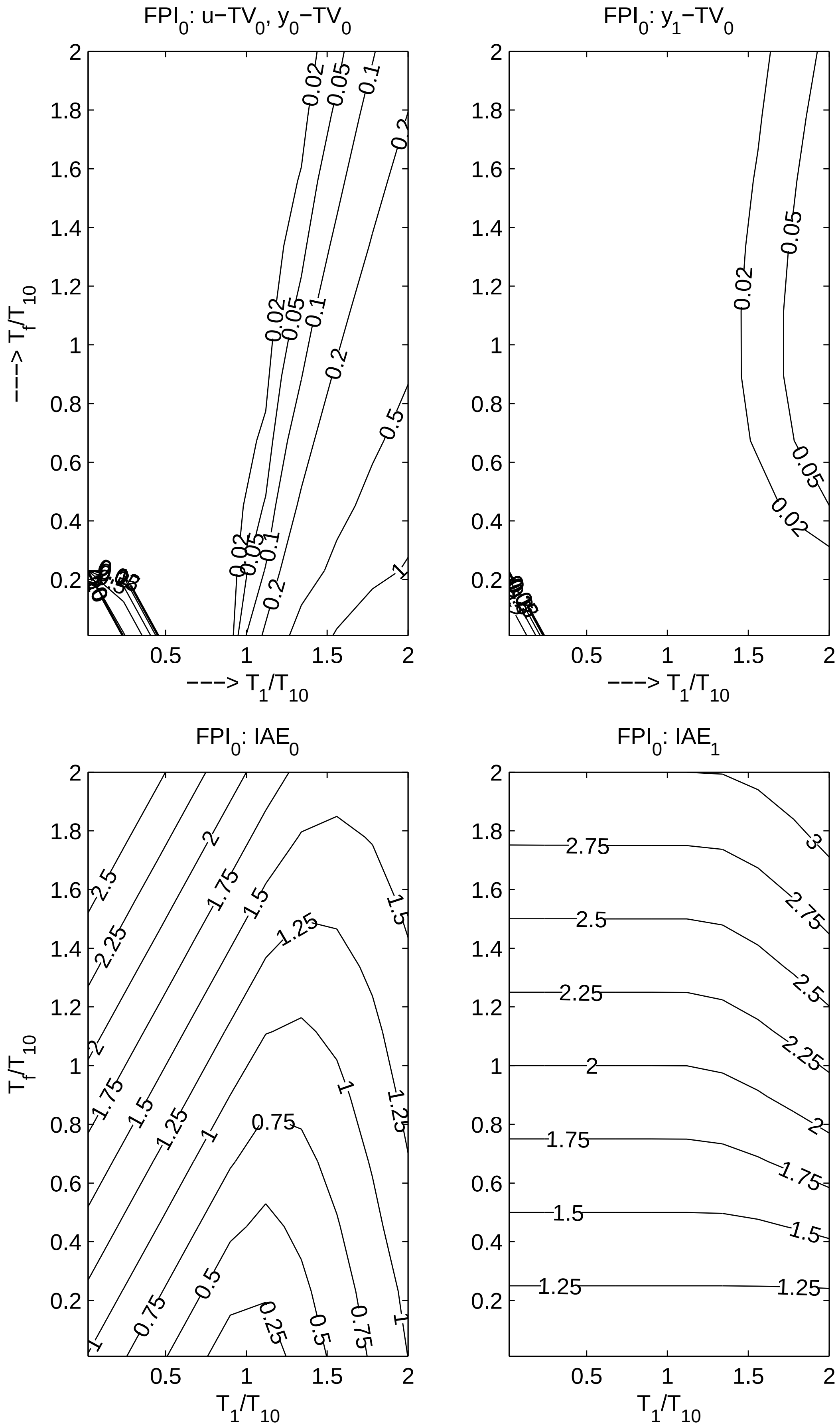

Figure 2.18: FPI_0 with $T_p = T_f$: Performance portrait for the TV_0 values of the setpoint step responses at the outputs y_0 (controller output) and y_1 and the equivalent IAE values

v_o it would be possible to extend this static feedforward control by the Disturbance Observer (DO) inspired by the IMC (Morari and Zafiriou, 1989). For the input disturbance it may similarly be used the DO based on the inverse plant model inspired by Ohnishi et al. (1996). Thereby, in both cases, estimate of the plant dead time T_{d0} was inserted into the DO branch from the controller output.
In controlling plant (2.68) both these alternatives are equivalent, but in order to be clear in controlling more complex plants and with sake of the brevity we will propose following:

Definition 2.14 (Predictive PrI$_0$ and FPrI$_0$ Controllers). Under the PrI$_0$ controller we will understand the static feedforward control with the gain $1/K_0$ extended by the input or output disturbance reconstruction and compensation (Fig. 2.19) with the DO filter time constant T_f and by the prefilter with the time constant T_p (in the simpest case with $T_p = T_f$) giving the resulting control law

$$U(s) = \frac{W(s)}{K_0\left(1+T_f s\right)} - \left[\frac{Y(s)}{K_0\left(1+T_f s\right)} - \frac{\mathrm{e}^{-T_{d0}s}U(s)}{K_0\left(1+T_f s\right)}\right] \tag{2.69}$$

2.3.1 Performance Portrait of the PrI$_0$ and FPrI$_0$ Controllers

Intuitively one could expect optimal behavior of this controller for $K_0 = K, T_{d0} = T_d$. However, how to choose K_0 and T_{d0} and what happens in the case of a parameter mismatch?
The setpoint-to-output closed loop transfer functions of the PrI$_0$ and FPrI$_0$ controllers corresponding to the output y_1 and a plant-model parameter dismatch are given as

$$\begin{aligned} F_{w1}(s) = \frac{Y_1(s)}{W(s)} &= \frac{K\left(1+T_f s\right)\mathrm{e}^{-T_d s}}{\left(1+T_p s\right)\left[K_0\left(T_f s+1-\mathrm{e}^{-T_{d0}s}\right)+K\mathrm{e}^{-T_d s}\right]} \\ F_{w1p}(s) &= \frac{K\mathrm{e}^{-T_d s}}{K_0\left(T_f s+1-\mathrm{e}^{-T_{d0}s}\right)+K\mathrm{e}^{-T_d s}}\,;\; T_p = T_f \end{aligned} \tag{2.70}$$

Similarly, the input disturbance-to-output y_1 closed loop transfer functions for the plant-model parameter dismatch is given as

$$F_{vi1}(s) = \frac{Y_1(s)}{V_i(s)} = \frac{KK_0\mathrm{e}^{-T_d s}\left(1+T_f s-\mathrm{e}^{-T_{d0}s}\right)}{K_0\left(T_f s+1-\mathrm{e}^{-T_{d0}s}\right)+K\mathrm{e}^{-T_d s}} \tag{2.71}$$

Obviously, $F_{w1}(0) = 1$ and $F_{vi1}(0) = 0$, what guarantees I-behaviour, i.e. rejection of piece-wise constant disturbances also for $K_0 \neq K$ and $T_{d0} \neq T_d$.

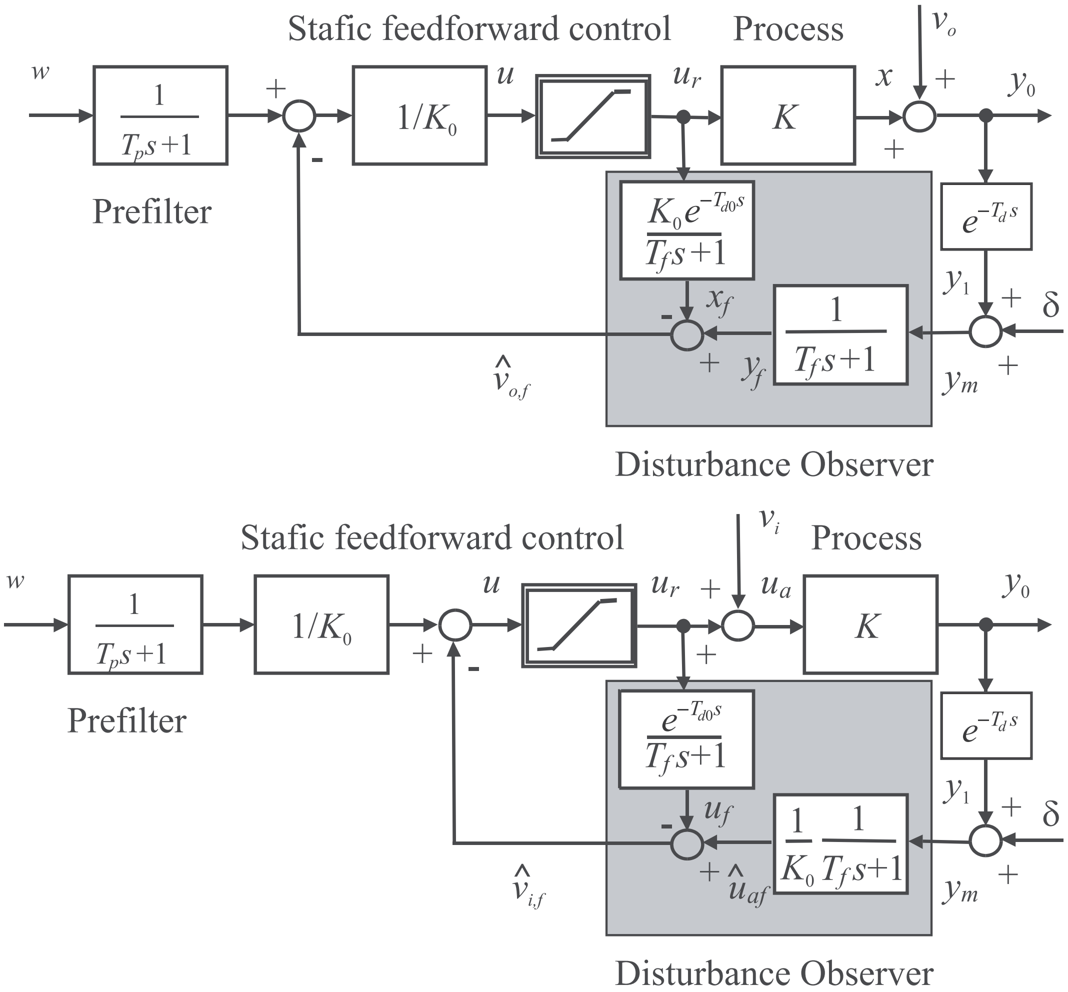

Figure 2.19: a) Fundamental $\mathrm{FPrI_0}$-PM controller ($\mathrm{FPrI_0}$-IMC controller) controller designed as static feedforward control extended by input disturbance reconstruction and compensation using Parallel plant Model (above) and the $\mathrm{FPrI_0}$-IM controller with Inverse Model of the invertible dynamics reduced to $1/K_0$ (below); in both cases the disturbance reconstruction was balanced by including dead time estimate into the DO channel from the controller output; both structures are extended by a prefilter (in the simpest case with $T_p = T_f$) to $\mathrm{FPrI_0}$ controllers

These properties hold also for the output y_0

$$F_{w0}(s) = F_{w1}(s)\,\mathrm{e}^{T_d s}\,;\;\; F_{vi0}(s) = F_{vi1}(s)\,\mathrm{e}^{T_d s} \tag{2.72}$$

To be able to use the generated PP for any plant (2.68), the setpoint step responses will be mapped by using 3D coordinate system (κ, τ_f, τ_d) with normalized variables

$$\kappa = K_0/K\,;\;\; \tau_f = T_f/T_{d0}\,;\;\; \tau_d = T_d/T_{d0}\,;\;\; \tau_p = T_p/T_{d0}\,;\;\; p = T_{d0}s \tag{2.73}$$

that yield

$$\begin{aligned}
F_{w1}(p) = \frac{Y_1(p)}{W(p)} &= \frac{(1+\tau_f p)\,\mathrm{e}^{-\tau_d p}}{(1+\tau_p s)\left[\kappa\left(\tau_f p + 1 - \mathrm{e}^{-p}\right) + \mathrm{e}^{-\tau_d p}\right]} \\
F_{w1p}(p) &= \frac{\mathrm{e}^{-\tau_d p}}{\kappa\left(\tau_f p + 1 - \mathrm{e}^{-p}\right) + \mathrm{e}^{-\tau_d p}}\,;\;\; \tau_p = \tau_f \\
F_{vi1}(p) = \frac{Y_1(p)}{V_i(p)} &= \frac{K_0 \mathrm{e}^{-\tau_d p}\left(1 + \tau_f p - \mathrm{e}^{-p}\right)}{\kappa\left(\tau_f p + 1 - \mathrm{e}^{-p}\right) + \mathrm{e}^{-\tau_d p}}
\end{aligned} \tag{2.74}$$

The transfer functions corresponding to the output y_0 may similarly be derived by means of

$$F_{w0}(p) = F_{w1}(p)\,\mathrm{e}^{\tau_d p}\,;\;\; F_{vi0}(p) = F_{vi1}(p)\,\mathrm{e}^{\tau_d p} \tag{2.75}$$

Examples of one layer of the 3D PP of the PI_0 and FPI_0 in Fig. 2.20 that by introducing the prefilter the ϵ_y-areas reasonably enlarger and that e.g. the 10^{-5}-MO area is close to the area with $\mathrm{TV}_0 = 10^{-4}$, what again points out possibility to work with the numerically simpler measures for the integral deviations from strictly monotonic control at the plant input and output. By being based on the 2-parameter plant model the PrI controllers represent alternatives to the PI controllers. For systems with the dominant time delays they enable substantial quality improvement. First version of PrI controller was proposed by Reswick (1956). Yet before the well known Smith Predictor (SP) he proposed active compensation of the whole identified dead time ($T_{d0} = T_d$ and $K_0 = K$) corresponding $T_f = 0$. This caused, however, enormous sensitivity to parameters uncertainty, because for $T_f \to 0$ the monotonicity areas shrink (Fig. 2.21). Because of lacking method for a reliable controller tuning, it was practically forgotten and newer works (Åström & Hägglund, 1995; 2005, Guzman et al., 2008; Normey-Rico et al., 2009) mention just the Smith Predictor. The PP based analysis enables to explain the high sensitivity of the Reswick's solution and importance of the choice of Tf and gives also possibility of robust tuning of these simplest possible predictive controllers.

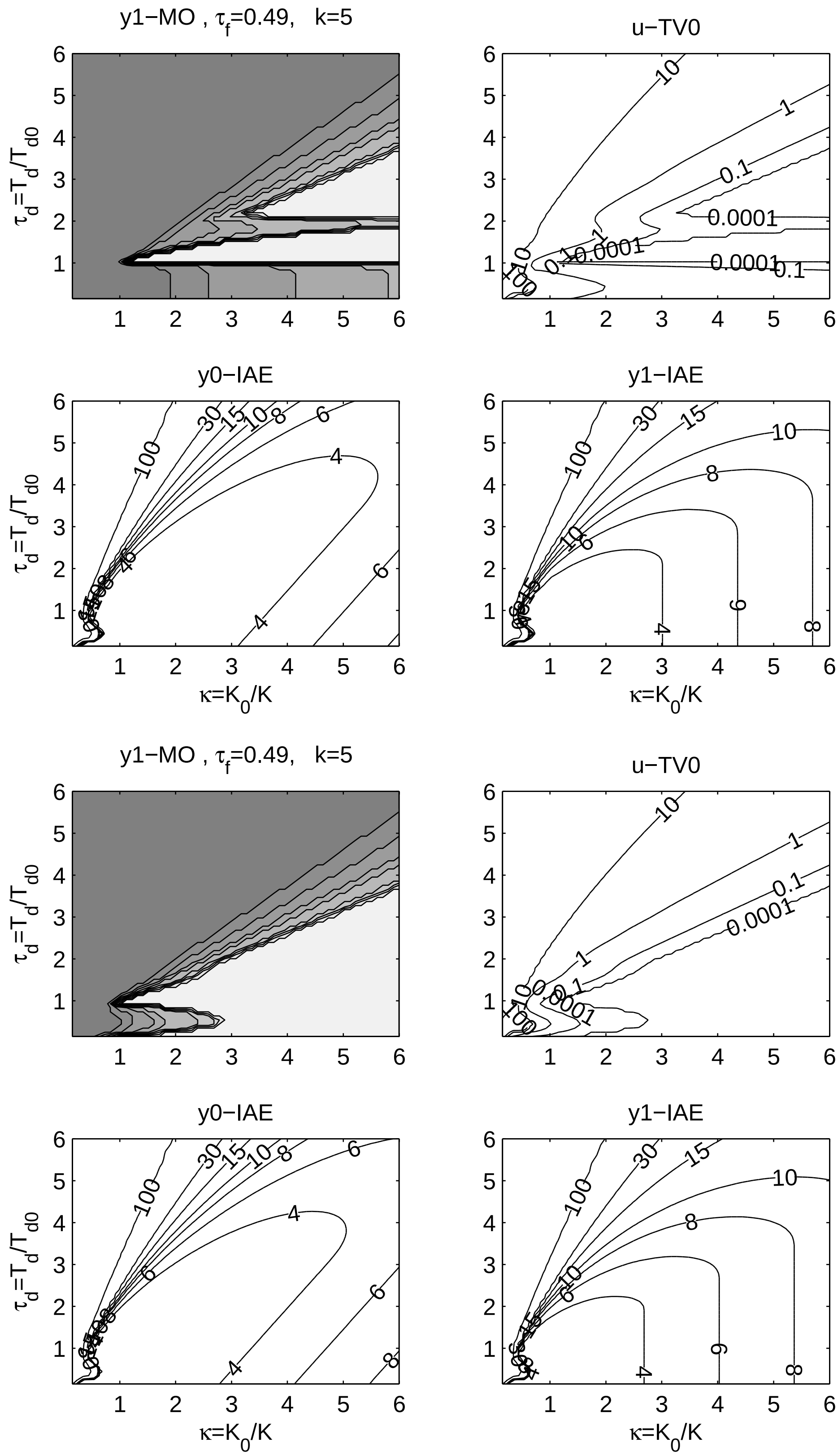

Figure 2.20: One layer of the PP of the plant (2.68) with the PrI_0 (above) and FPrI_0 controller (below) corresponding to $\tau_f = 0.49$ generated over 61x61x11 points and showing ϵ_y-MO areas (left above) for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$, white denoting the best performance, the u-TV_0 contours (right above) and the IAE0 and IAE1 levels (below) in the plane (κ, τ_d)

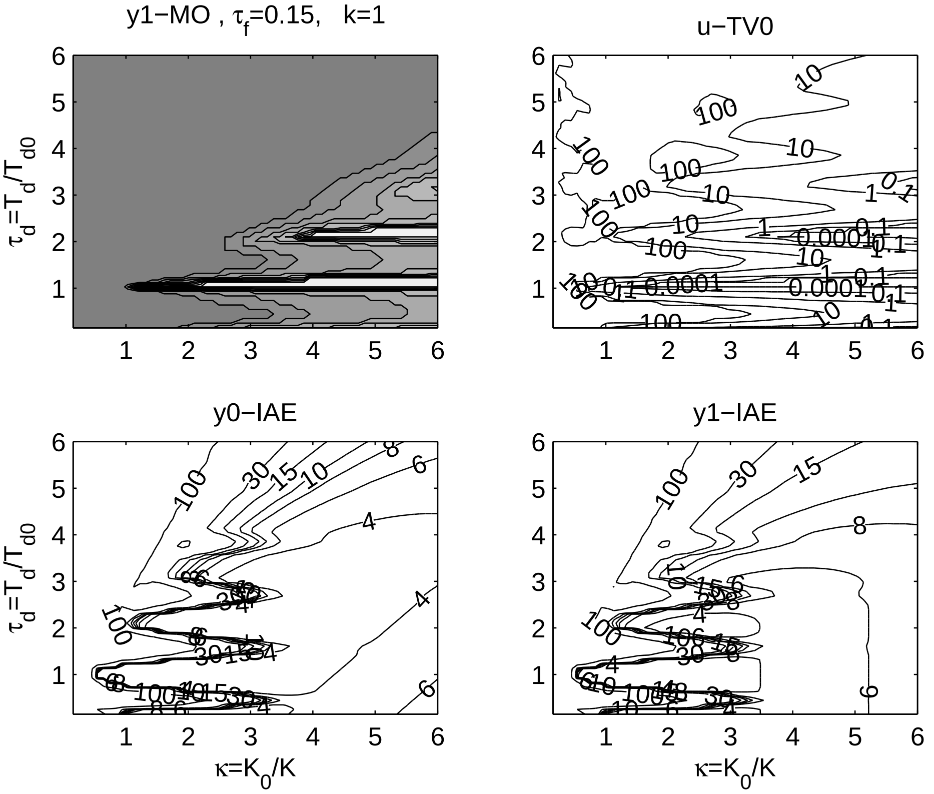

Figure 2.21: Layer of the PP of the plant (2.68) with the FPrI_0 controller corresponding to $\tau_f = 0.15$ generated over 61x61x11 points and showing ϵ_y-MO areas (left above) for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$, white denoting the best performance, the u-TV_0 contours (right above) and the IAE0 and IAE1 levels (below) in the plane (κ, τ_d)

2.3.2 Robust Tuning of the PrI$_0$ and FPrI$_0$ Controllers

In a subplane (κ, τ_d) with a given τ_f the robust design corresponding to plant (2.68) means to locate Uncertainty Box of all possible operating points

$$UB = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \kappa_{min}, \tau_{d,max} & \kappa_{max}, \tau_{d,max} \\ \kappa_{min}, \tau_{d,min} & \kappa_{max}, \tau_{d,min} \end{bmatrix} \qquad (2.76)$$

with vertices corresponding to combinations of the limit values of κ and τ_d by specifying K_0, T_{d0} and $T_f$in such a manner that will guarantee the fastest possible transients (with minimal average IAE value). Examples of sweeping parameter area corresponding e.g. to ϵ_y-MO output y_1, $\epsilon_y = 0.02$ and looking for appropriate UB lying completely in it are in Fig. 2.22.
Due to the relatively rough quantization, the achieved overshooting (Fig. XXX3 below) is not absolutely close to the tolerable value. It is to note that the found "optimal" tuning of this controller is very close to the expected value of the gain $K_0 = K_{max}$) that may reasonably simplify the tuning process by reducing the task to 2D space of parameters (τ_d, τ_f).
In the case with single uncertain parameter the task reduces to finding optimal position of a horizontal (uncertain gain K), or vertical (uncertain T_d) uncertainty line segment (ULS).
By using uncertainty information represented by (2.68) it is necessary to sweep over all possible values of T_f for UB (2.76) or ULS defined by ratios of extreme values of uncertain parameters c_K, or c_d and lying in the required performance area. During this step, from identified values of particular UB (2.68) one has to recalculate the task from fixed controller tuning $K_0, T_{d,0}$ and variable plant parameters K, T_d to fixed limit loop values (2.68) and variable controller tuning T_f, K_0 and $T_{d,0}$ corresponding to the optimal position of UB according to

$$K_0 = \kappa_{min}^{opt} K_{max}\,; \; T_{d,0} = T_{d,min}/\tau_{min}^{opt}\,; \; IAE_{mean} = T_{d,0} IAE_{mean}^{opt} \qquad (2.77)$$

By increasing the tolerable deviation from monotonicity to $\epsilon_y = 0.05$ (Fig. 2.23), the transient run faster, but simultaneously the additional control effort expressed by increased u-TV$_0$ value occurs.
In both analyzed cases, due to the relatively rough quantization, the achieved overshooting of the step responses is not absolutely close to the tolerable value. A direct increase of points in one dimension in generating the the performance portrait leads in 3D PP to cubic increase of the total number of points. But, from the shapes of ϵ_y-MO areas of the PP it is evident that it is allways possible to set $K_0 = K_{max}$ and reduce the whole design procedure to 2D space (τ_d, τ_f).

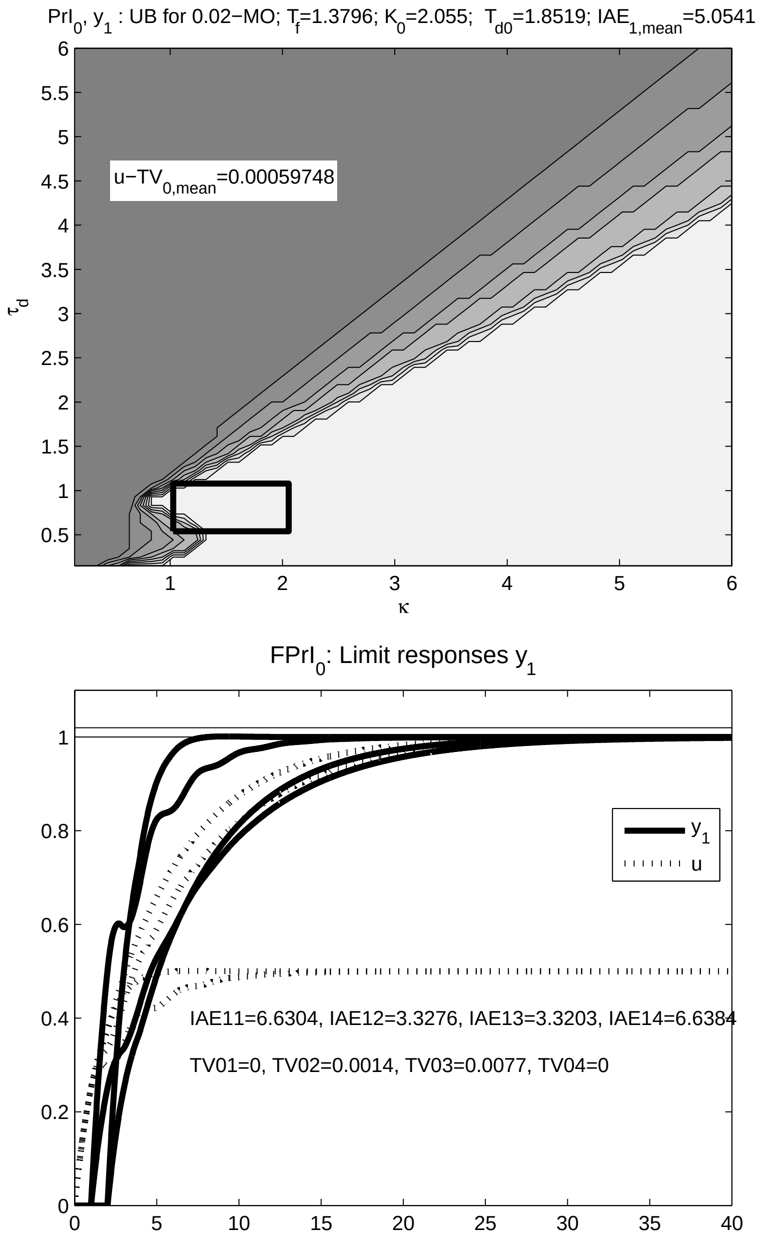

Figure 2.22: Result for seeping an optimal UB of the plant (2.68) with $K_{min} = 1, K_{max} = 2, T_{d,min} = 1, T_{d,max} = 2$ over PP of 61x61x11 points, areas of ϵ_y-MO output step responses identified for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$, white denoting the best performance, with identified parameters $T_f = 1.3796, K_0 = 2.005$ and $T_{d,0} = 1.8519$ (above) and the corresponding transients (below)

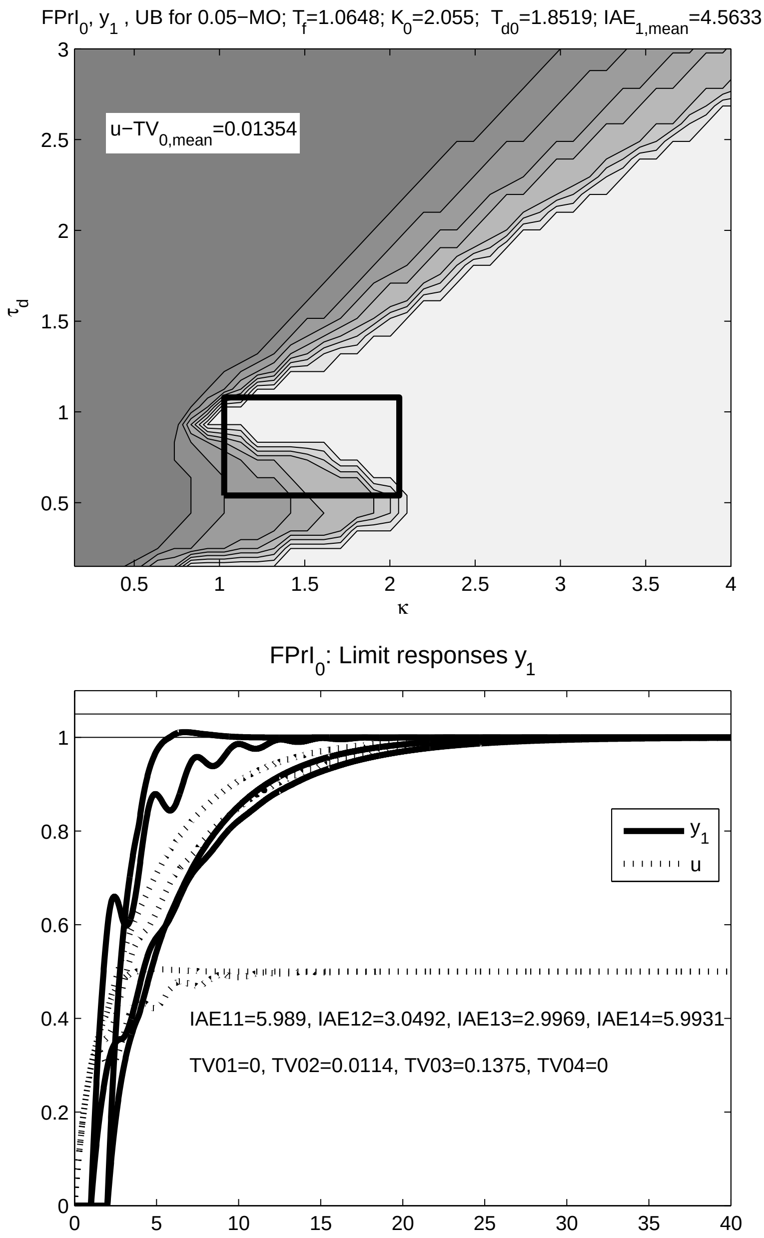

Figure 2.23: Result for seeping an optimal UB of the plant (2.68) with $K_{min} = 1, K_{max} = 2, T_{d,min} = 1, T_{d,max} = 2$ over PP of 61x61x11 points with identified parameters $T_f = 1.0648, K_0 = 2.005$ and $T_{d,0} = 1.8519$; areas of ϵ_y-MO output step responses identified for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$, white denoting the best performance (above) and the corresponding transients (below)

Example 2.15 (PrI$_0$ Controler for Plant from Example 2.9).
This illustrative example compares robust design of the PrI$_0$ controller with the FSP (Normey-Rico and Camacho (2007); Example 6.1) with the robust design of I controller in Example 2.9. The uncertain plant to be controlled is (2.38). Its Performance Portraits achieved by the FSP and I controller are given by Fig. 2.10 and Fig. 2.11. For tuning the FPrI$_0$ controller, similarly as in (2.42) the equivalent dead time will be determined by using information about the sum of the plant time constants as

$$T_d = L + S\,;\; S = \sum T_i = 1.875\,;\; T_d \in \langle 10.875, 13.875\rangle \tag{2.78}$$

According to this, the optimal UB (Fig.) was specified in 3D PP of the plant (2.68) by tuning parameters

$$T_f = 4.7359\,;\; K_0 = 1.584\,;\; T_{d0} = 11.6935 \tag{2.79}$$

Due to the relatively rough quantization, the calculated gain K_0 is rather overestimated with respect to $K_{max} = 1.2$ and so the achieved overshooting of the step responses is not absolutely close to the tolerable value. Despite to this the achieved results are comparable with those achieved with retuned Filtered Smith Predictor based on the first order plus dead time model (Huba, 2011). Also now, explanation for this surprising result, when the extremely simple model gives excellent results, may be taken from the same source as this example (Normey-Rico and Camacho (2007), pp. 174): "when the dead-time is dominant, the contribution of the open loop poles to the closed loop response will be small thus their elimination will contribute with a small increment in the speed of the transients". PrI$_0$ and FPrI$_0$ fully respect this fact and will surelly find top position in many industrial applications.

2.4 Summary

1. Known input disturbances may be compensated by opposite signal at the controller output. Output disturbances may be compensated by opposite signal correcting the reference setpoint value of the controller.

2. By extending the static feedforward control of a memoryless plant by disturbance observer (DO) for reconstruction of disturbances one gets the generic structure of the I$_0$ controller. Different stable low-pass filter can be chosen with respect to the measurement noise filtration and loop robustness. Continuity of the setpoint step response may be achieved by using prefilter for the setpoint variable.

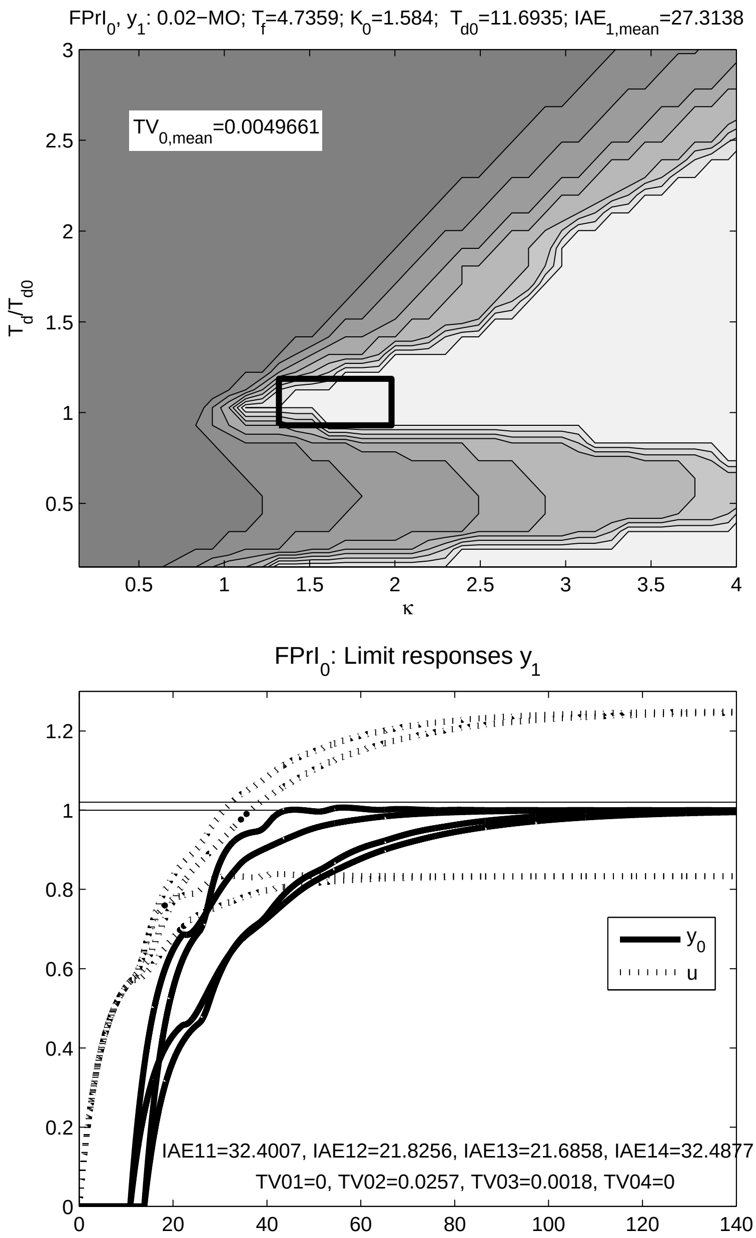

Figure 2.24: Result for sweeping IAE optimal UB for y_1 and 0.02–MO of the plant (2.38) with $K_{p,min} = 0.8, K_{p,max} = 1.2, L_{min} = 9, L_{max} = 12$ approximated by T_d (2.78) over PP of 61x61x11 points with identified parameters $T_f = 4.7359, K_0 = 1.584$ and $T_{d,0} = 1.8519$; areas of ϵ_y-MO output step responses of y_1 identified for tollerances $\epsilon_y = \{0.1, 0.05, 0.02, 0.01, 0.001, 0.0001, 0.00001\}$, white denoting the best performance (above) and transients corresponding to the limit uncertain parameter values (below)

3. In loops with strictly memoryless plant represents the I_0 controller a fundamental solution – the DO filter time constant may be arbitrarily small (the gain of the equivalent I_0 controller infinitely large) and the corresponding transient responses infinitely fast.
4. In tuning real loops with memoryless plant it is important to estimate the every time present nonmodelled loop dynamics. This can be approximated by dead time, by time constant, or by more complex dynamics. Controller parameters corresponding to the fastest non-overshooting and monotonic control may be well approximated by analyzing conditions of double real dominant close doop pole (DRDP). Approximations by dead time usually lead to faster monotonic transients than approximations by time constant.
5. Tuning of the I_0 controller gain is equivalent to simultaneous tuning of the DO filter time constant T_f used in disturbance reconstruction and tuning of the reciprocal gain of the feedforward control. For achieving setpoint step responses with defined overshooting, maximal dead time values and minimal plant gains have to be identified.
6. Tuning of the I_0 controller brings several degrees of freedom. One can decide about dynamics of the control signal corresponding to a setpoint step that may either have stepwise character (achieved by using controller according to Fig. 2.1) or softer exponential one (given by controller in Fig. 2.12 with prefilter time constant $T_p = T_f$, or by controller in Fig. 2.13 without prefilter). The nonmodelled loop dynamics may be approximated by a dead time, by a time constant, or by more complex transfer function. The loop dynamics may be approximated by providing a step response experiment, by measurement on stability border, by relay experiment, etc.
7. In control loops with a memoryless plant and one (stable) dominant time constant it is possible to cancel its effect in DO by filtered inverse of this dominant loop dynamics that gives structure of the PI_0 controller. For a neglected nonmodelled dynamics it represents a fundamental solution – ideally, the DO filter time constant may be arbitrarily small (the equivalent I_0 controller gain may be infinitely large) and the corresponding transient responses may be infinitely fast. Different stable low-pass filters can be chosen with respect to the measurement noise filtration and loop robustness.
8. In nominal case, a reliable controller tuning has to respect the non-modelled loop dynamics that remains after cancelling the dominant

time constant. The dominant time plant constant effect on the reconstruction dynamics may also be balanced by adding its estimate into the branch leading from the controller output. In this way one gets IMC like structure of the PI_0 controller that has no more properties of fundamental solutions: its dynamics cannot be arbitrarily speeded up, just to a limit value given by the dominant loop time constant. This solution may, however, be interesting by low noise sensitivity and robustness against parameters uncertaintny.

9. Active compensation of the loop (plant) time constants by the inverse terms in the DO based PI_0 controllers may lead to increased sensitivity to the measurement noise. But, the loop sensitivity to parameters uncertainty may be decreased. Higher order models usually also give lower effect of the nonmodelled dynamics. Ideal controllers (corresponding to models with neglected nonmodelld dynamics) represent fundamental solutions enabaling to shift closed loop pole (observer pole) theoretically to minus infinity and so to speed up transients to stepwise changes of control signal and output variable. However, in all real loops it is necessary to limit admissible closed loop poles (filter poles) to values giving acceptable noise amplification, robustness to model uncertainty and nonmodelled dynamics.

10. For the closed loop with monotonic nonoverhooting control signal transients and for admissible inputs (reference signals and acting disturbances) the control saturation will never be activated and so it can be omitted from considered control structures. This enables to describe all problems considered within the dynamical class 0 (DC0) by linear control theory. Therefore, in dealing with linear PID control structures we will consider their use within the DC0, even in situations when for the sake of simplicity the index "0" was omitted.

11. Active compensation of dead time by inversion is not possible. In this case, the dead time introducing time shift of the measured output may be compensated by including estimate of dead time into the observer branch leading from the controller output. The disturbance will be reconstructed by the time delay, but its values will be not distorted by different time shifts of both DO branches. In this way it is possible to construct predictive I_0 controller (PrI_0) and its filtered version $FPrI_0$ equipped by a prefilter.

12. Introduction of DO based I action designed as reconstruction and compensation of input, or output disturbances plays a key role in designing

constrained integrating controllers for higher dynamical classes of control that do not exhibit integrator windup.

2.5 Questions and Exercises

1. Which controller is more sensitive to the measurement noise: the PI_0, or the PrI_0 one?
2. How could you define PID_0 controller for active compensation of two time constants?
3. Could you formulate alternative solutions to this problem?
4. Which criteria must fulfill proposed controllers to be considered as the fundamental ones? Do all solution proposed by you fulfill these requirements?
5. What does characterize index "0" of the dynamical class DC0?
6. How could you define $PrPI_0$ controller for active compensation of one time constant and of long dead time?
7. Could you formulate alternative solutions to this problem?
8. Which criteria must fulfill propoes controllers to be considered as the fundamental ones? Do all solution proposed by you fulfill these requirements?

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Chapter 3

Tasks for Controlling the Thermo Optical Plant

Tuning of simple controllers respecting input constraints will be considered in this chapter and verified by controlling simple physical device of thermo-optical plant. The chapter starts with short user's manual and installation guide to the uDAQ28/LT device which will be used as a real plant to apply the control on. In the introduction, several fundamental controllers of the Dynamical class 0 (DC0) will be considered that avoid control saturation by guaranteeing monotonic transients among steady states at the controller output. Processes of the DC0 are typically met in situations, where the dynamics of transients may be neglected, i.e. it is not connected with a reasonable energy accumulation. In such cases, the ideal control response following a setpoint step may also converge to step function (not having a saturation phase). Controllers of the DC0 may also be successfully applied to any stable plant, but in such situations it is no more possible to speed up the control signal transient up to the step function, just to keep it monotonic that guarantees that all such controllers may again be successfully treated by the linear theory as well. In the second part, basic structures of the DC1 will be introduced that may already typically have one constrained period in their control signal step responses. Here, control structures with integral action based on disturbance observers will be introduced that do not exhibit windup phenomenon and so enable simpler one-step tuning that in the case of traditional linear controllers extended by the anti-windup circuitry.

3.1 Thermo-optical Plant uDAQ28/LT – Quick Start

This section gives a short guide to uDAQ28/LT plant. Get familiar with thermo-optical plant interface. For more information on the device please refer to the user's manual. This device offers measurement of eighth process variables (temperature and its filtered value, ambient temperature, light intensity, its filtered value and its derivative, the ventilator speed of rotation and its motor current). The temperature and the light intensity control channels are interconnected by three manipulated variables: the bulb voltage (the heat and light source), the light-diode voltage (the light

Figure 3.1: Found new hardware wizard

source) and the ventilator voltage (the system cooling). The plant can be easily connected to standard computers via USB, when it enables to work with the sampling periods 40-50 ms and larger. Within the Matlab/Simulink scheme the plant is represented as a single block, limiting use of costly and complicated software package for the real time control.

3.1.1 Installation in Windows Operating System

Device Driver Installation

New hardware is detected automatically by operating system and *Found New Hardware Wizard* will start after plugging device into electrical power network and connecting it to PC by USB cable (Fig. 3.1). Choose driver installation from specific location (Fig. 3.2)
Directory *driver* from the package of supporting software is needed to select as device driver location path (Fig. 3.3). Complete installation requires two cycles of adding new hardware (*USB Serial Converter* and *USB Serial Port* is installed into operating system, *Found New Hardware Wizard* starts automatically second time). The user is informed on successful completion of installation, see the window on the Fig. 3.4.
When the device has been installed, (virtual) serial port is available on the list of hardware devices, labeled as USB Serial Port. One can get to its settings through Device Manager (Start / Settings / Control Panel / System / Hardware / Device Manager) if Ports (COM & LPT) / USB Serial Port is selected (Fig. 3.5). The port has automatically assigned by the system one of com port numbers (which is not in use). If the assigned

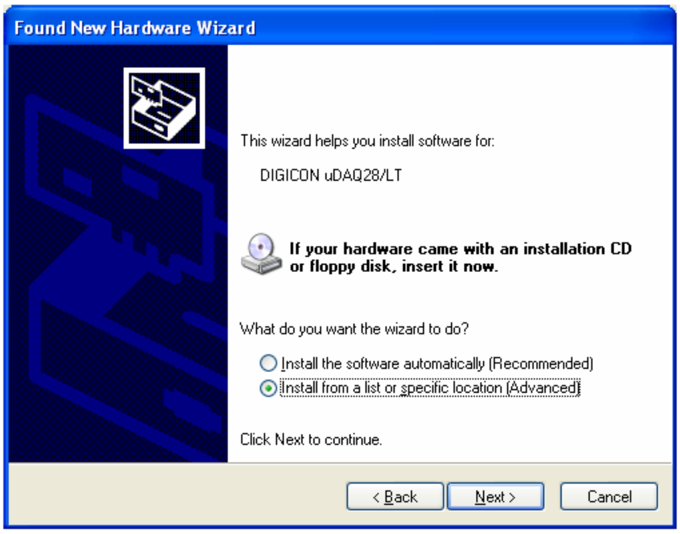

Figure 3.2: Install driver from specific location selection

Figure 3.3: Driver location selection

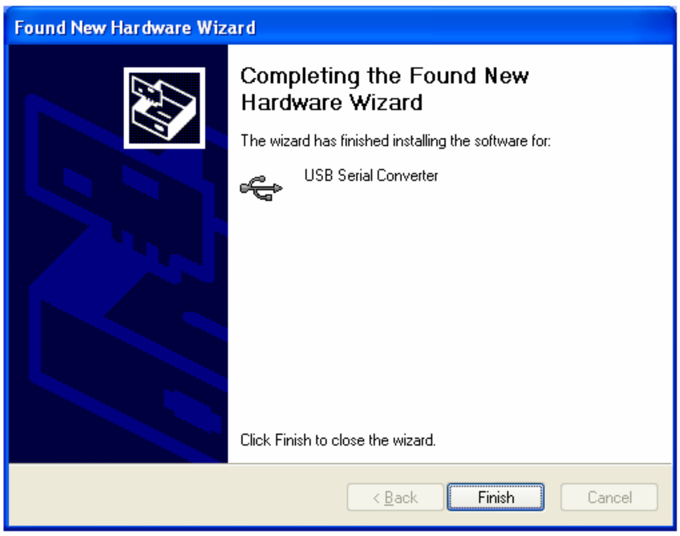

Figure 3.4: Successful completion of installation

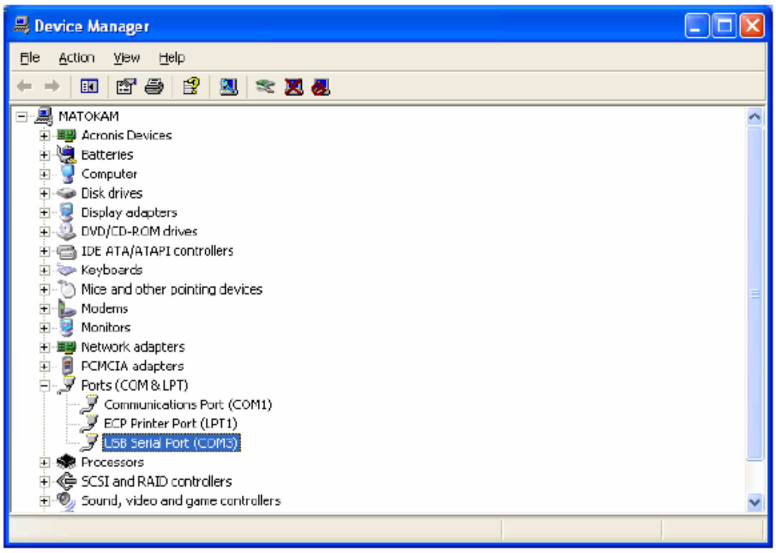

Figure 3.5: Device manager – USB serial port

number is bigger than 4, it is necessary to change it to different com port number from the range 1-4, which is not in use at the moment. It is possible to perform on the tab Port Settings / Advanced / COM Port Number (Fig. 3.6). The last action after installation to be done is setting parameter Latency Timer (msec) on value 1, which minimizes data loss during data transfer. Parameter Latency Timer (msec) is available on the same tab as COM Port Number (Fig. 3.6). In the case that all com port numbers from the range 1-4 are already in use by other applications or programs, first assign (change) com port number bigger than 4 to one of those programs and then assign available number from 1 to 4 to USB Serial Port.

Driver and Software Package Installation in Matlab

To install driver and software package in Matlab, follow these instructions:

- Before installation it is good to make sure that any unterminated process matlab.exe is not in the memory; if it is, terminate it (tab Processes in Task Manager, activated by keys CTRL+ALT+DEL)
- Run Matlab
- Change the working directory to the directory from the package of supporting software where installation file udaq_setup.p is located (e.g. by typing command cd e:\dirname)
- Run file udaq_setup.p by typing command 'udaq_setup'

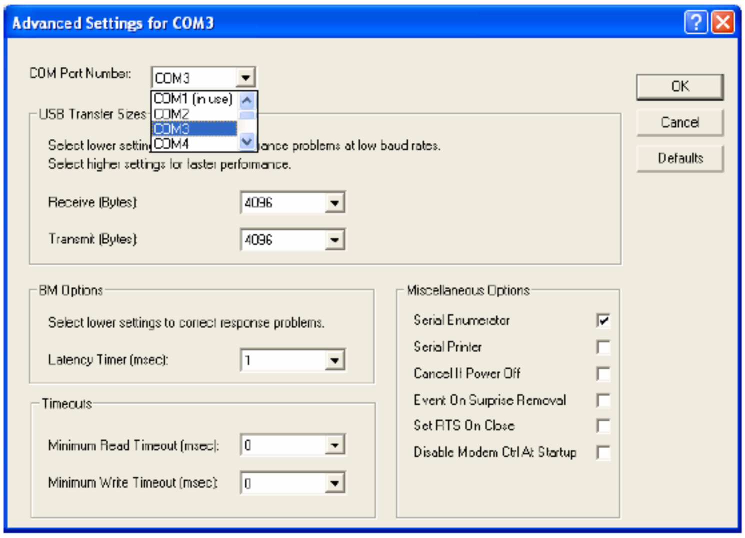

Figure 3.6: Assignment COM port number to USB Serial Port, Latency Timer setting

After successful completion of installation there is directory *matlabroot/udaq/udaq28LT* and particular subdirectories and files copied on the local hard drive. (*matlabroot* represents the name of the directory displayed after typing command 'matlabroot' in Matlab command window) Two mdl files are open after installation in Matlab: *udaq28LT_iov2.mdl*, located in *matlabroot/udaq/udaq28LT/examples* and the library containing two blocks (drivers), represented by mdl file *matlabroot/udaq/udaq28LT/blks/udaq28LT_lib.mdl*. You can create your own simulation mdl file that communicates with thermo-optical plant by copying the driver block from the library (or from other functioning mdl file) into your own mdl file.

Thermo-optical Plant Communication Interface

Thermo-optical plant communication interface is represented in Matlab by one of the blocks udaq28LT_v1R13 (Fig. 3.7) or udaq28LT_v2R13 located in the library matlabroot/udaq/udaq28LT/blks/udaq28LT_lib.mdl. Double clicking on the udaq28LT_v1R13 block brings up the block parameters menu (Fig. 3.8).

Measurement and Communication System

The inputs and outputs of the communication interface refer to these signals.

Inputs:	Bulb	0-5 V to 0-20 W of light output
	Fan	0-5 V to 0-6000 fan rpm

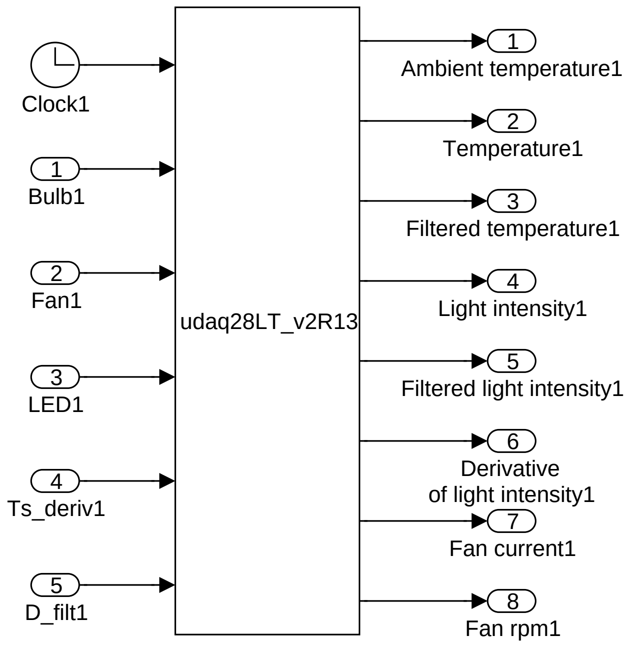

Figure 3.7: Communication interface block in Simulink

LED	0-5 V to 0-100% of LED light output
T, D	microprocessor inputs for the purpose of calculation of the first light light channel derivative (sample period the microprocessor samples light channel with – minimal possible value is 1 ms and coefficient of actual sample for the discrete filter of the first order with accuracy of 3 decimal positions)

Outputs:	Ambient temperature	
	Temperature	sensor PT100 range $0 - 100\,^\circ$C accuracy: better than 99%
	Filtered temperature	(1st order filter with time constant cca 20 s)
	Light intensity	
	Filtered light intensity	(1st order filter with time constant cca 20 s)
	Filtered derivative of the first light intensity channel	
	Current consumption by fan (0-50 mA)	
	Fan revolutions (0-6000 rpm)	

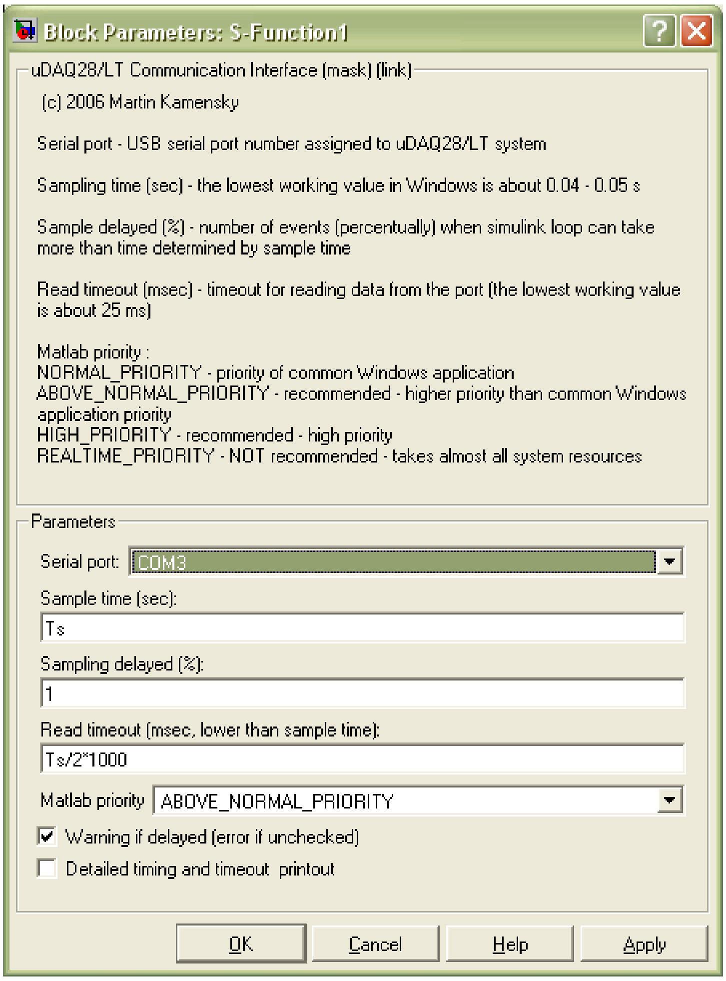

Figure 3.8: User dialog window of communication interface

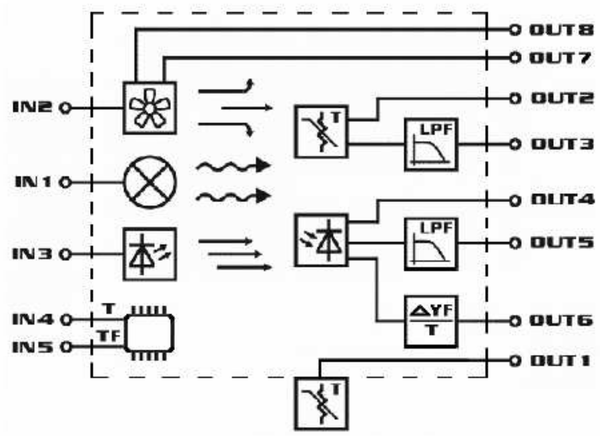

Figure 3.9: Basic electrical diagram of thermo-optical plant uDAQ28/LT

Operating temperatures range:	$0 - 70\,^{\circ}\mathrm{C}$
Power supply:	12V/2A DC (external adapter)
Communication interface:	USB – virtual serial port
Data transfer speed:	250 kbit/s

3.2 Light Channel Control

The non-filtered and filtered light channels of uDAQ/28LT plant are going to be controlled in this section. Simple alternatives to linear I-controllers will be practiced.

3.2.1 Feedforward Control

Tasks:

- Identify non-filtered light channel parameters.
- Control non-filtered light channel using inverse process gain.
- Analyze the steady state error for various setpoint changes.
- Measure the I/O characteristics of the non-filtered light channel.

Let us start with getting to know the light channel characteristics. The non-filtered light channel represents a very fast process which can be approximated as memoryless plant. In an ideal case static feedforward control with inverse process gain should be sufficient for such process. Measure one point of the I/O characteristic to obtain the process gain.

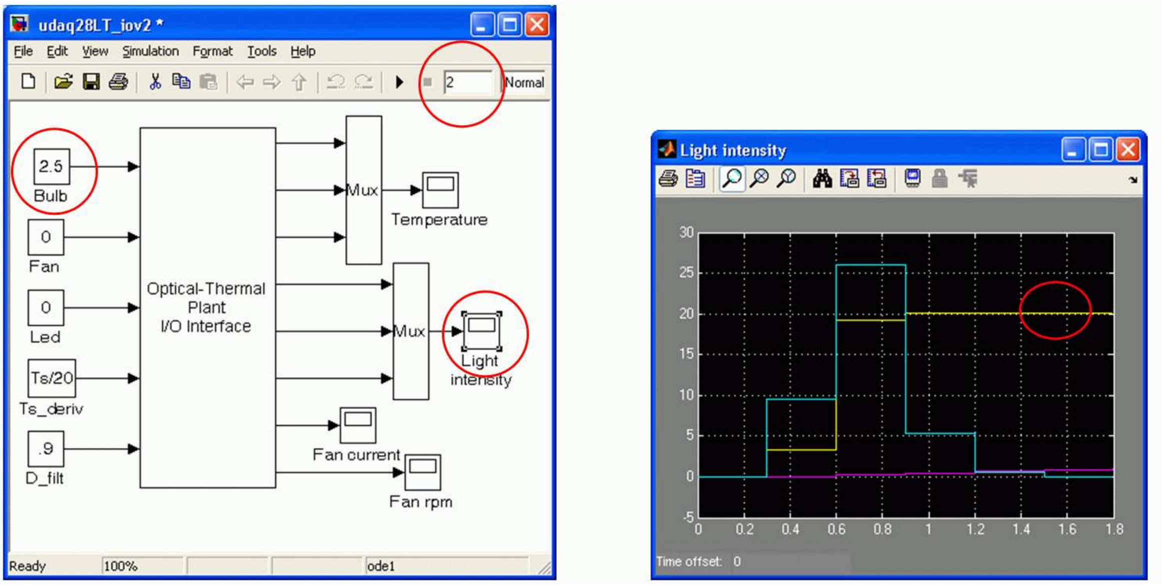

Figure 3.10: Basic Simulink model for the udaq28/LT plant

The basic I/O Simulink model (matlabroot/udaq/udaq28LT/examples/udaq28LT_iov2.mdl) can be used. As other alternative use the *exnum* command and choose experiment no.1 which opens up basic I/O Simulink model of the plant. Set the bulb voltage to $u_s = 2.5\,\mathrm{V}$ and run the experiment for 2 s. Put down the steady state value of the light intensity. By default it is represented by the yellow transient in the light intensity scope (Fig. 3.10).

The steady state value of the light intensity in this example is approximately $y_s = 20$. The process gain can be than computed as $K = y_s/u_s$. In this example it gives

$$K = \frac{y_s}{u_s} = \frac{20}{2.5} = 8 \tag{3.1}$$

Modify the Simulink model to use the inverse process gain to control the plant (Fig. 3.11). Do not forget to add the input saturation in the model, because the bulb voltage is limited from 0 V to 5 V. Add the setpoint signal to the light intensity scope. Set the simulation time to infinity.

Make multiple setpoint steps in a wide operational range. It can be done while the experiment is running.

You should observe a steady state error in several working points. The smallest steady state error can be seen around the point where the process gain was measured. It is not difficult to conclude that the process parameters vary through the operational range, in other words the I/O characteristics of the non-filtered light channel is not linear.

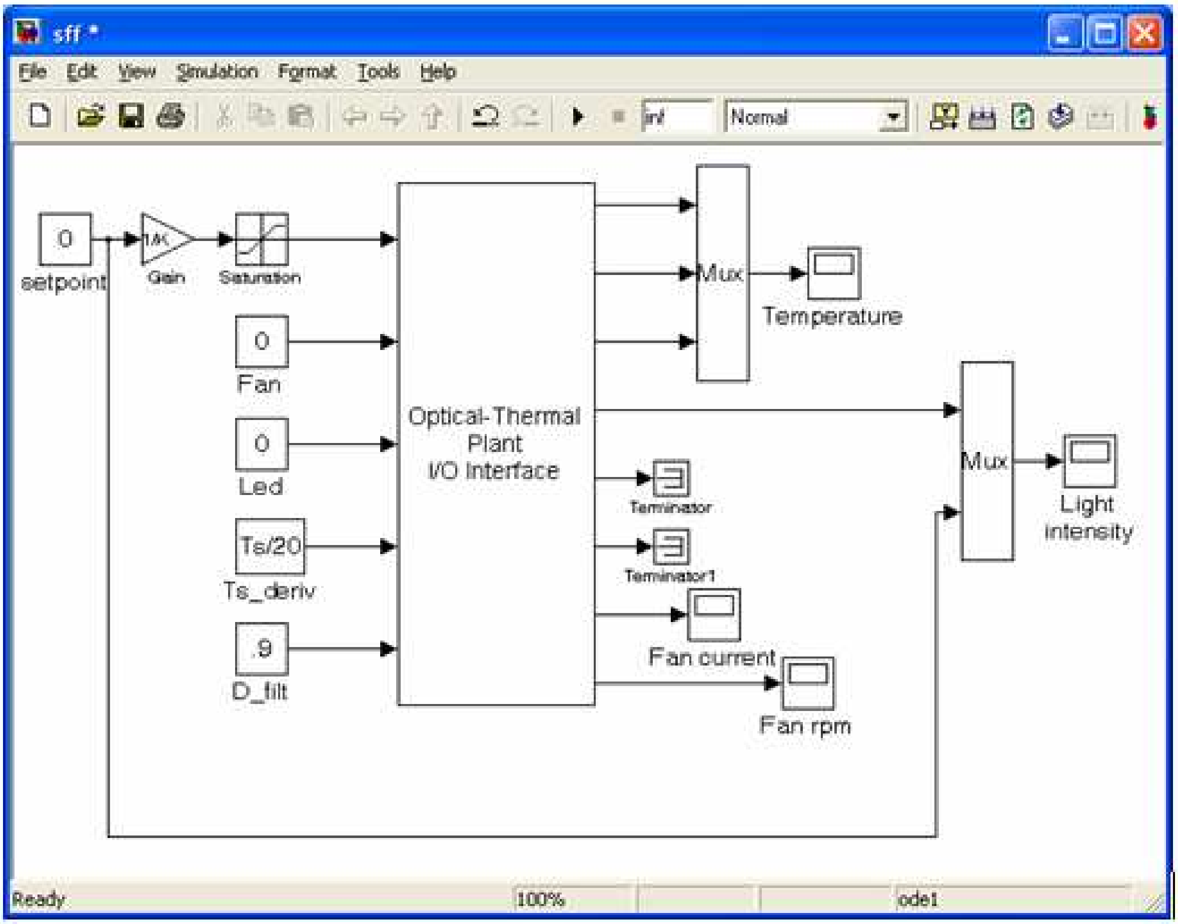

Figure 3.11: Static feedforward control

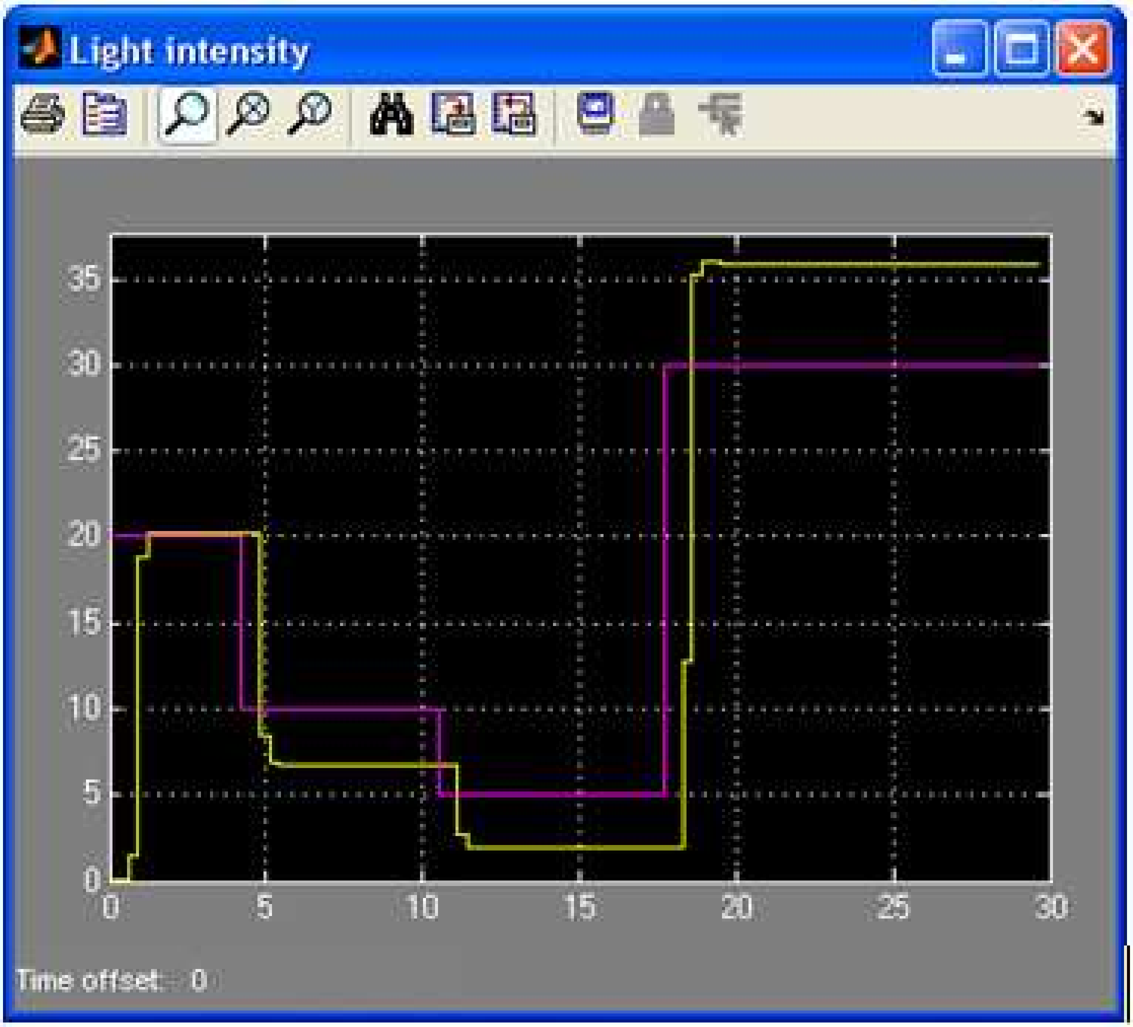

Figure 3.12: Experimental results – Light intensity scope

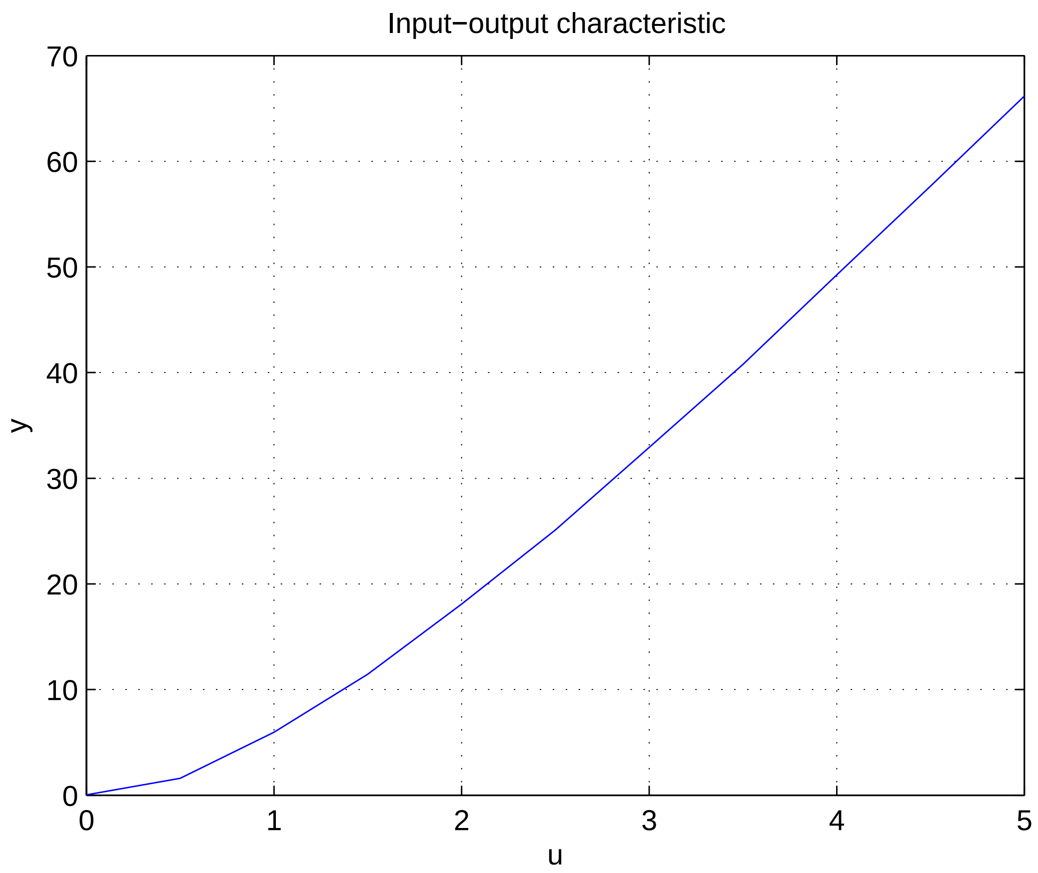

Figure 3.13: I/O characteristics measurement results

The exnum command can be used to measure I/O characteristics of the plant and to obtain the process parameters in multiple working points. Choose the experiment no.2 for I/O characteristics measurement. The following figures will give you the information on the I/O characteristic, the process gain and dead time through the operational range. Short delay in a light intensity change can be observed after a bulb voltage step. The uDAQ28/LT device converts bulb voltage steps into a steep ramp to prevent undesired disturbances in the plant. Let us approximate this delay as a dead time. After running the I/O characteristics measurement plot the step responses from which the I/O characteristics of the light channel was obtained by the following commands:

```
stairs(yl(:,1),yl(:,2))
xlabel('t[s]')
ylabel('light intensity')
title('Step responses of non-filtered light channel')
```

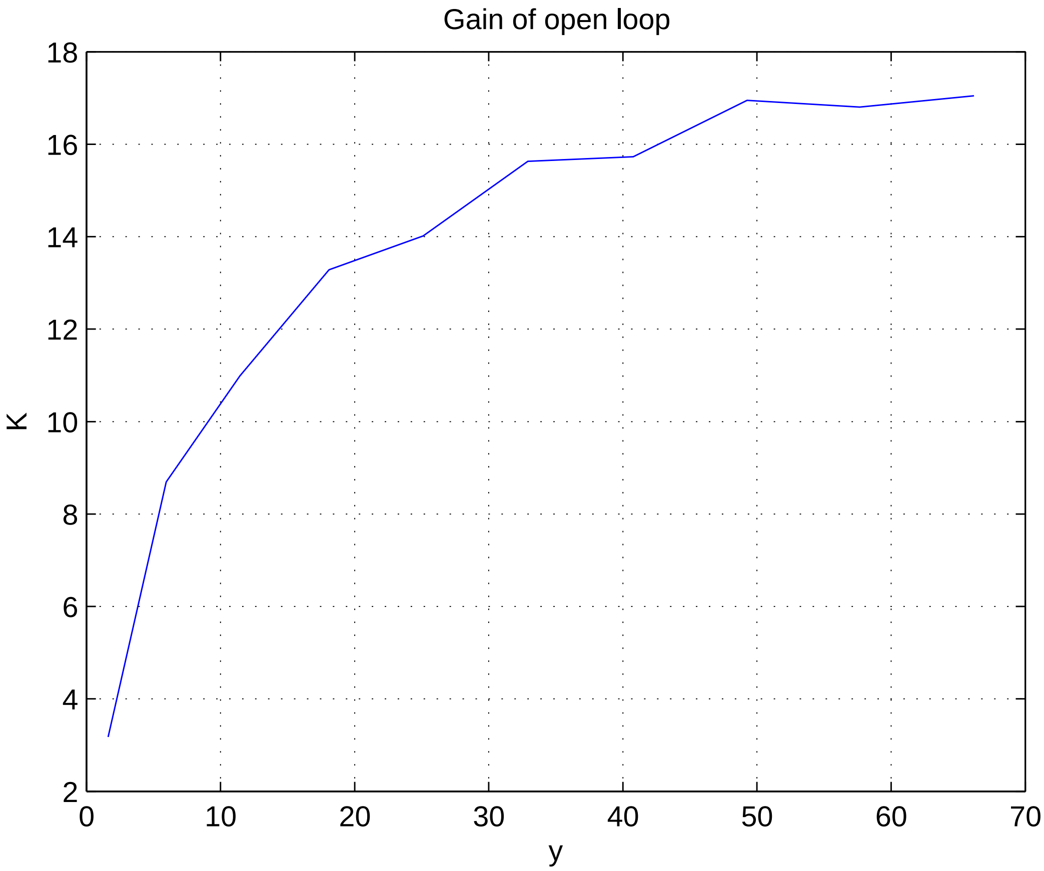

Figure 3.14: Process gain in multiple operating points as function of the output variable

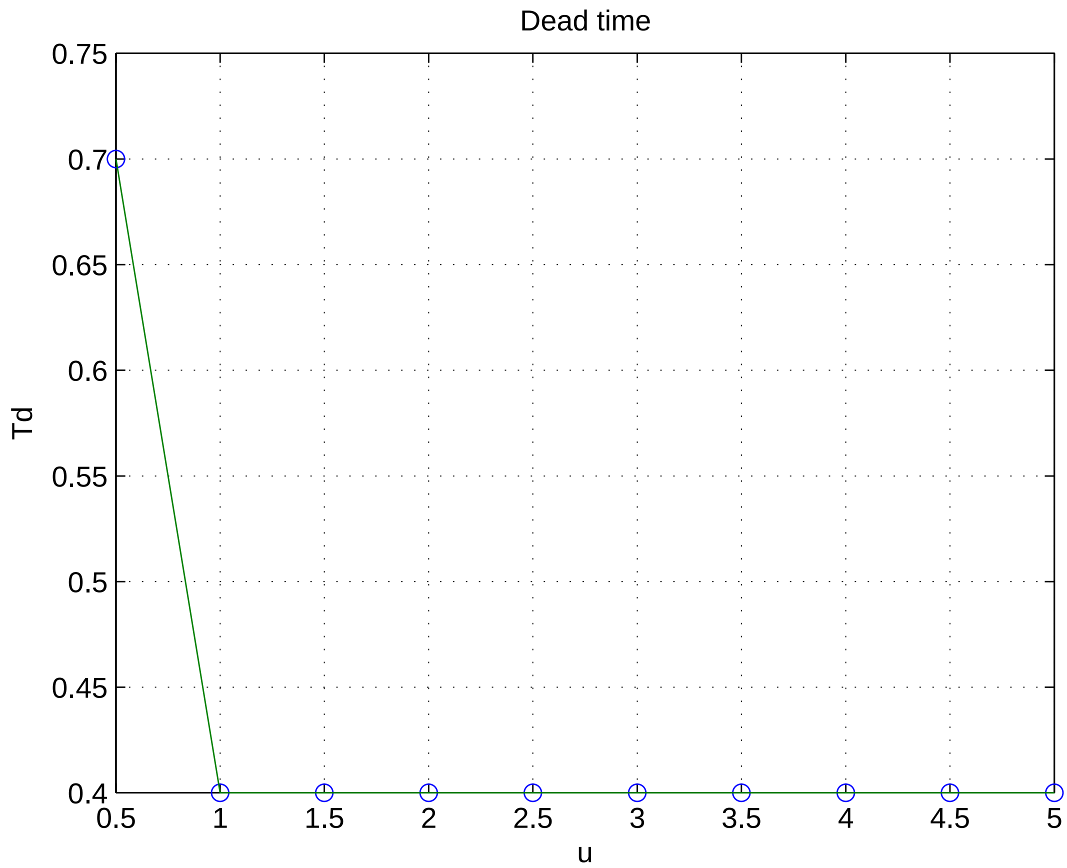

Figure 3.15: Dead time in multiple operating points as function of the input variable

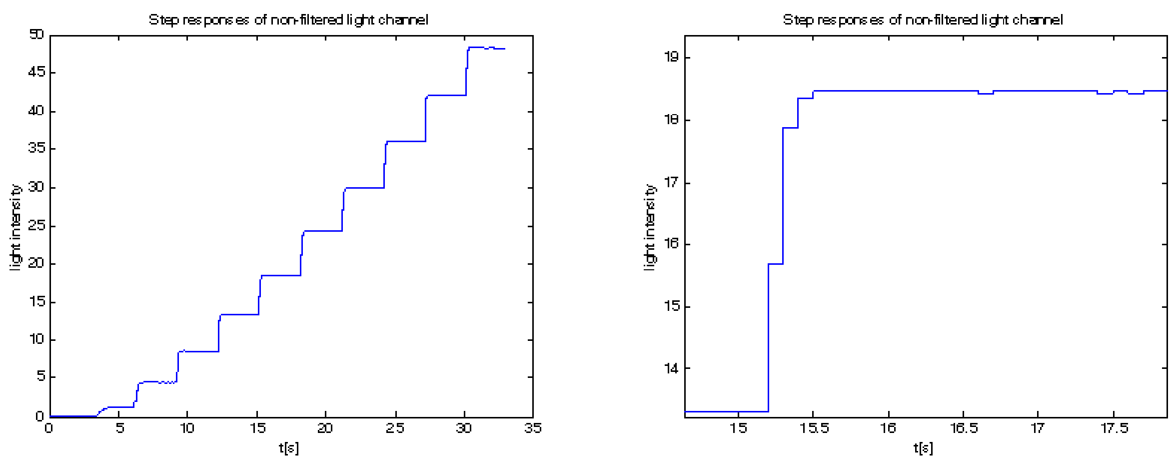

Figure 3.16: Step responses of the non-filtered optical channel – overview and a detail of one step response. These are the responses to 0.5V bulb voltage steps made in 3s intervals.

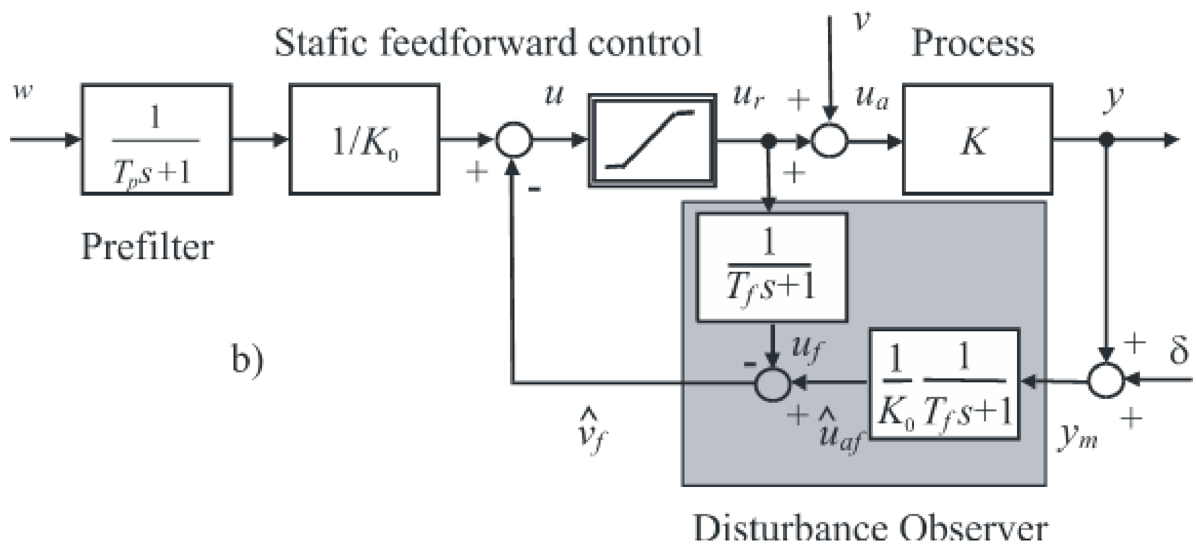

Figure 3.17: FI_0-controller, structure equivalent for $T_p = T_f$ to I-controller

3.2.2 I_0 Controller

Tasks:

- Add a disturbance observer to the feedforward control to compensate steady state error.
- Use the I/O characteristics measurement results to tune the controller by hand.
- Tune the controller using the performance portrait method.
- Make a step disturbance by a LED voltage step during the experiments.
- Compare the results with various controller tuning.

The disturbance observer can be added to the static feedforward control to compensate the steady state error. To obtain a structure equivalent to I-controller, the pre-filter with time constant equal to the observer time constant has to be added as well. Let us denote the controller as I_0-controller (Fig. 3.17) and the controller with pre-filter as FI_0.

Tuning of the I_0 and FI_0-controller requires information on a process gain and approximation of the non-modeled dynamics – usually by the dead time. The approximation of the delay in this example is $T_d = 0.4$. The filter time constant of the disturbance observer is restricted by the parasitic time delays of the non-modeled dynamics in the process. Use following formula to set up the disturbance observer filter time constant:

$$T_{fil} = \mathrm{e}T_d \tag{3.2}$$

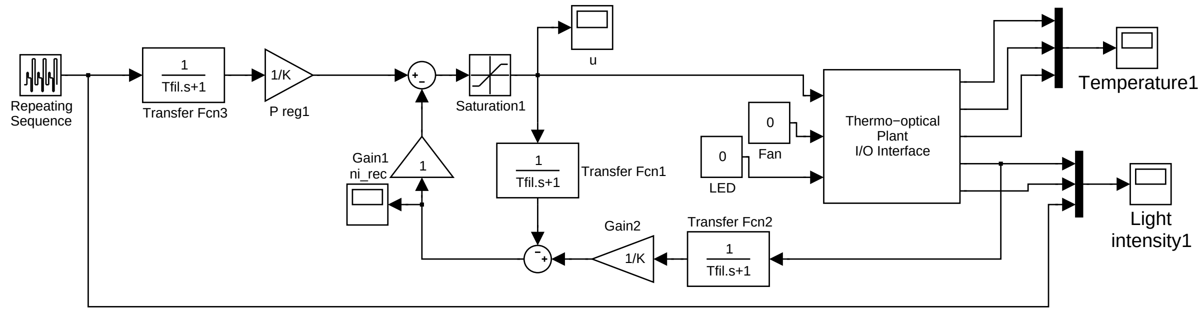

Figure 3.18: FI_0-controller – Simulink model

Try to use multiple process gains for controller tuning, to improve control quality. Use the lowest, the average and the maximum process gain from Fig. 3.13. Choose experiment no.3 with the exnum command to control non-filtered optical channel by FI_0-controller. You will be prompted for the process gain value and the delay time constant. In Fig. 3.18 there is the Simulink model which should pop up when the correct experiment starts. The experiment will run with multiple setpoint steps. Feel free to modify the model to make your own setpoint steps sequence. The goal is to achieve quick non-overshooting transients.

The experimental results can be plot using following commands.

```
figure
stairs(yl(:,1),yl(:,2), 'k')
hold on
stairs(yl(:,1),yl(:,4), 'k:')
xlabel('t[s]')
ylabel('light␣intensity')
legend('system␣output', 'setpoint')
```

Using the recommended three process gains in the experiment you should obtain similar results to Fig. 3.19, 3.20 Fig. 3.21,3.22, and Fig. 3.23,3.24.

For the plant used in this example the desired control quality was achieved when the process gain $K = 17$ was used. The control results can be seen in Fig. 3.23.

From the experience with the previous experiments one can assume that a higher value of the process gain leads to lower overshoot with slower transients and vice versa. After getting some experience with the controller, let us practice robust controller tuning. At first put down the intervals in which the process gain and the dead time range. The data from Fig. 3.14 and Fig. 3.15 can be used. In this example the dead time T_d ranges in interval [0.4,0.7] and the process gain ranges in interval [3,17]. Now de-

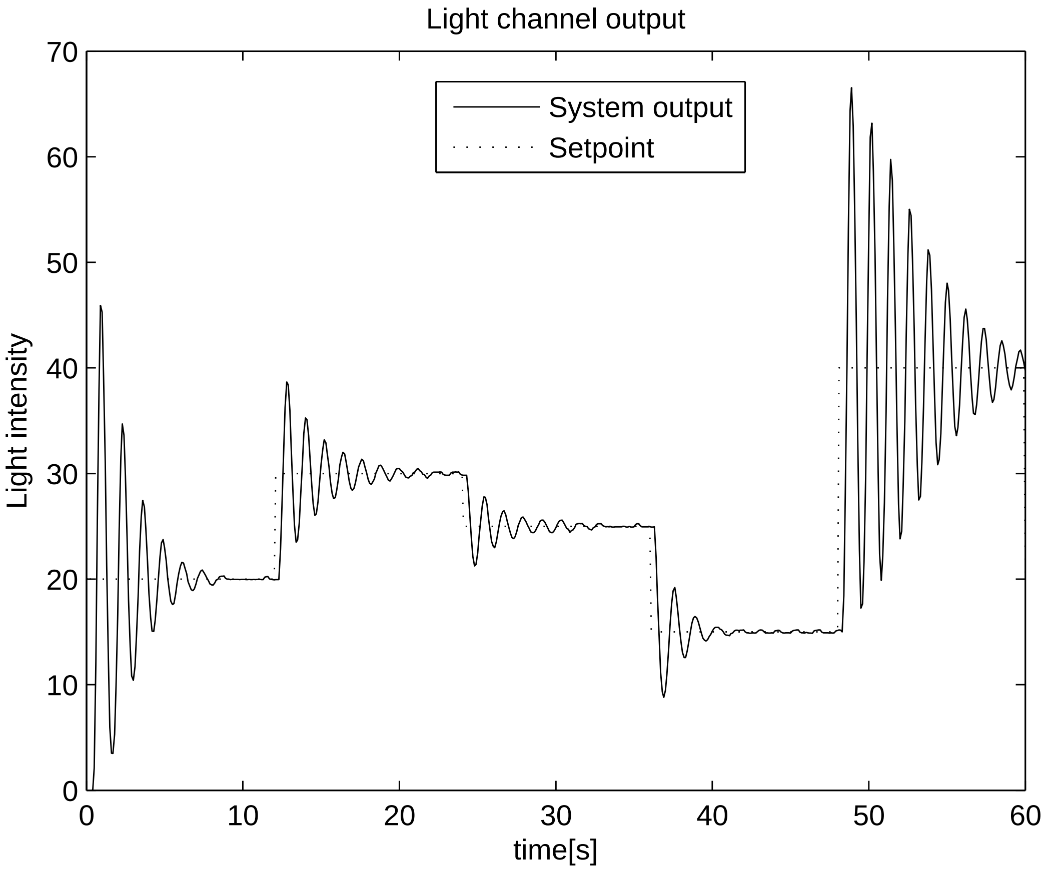

Figure 3.19: Experimental results for K=3

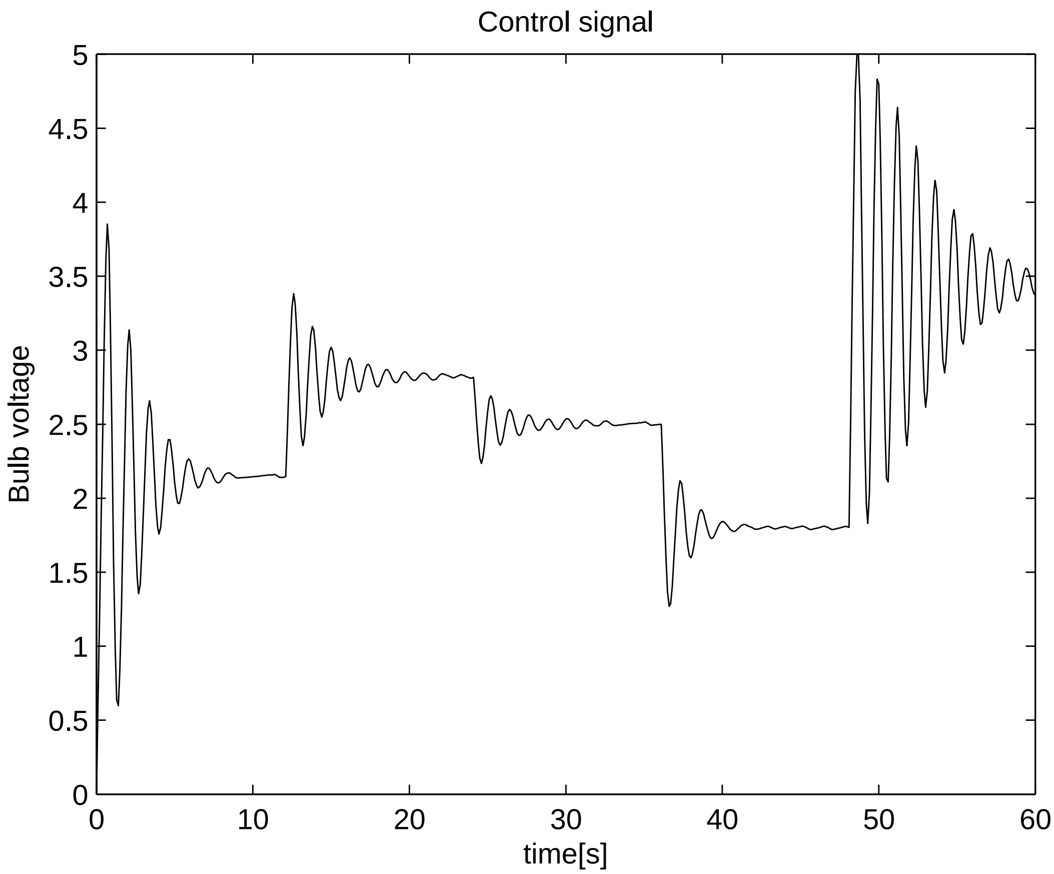

Figure 3.20: Experimental results for K=3

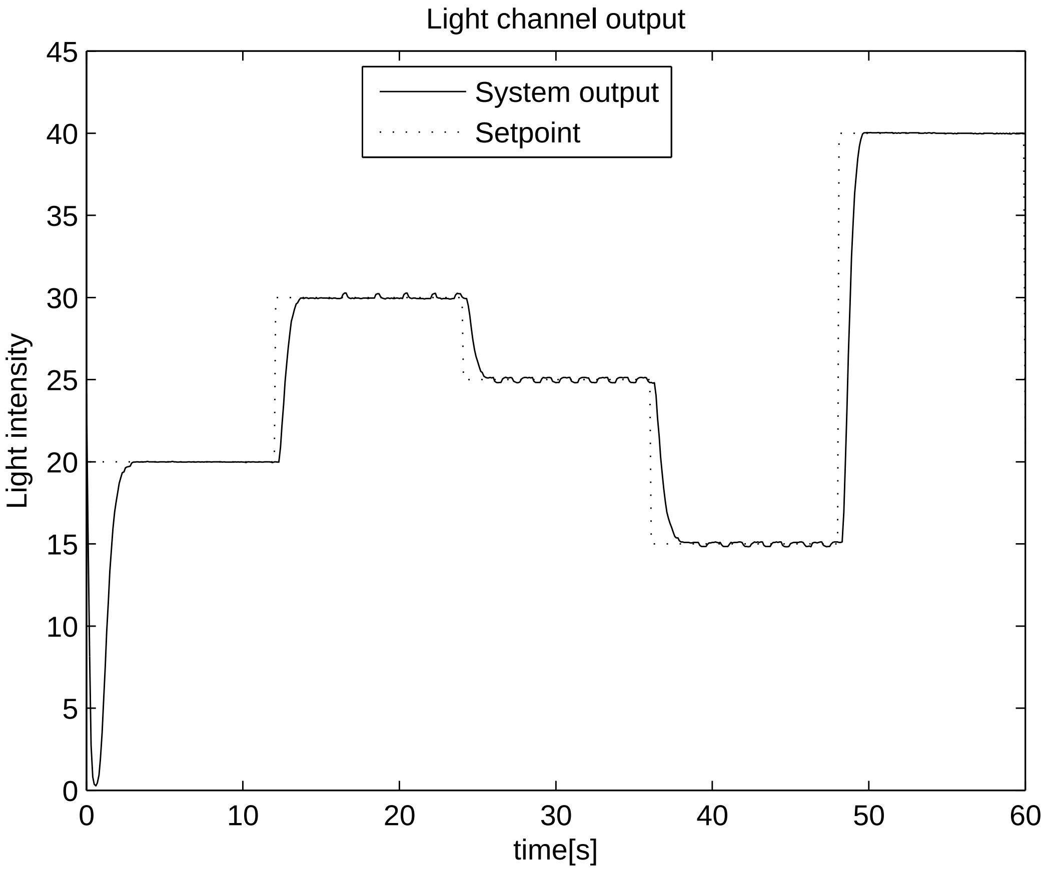

Figure 3.21: Experimental results for K=10

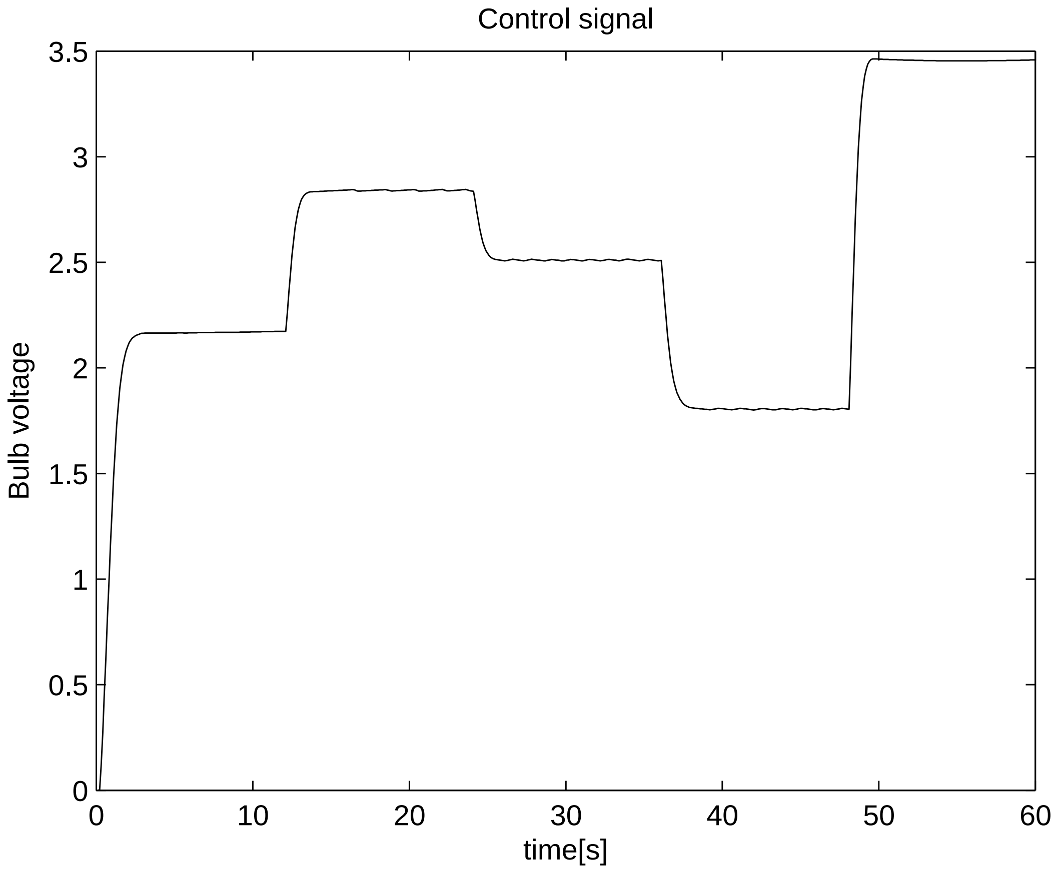

Figure 3.22: Experimental results for K=10

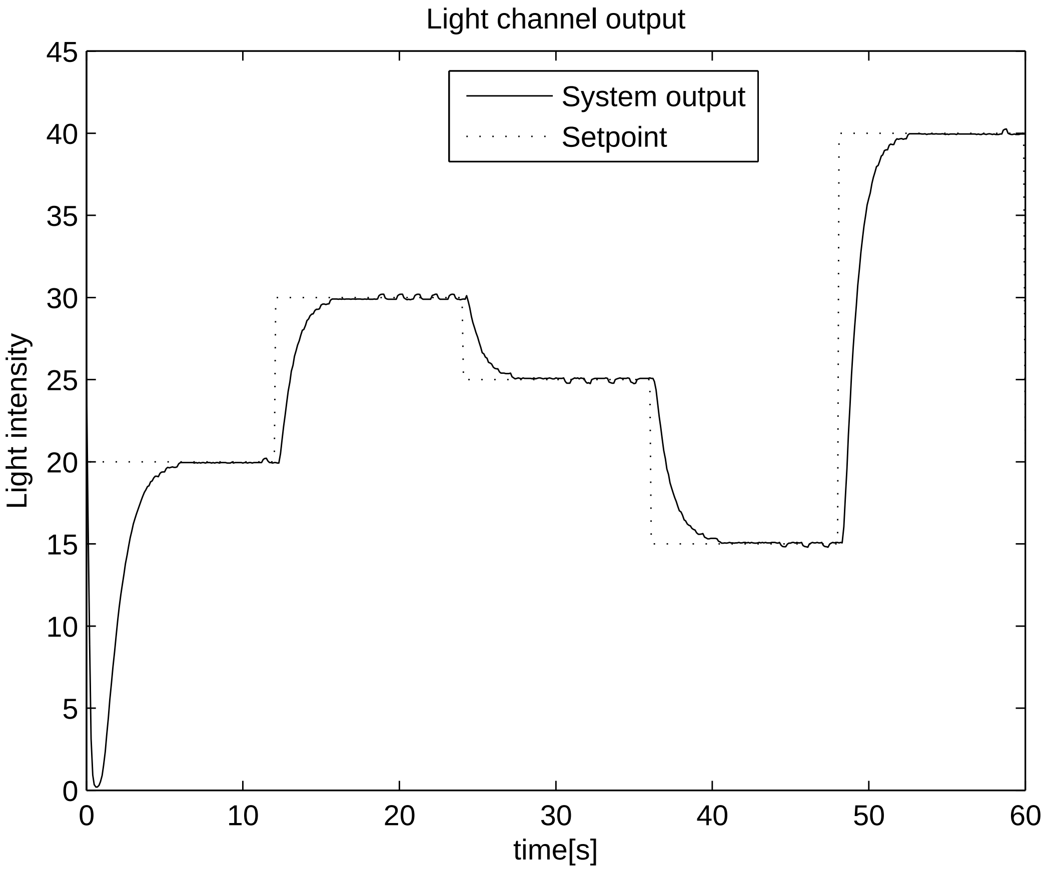

Figure 3.23: Experimental results for K=17

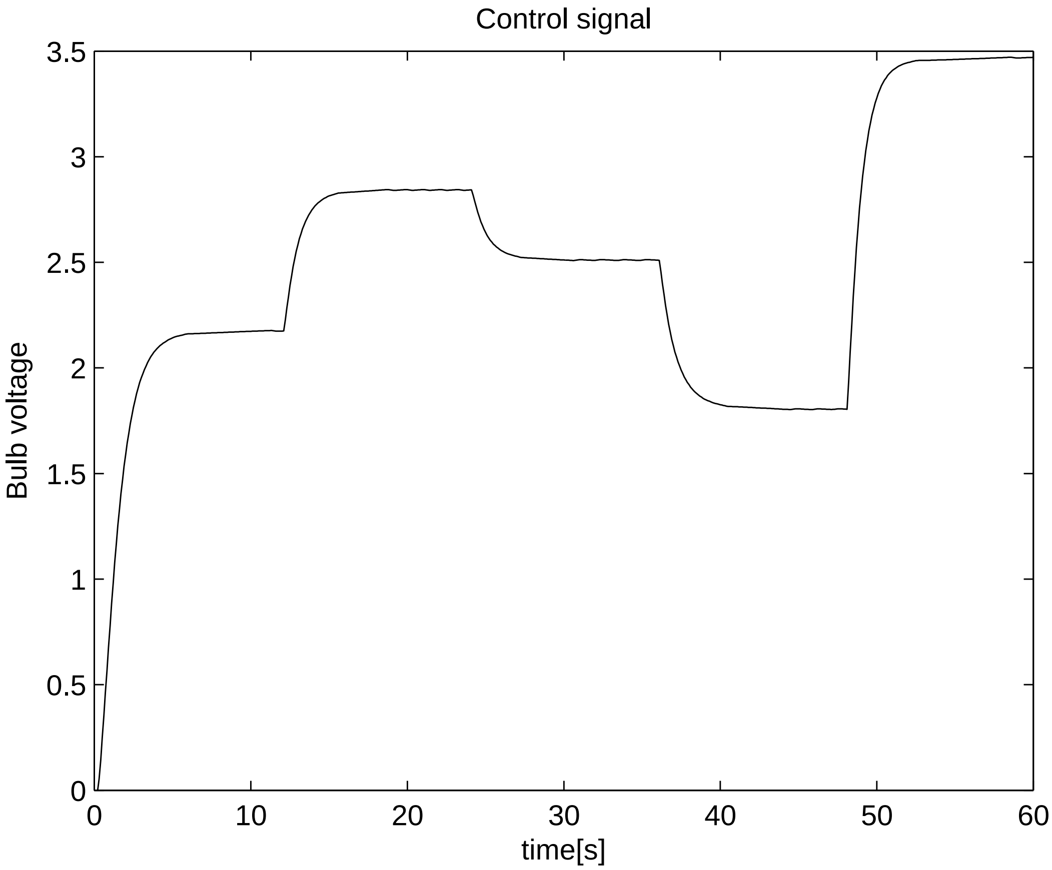

Figure 3.24: Experimental results for K=17

Table 3.1: Controller tuning

$\epsilon_y[\%]$	10	5	4	2	1	0.1	0.01	0.001	0	0
$\tau = \kappa/\Omega$	1.724	1.951	2.0	2.162	2.268	2.481	2.571	2.625	2.703	2.718
$q = \Omega/\kappa$	0.58	0.515	0.5	0.465	0.441	0.403	0.389	0.381	0.37	0.368

termine the interval in which normalized variable κ ranges. κ represents normalized variable

$$\kappa = K_0/K \tag{3.3}$$

where K_0 stands for the process gain used in the controller, K corresponds to real process gain, which varies through the operational range. The goal is to fit the uncertainty box into area of performance portrait (Fig. 3.25) where the controller gives monotonic transients. Try to use two limit values of process gain for K_0, in this example it gives

$$K_0 = K_{min} = 3 \tag{3.4}$$

which leads to $\kappa = [3/17, 1]$.
For the maximal value of K_0

$$K_0 = K_{max} = 17 \tag{3.5}$$

it gives $\kappa = [1, 17/3]$.

$$\Omega = T_d/T_f \tag{3.6}$$

Calculate filter time constant T_f for both selections of K_0. For K_0 (3.4) it yields

$$\Omega = 0.0649 \tag{3.7}$$

$$T_f = T_d/\Omega = 0.7/0.0649 = 10.7825 \tag{3.8}$$

For K_0 (3.5) it yields

$$\Omega = 0.3679 \tag{3.9}$$

$$T_f = T_d/\Omega = 0.7/0.3679 = 0.7e = 1.9028 \tag{3.10}$$

Verify controller tuning by real experiment for various setpoint steps. Add a disturbance using a LED to verify input disturbance compensation. You should observe similar transients using both controller tunings (Fig. 3.26). The 2 V LED voltage step was made at time 105 s. Following table summarizes the robust FI_0-controller tuning for various overshooting tolerances.
Questions:

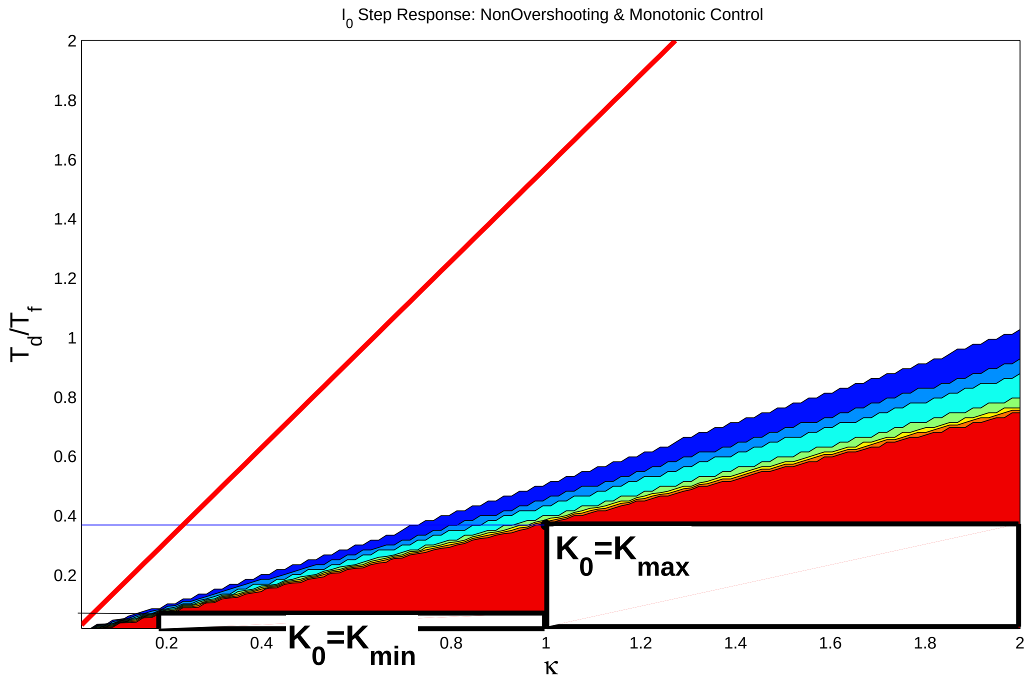

Figure 3.25: Performance portrait for FI_0-controller

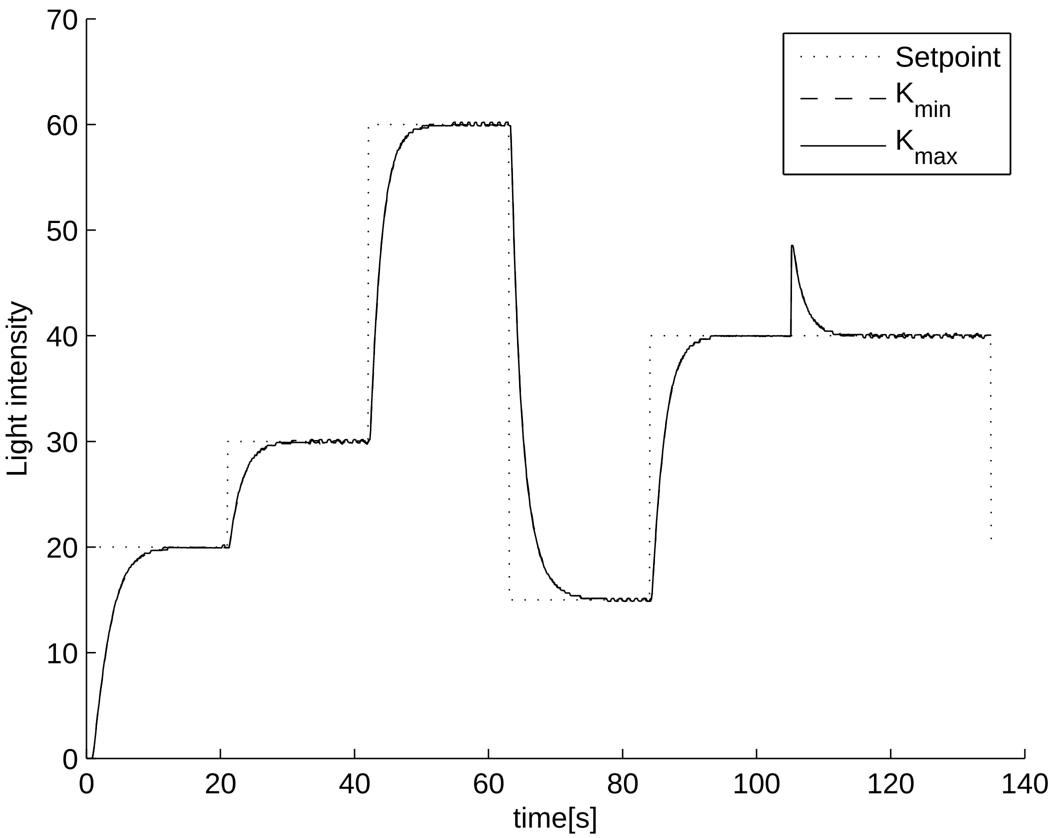

Figure 3.26: Real experimental results for FI_0-controller

- What type of plant is FI_0-controller suitable to control for?
- Which plant's parameter restricts the controller's dynamics?
- Was there any overshooting in the experiments?
- Why the performance of different controller tunings in Fig. 3.26 is almost the same?

3.2.3 Filtered Predictive I_0 Controller

Tasks:

- Add a dead time to the non-filtered light channel output.
- Modify the I_0-controller to compensate the delay.
- Tune the controller using a performance portrait.
- Make a disturbance by a LED voltage step during the experiments.
- Compare the control quality with the FI_0-controller using a real experiment.

Tuning of the closed loop systems involving dead-time still represents a challenging domain of control research. an increase of the dead-time values with respect to the dominant plant time constant leads in the loops with PID controllers to rapid performance deterioration. Therefore filtered predictive I_0 ($FPrI_0$)-controller will be used in this exercise. Under the $FPrI_0$-controller controller we will understand the static feedforward control with the gain $1/K_0$ extended by the input disturbance reconstruction and compensation (Fig. 3.27) with the disturbance observer filter time constant T_f and by the pre-filter with the time constant $T_p = T_f$.
Robust tuning of the $FPrI_0$-controller may again be done by the performance portrait. The information on plant parameters is needed. For the plant used in this exercises the performance portrait for undelayed plant output is in Fig. 3.29, the delayed plant output performance was analyzed in Fig. 3.30. Compare the performance of filtered predictive I_0-controller vs FI_0-controller. Use additional transport delay when controlling a non-filtered optical channel. In the following example 5 s transport delay was added to the non-filtered light channel output, so the transport delay of the plant ranges from 5.4 to 5.7 s. The process gain still range in interval [3,17]. For 1% overshooting tolerance it yields to filtered predictive I_0-controller

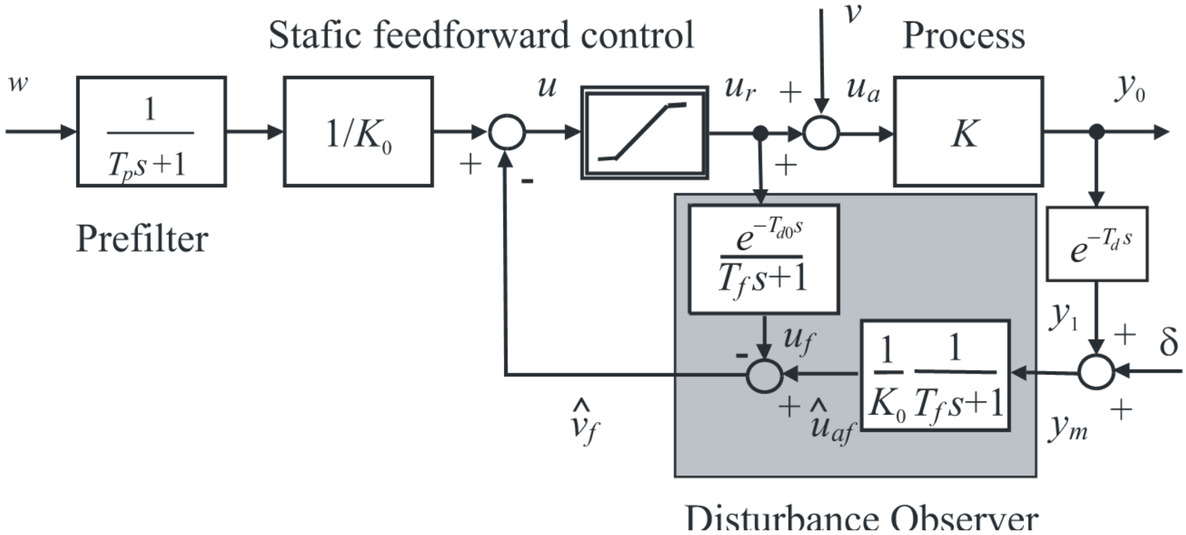

Figure 3.27: $FPrI_0$-controller

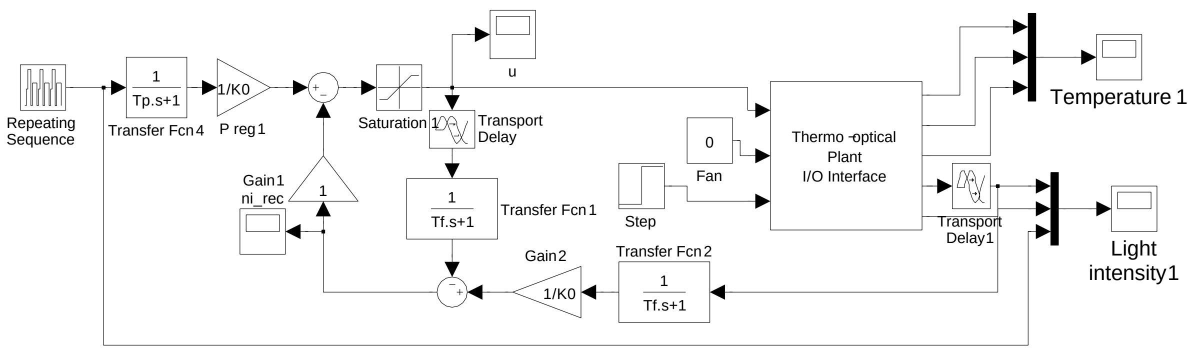

Figure 3.28: $FPrI_0$-controller - Simulink model

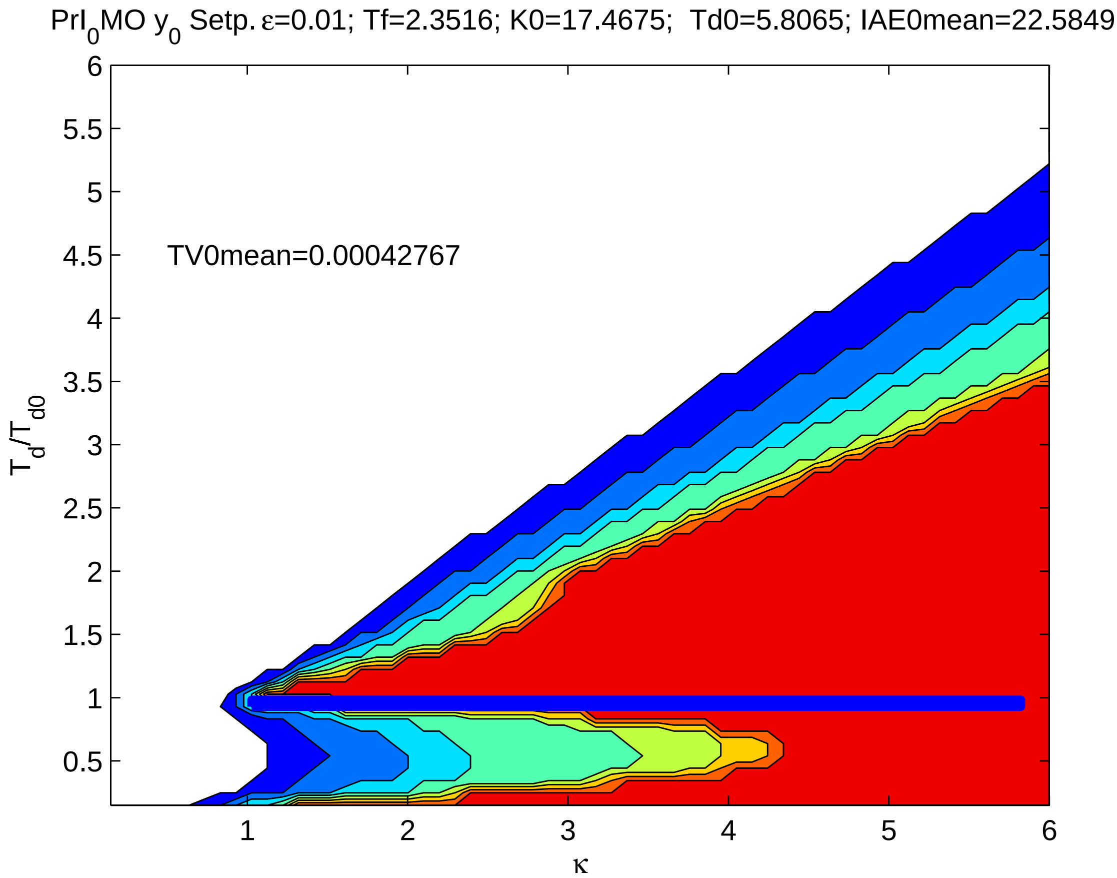

Figure 3.29: Performance portrait – FPrI$_0$-controller

parameters:

$$T_{d0} = 6.48 \tag{3.11}$$
$$T_f = 4.28 \tag{3.12}$$
$$K_0 = 14.15 \tag{3.13}$$

For FI$_0$-controller it gives

$$T_{d0} = 5.7 \tag{3.14}$$
$$T_f = \mathrm{e}T_{d0} \approx 16 \tag{3.15}$$
$$K_0 = 17 \tag{3.16}$$

The performance of both controllers is compared in Fig. 3.31, 3.32. Feel free to make this experiment with larger dead time e.g. 10 s.
Questions:

- What type of plant is FPrI$_0$-controller suitable to control for?
- Was there any overshooting in output and input transients during the experiments?
- Which controller performed better in disturbance rejection?

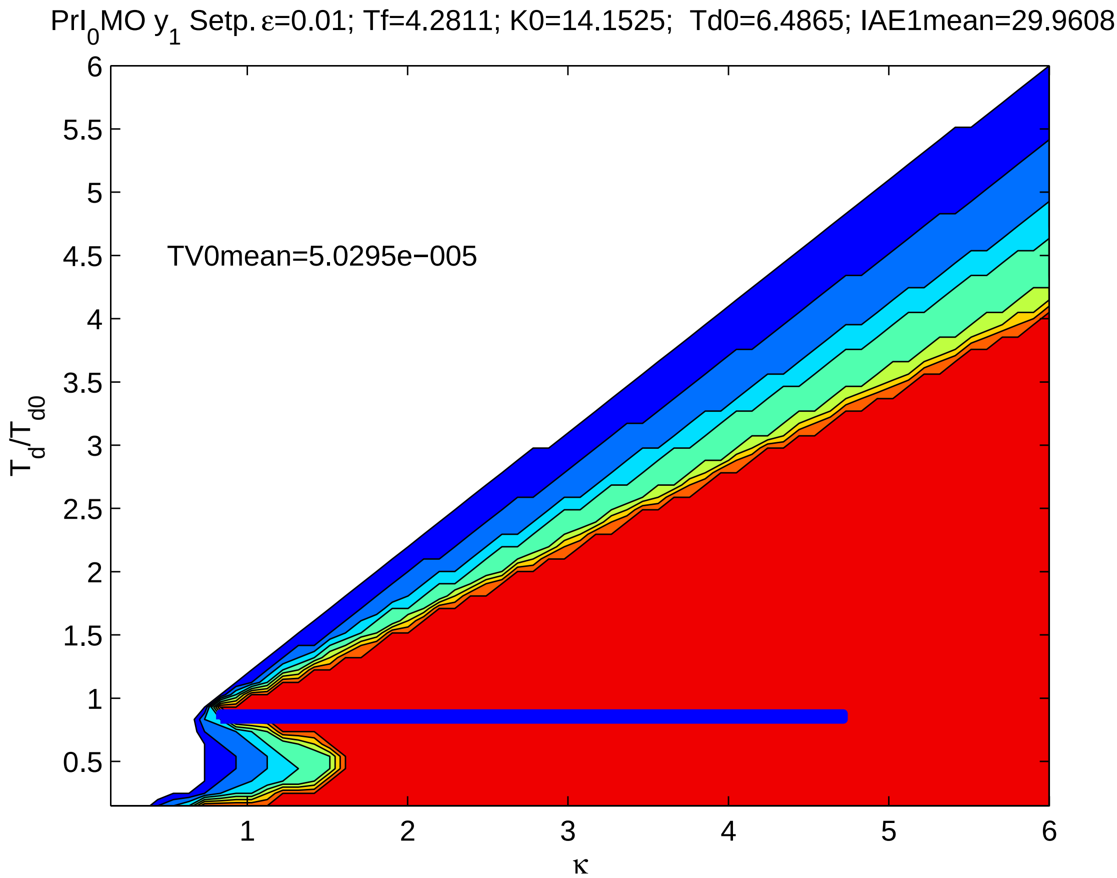

Figure 3.30: Performance portrait – FPrI_0-controller

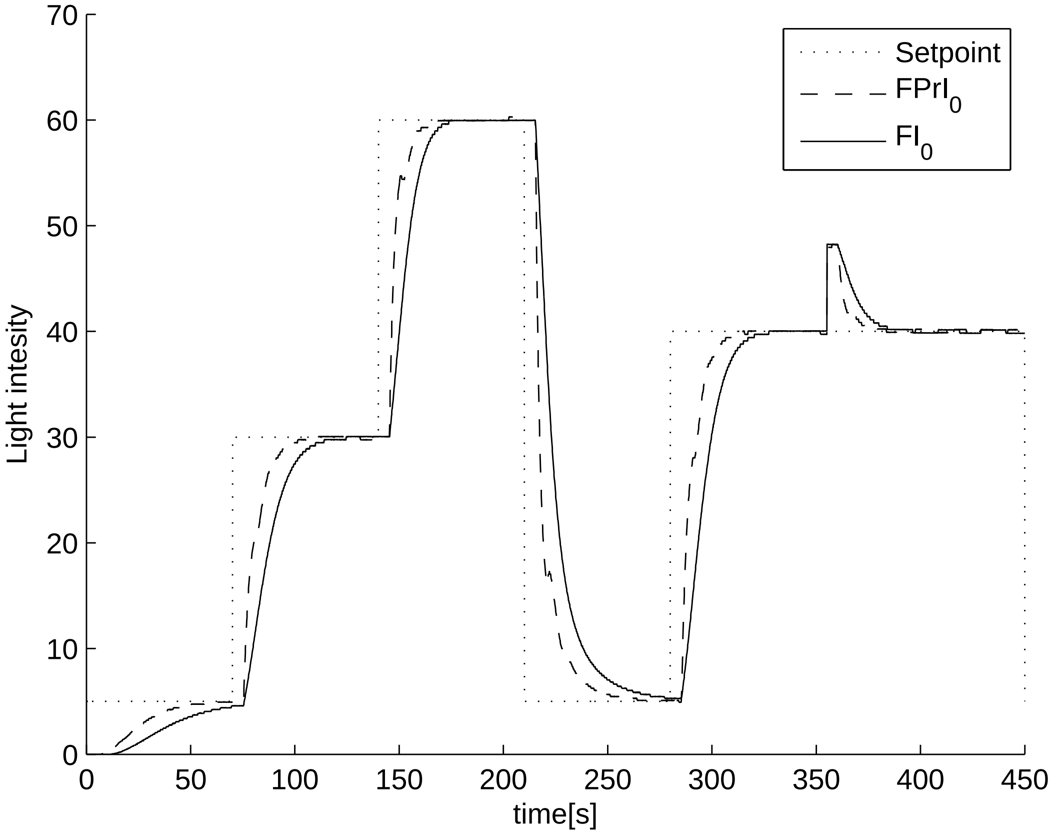

Figure 3.31: FPrI_0 – controller vs FI_0-controller under $5s$ transport delay, plant output

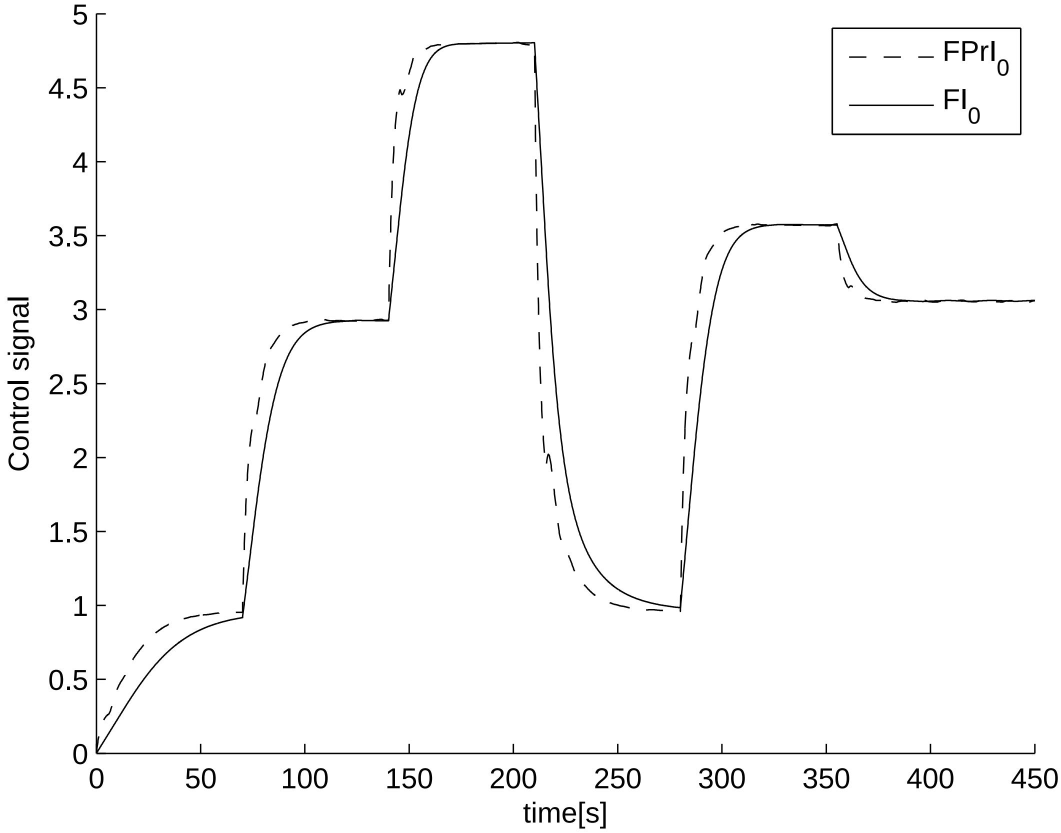

Figure 3.32: FPrI_0-controller vs FI_0-controller under $5s$ transport delay, control signal

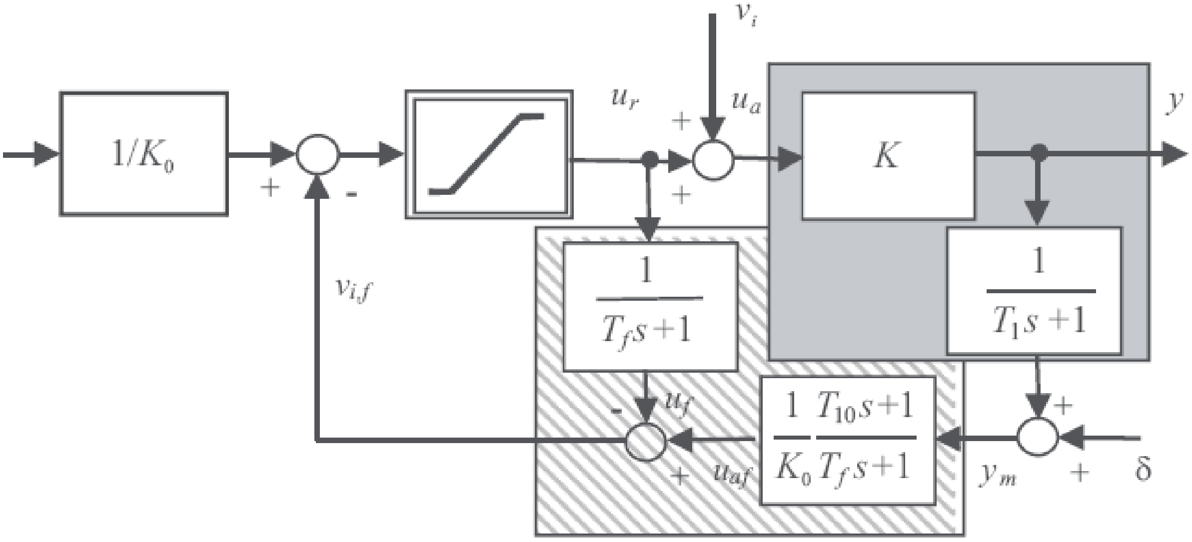

Figure 3.33: FPI_0 controller

3.2.4 PI_0 and FPI_0 Controllers

Tasks:

- Identify the parameters of filtered light channel.
- Use PI_0-controller to compensate the delay of the filtered light channel.
- Tune the controller using a performance portrait.
- Make a disturbance by a LED voltage step during the experiments with the filtered light channel.
- Compare the control quality with the I_0-controller using a real time experiment.

In practice it is often not efficient and sufficient to compensate large time constant's influence just by restricting the closed loop bandwidth. Active compensation of dominant loop time constant leads to control structures such as PI_0-controller (Fig. 3.33, 3.34).
The output of filtered optical channel will be used to practice FPI_0-controller tuning. An analogue first order filter is used for non-filtered light channel filtering. The process can so be approximated as

$$G(s) = \frac{K}{T_1 s + 1} e^{-T_d s} \tag{3.17}$$

The time constant T_1 represents a analogue filter time constant, dead time T_d is used to approximate the lag between a bulb voltage step and the corresponding change in the light intensity. Obtain the parameters of filtered light channel. The exnum command with experiment no.5 can be used.

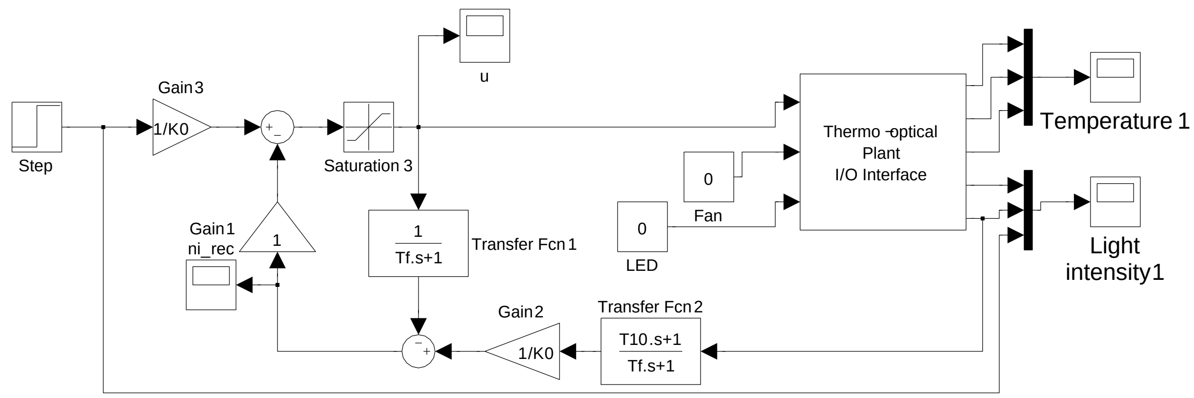

Figure 3.34: PI_0-controller: Simulink Model; to get FPI_0 controller you need to add pre-filter to the reference setpoint input

For the plant used in this example it gives

$$\begin{aligned} K &\in [8.8728, 19.8939] \\ T_1 &\in [17.0096, 25.8687] \\ T_d &\in [0, 0.6] \end{aligned}$$

The dead time T_d is relatively small comparing to the process time constant T_1 (we have so-called lag dominant plant), thus it can be neglected in further calculations. Figs. 3.35 and 3.36 show how the process gain and time constant vary through the operational range independence on the plant input.
Again the controller tuning can be done using the performance portrait method (Fig. 3.37). It is best to choose

$$\begin{aligned} T_{10} &= \max(T_1) \\ K_{10} &= \max(K) \end{aligned} \tag{3.18}$$

which for this plants is

$$\begin{aligned} T_{10} &= 25.8687 \\ K_0 &= 19.8939 \end{aligned} \tag{3.19}$$

Try multiple disturbance observer filter time constants T_f. In this example following T_f/T_{10} ratios were used:

$$T_f/T_{10} = \{0.8, 0.6, 0.06\} \tag{3.20}$$

$T_f/T_{10} = 0.8$ and $T_f/T_{10} = 0.6$ should give transients of the undelayed system output with up to 2% overshooting and the delayed system output should not overshoot. $T_f/T_{10} = 0.06$ corresponds to controller tuning where

$$T_f = \mathrm{e}^1 \cdot \max(T_d) = 1.6310 \tag{3.21}$$

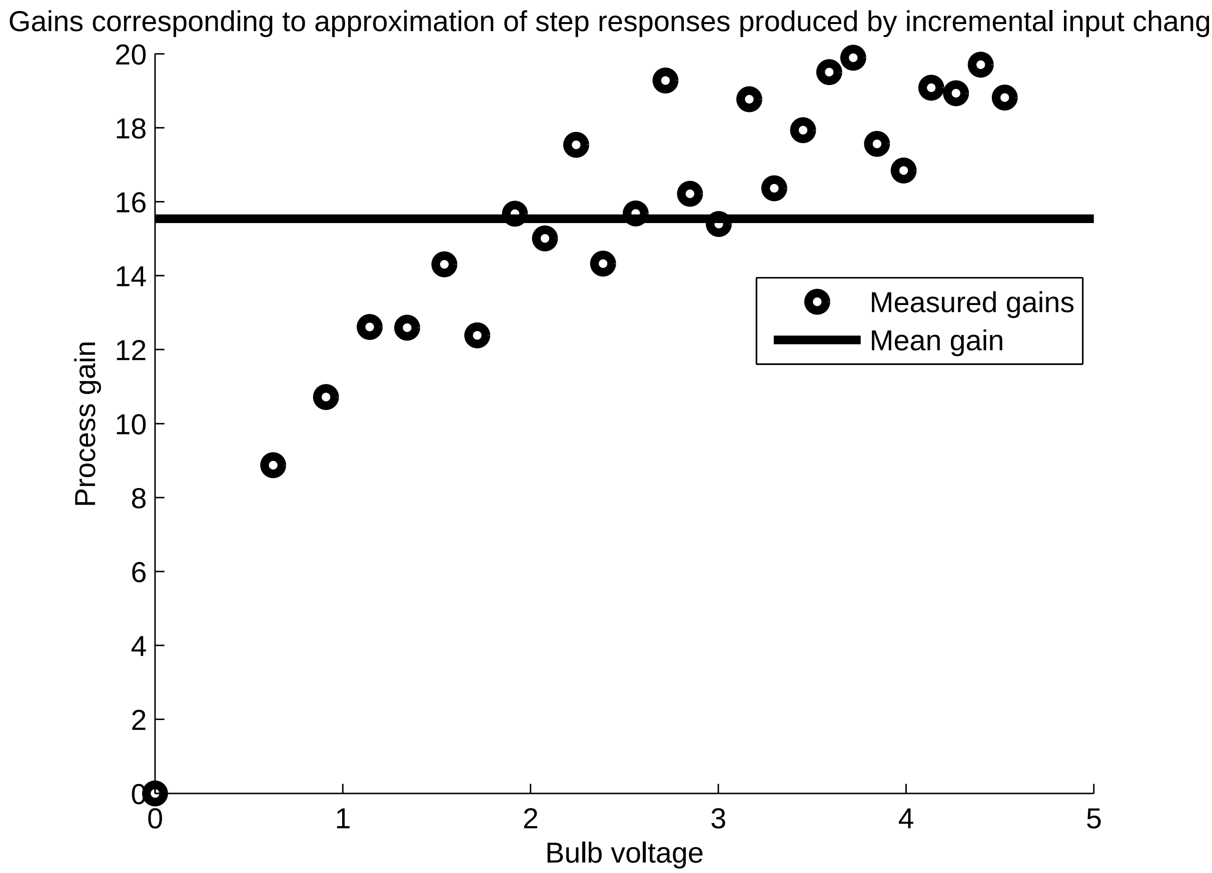

Figure 3.35: Process gain in several operation points as function of the plant input

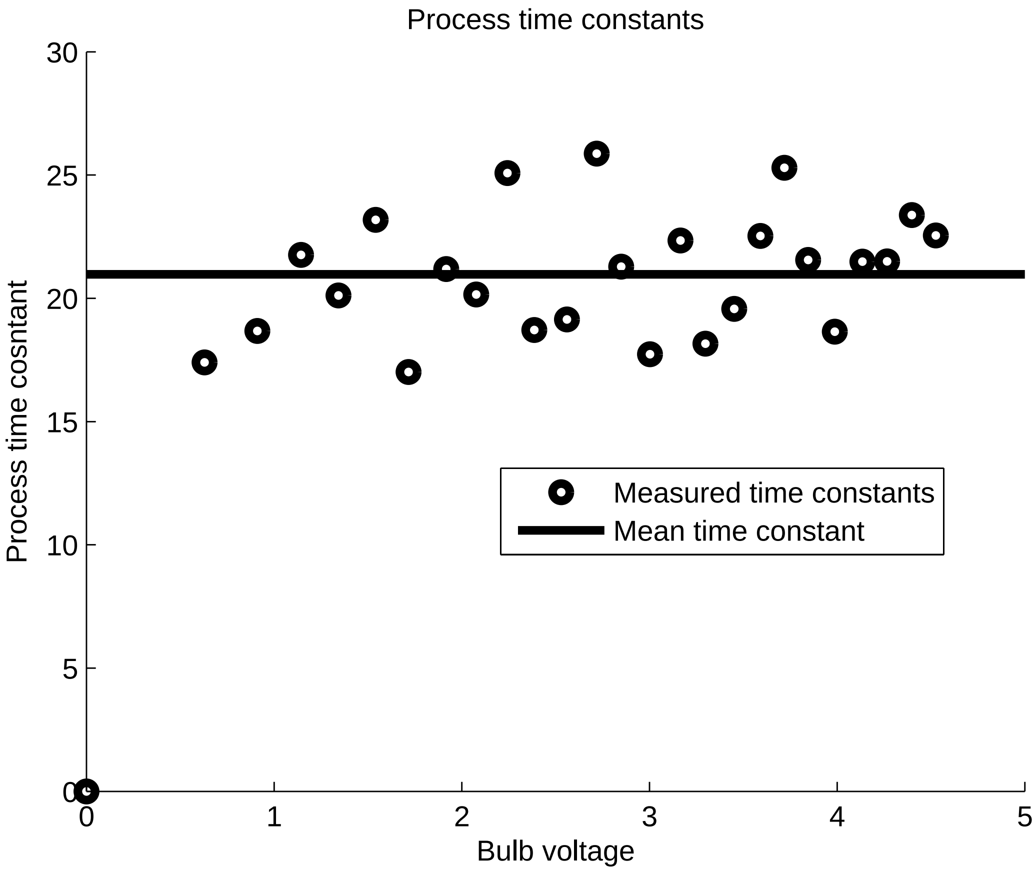

Figure 3.36: Process time constant in several operation points as function of the plant input

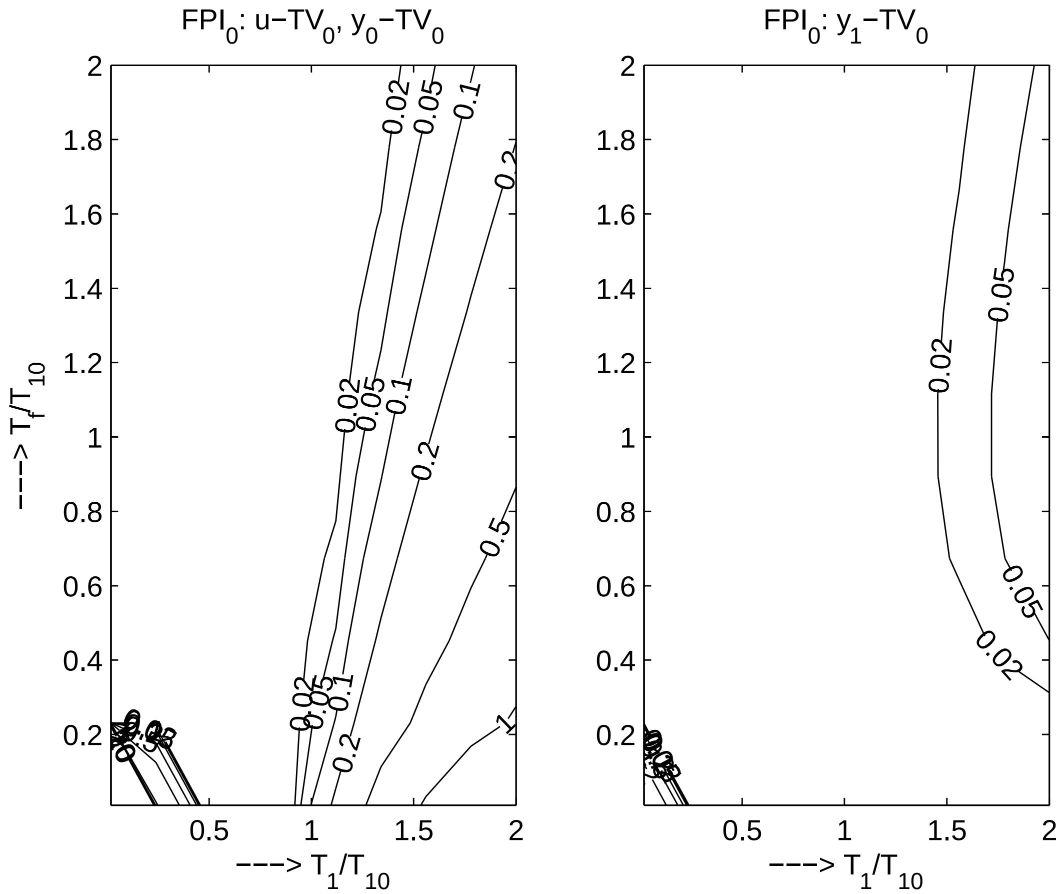

Figure 3.37: Performance portrait of the FPI_0-controller.

This tuning can yield to transients with approximately 10% overshooting of the undelayed system output and the delayed system output should not overshoot (see the performance portrait in Fig. 3.37).
Compare the results with the FI_0-controller (Fig. 3.17). Use the following FI_0-controller tuning:

$$\begin{aligned} K_0 &= \max(K) \\ T_f &= \mathrm{e}^1 \cdot \max(T_1) \end{aligned} \tag{3.22}$$

Make several setpoint steps, make a LED voltage step as well when the system output is settled. Do not forget to keep more time between setpoint steps when using I_0-controller. It is useful to make a simulation first to determine setpoint step time interval sufficient to settle the system output between them. The experiments results are shown inf Figs. 3.38, 3.39, 3.40. Compare the overshooting for setpoint step and disturbance step. The detailed view on these transients is shown in Figs. 3.41, 3.42, 3.43, and 3.44.
Questions:

- What type of plant are PI_0 and FPI_0-controllers suitable to control

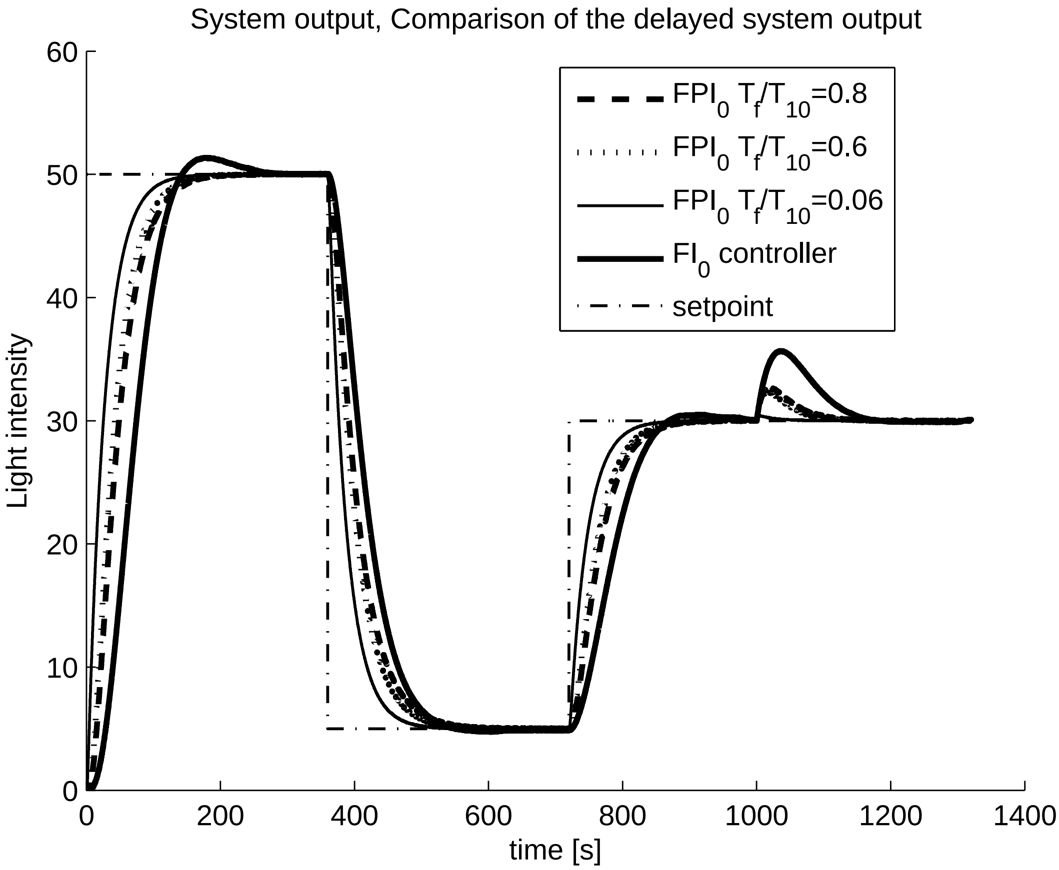

Figure 3.38: Experimental results – delayed system output

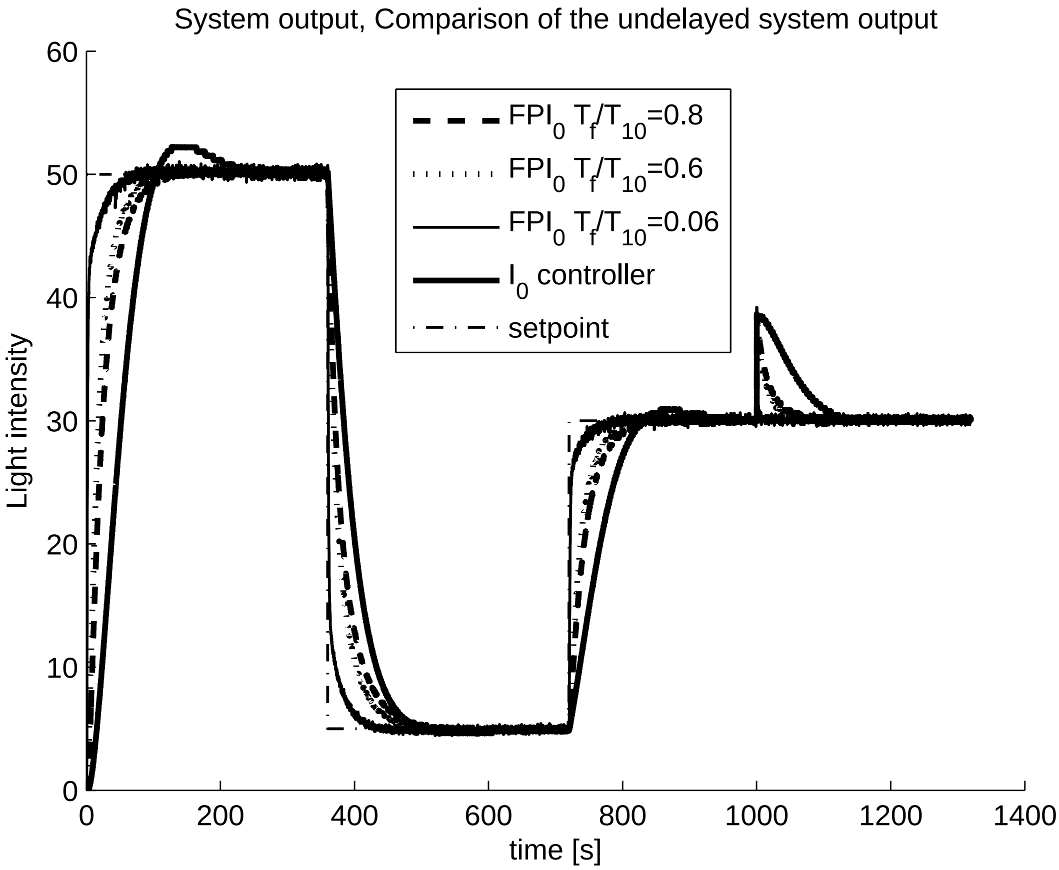

Figure 3.39: Experimental results – undelayed system output

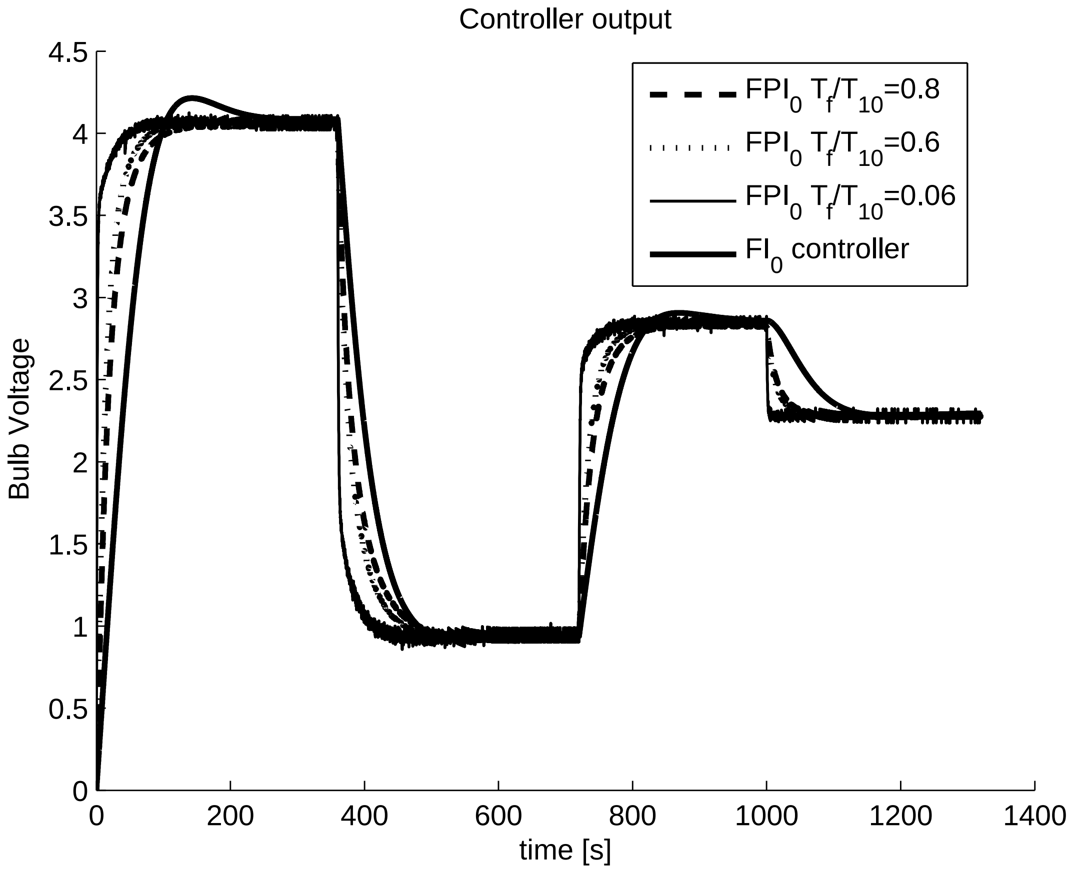

Figure 3.40: Experimental results – control signal

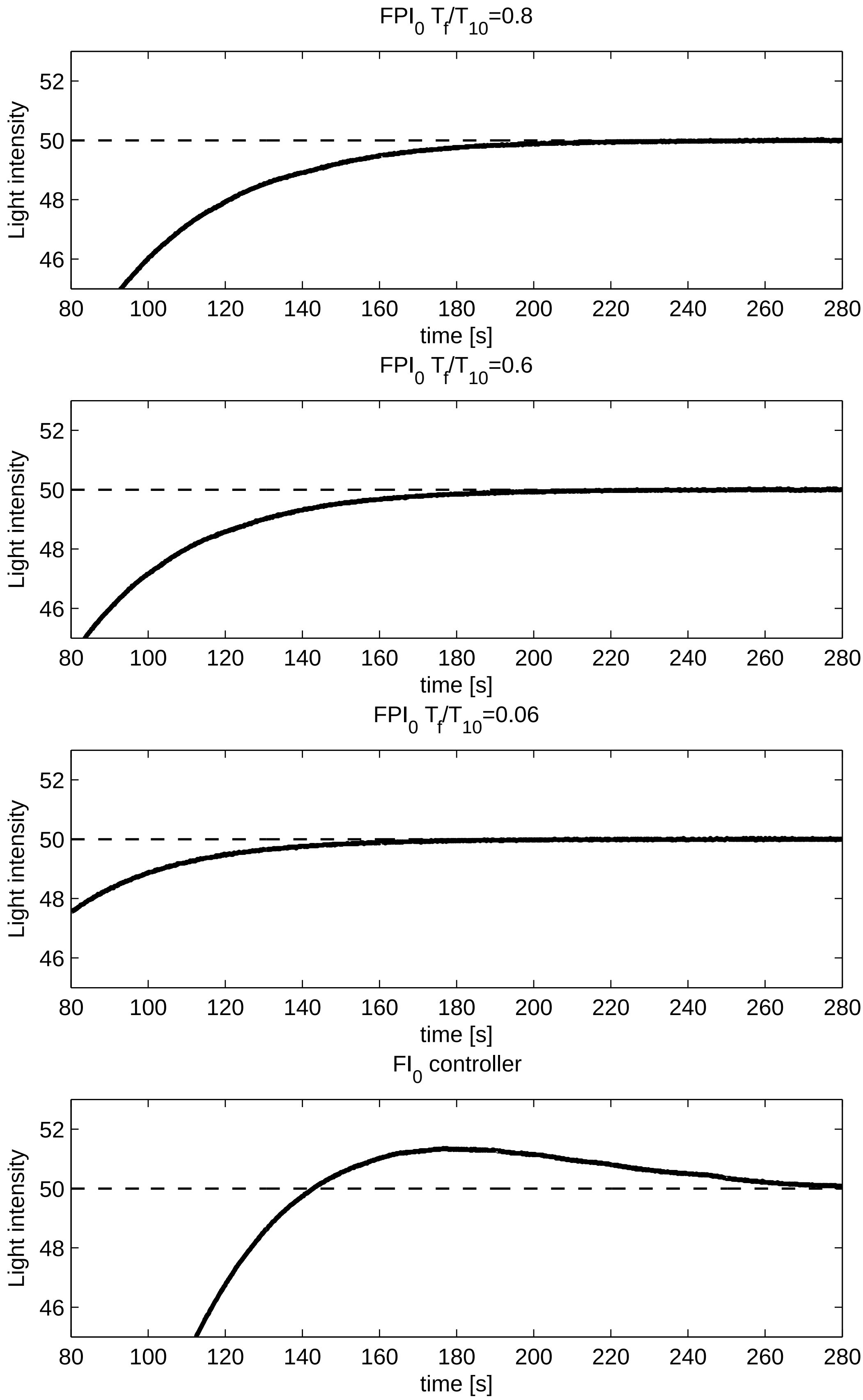

Figure 3.41: Experimental results detail – delayed system output, setpoint step

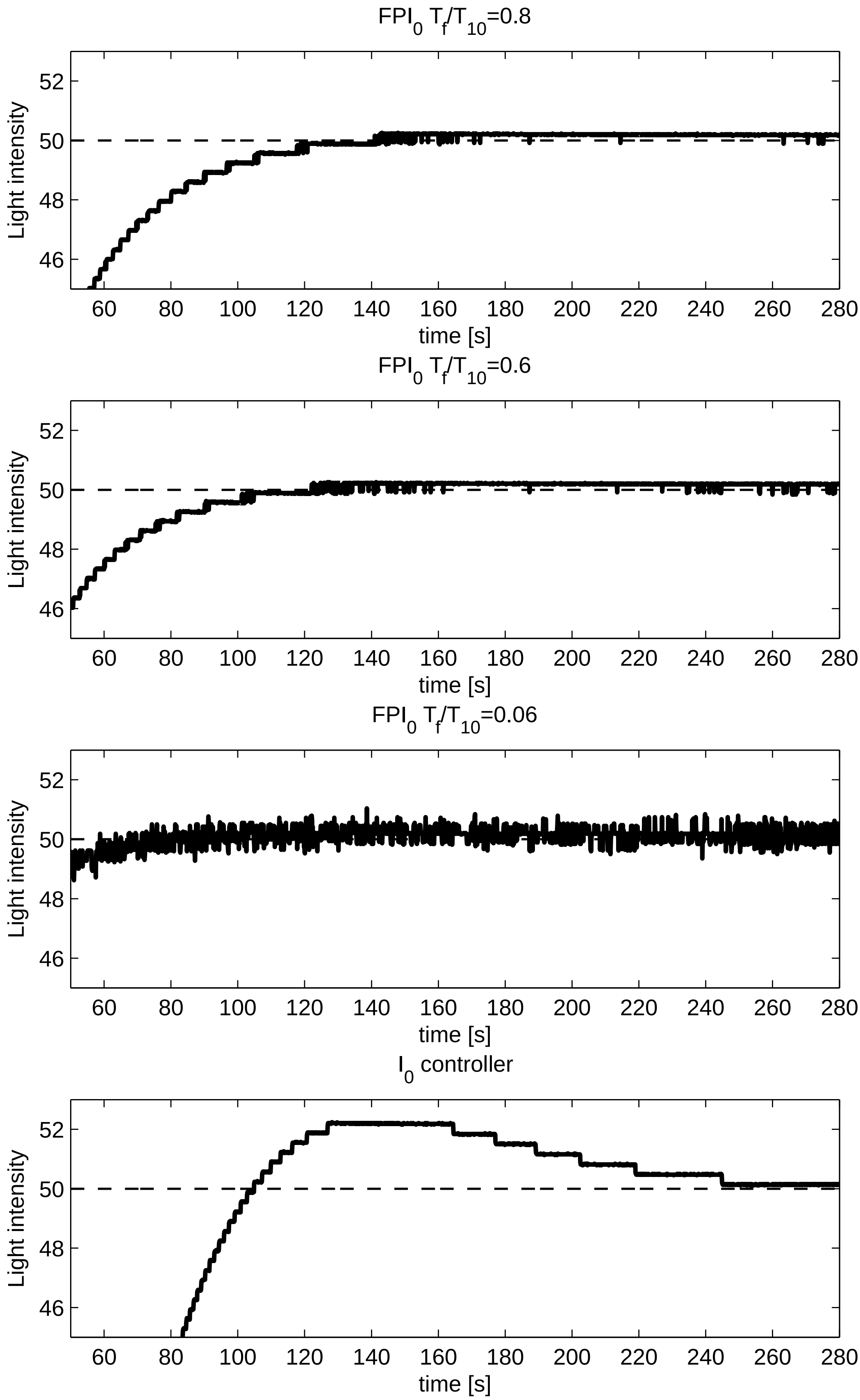

Figure 3.42: Experimental results detail – undelayed system output, set-point step

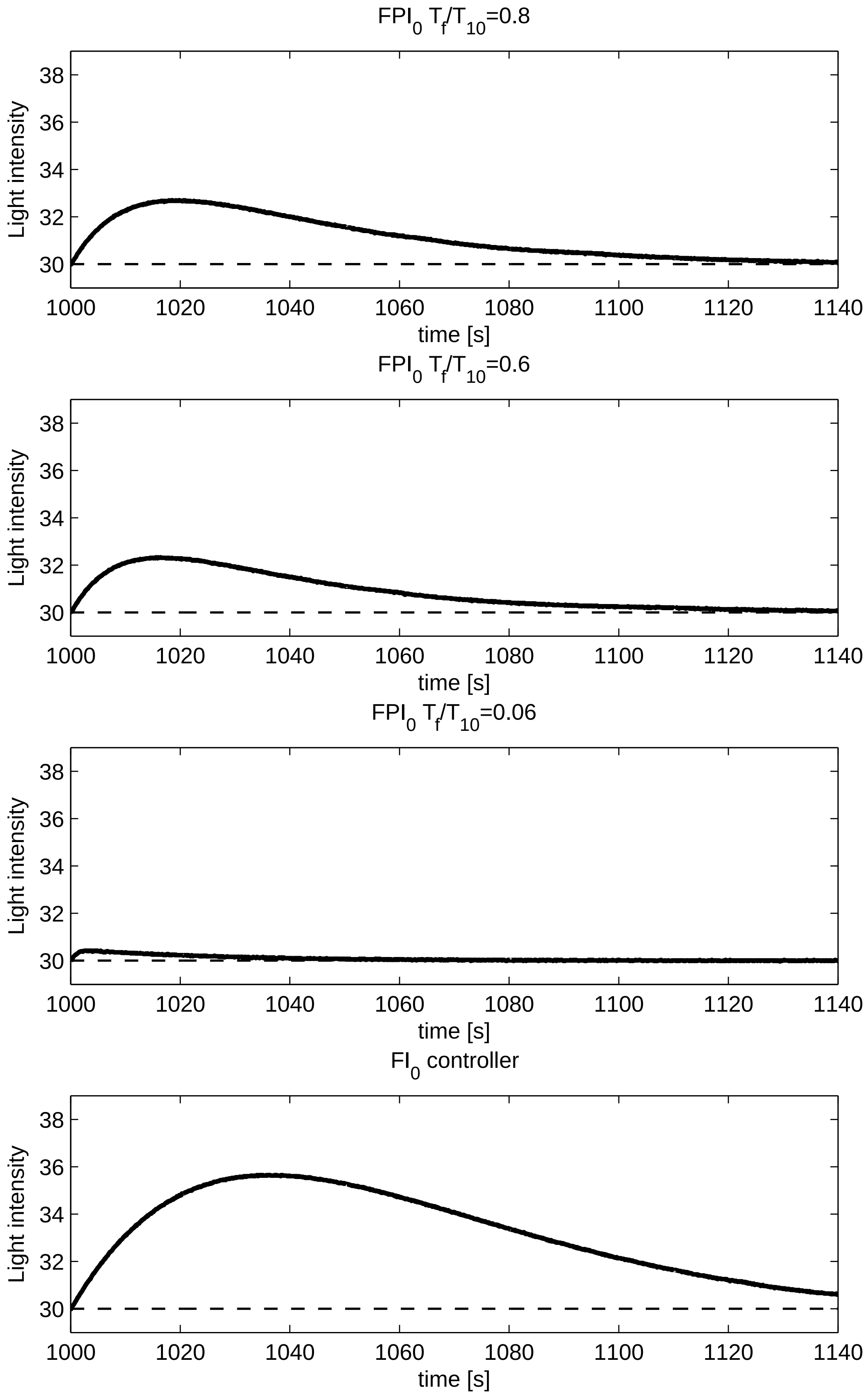

Figure 3.43: Experimental results detail – delayed system output, disturbance step.

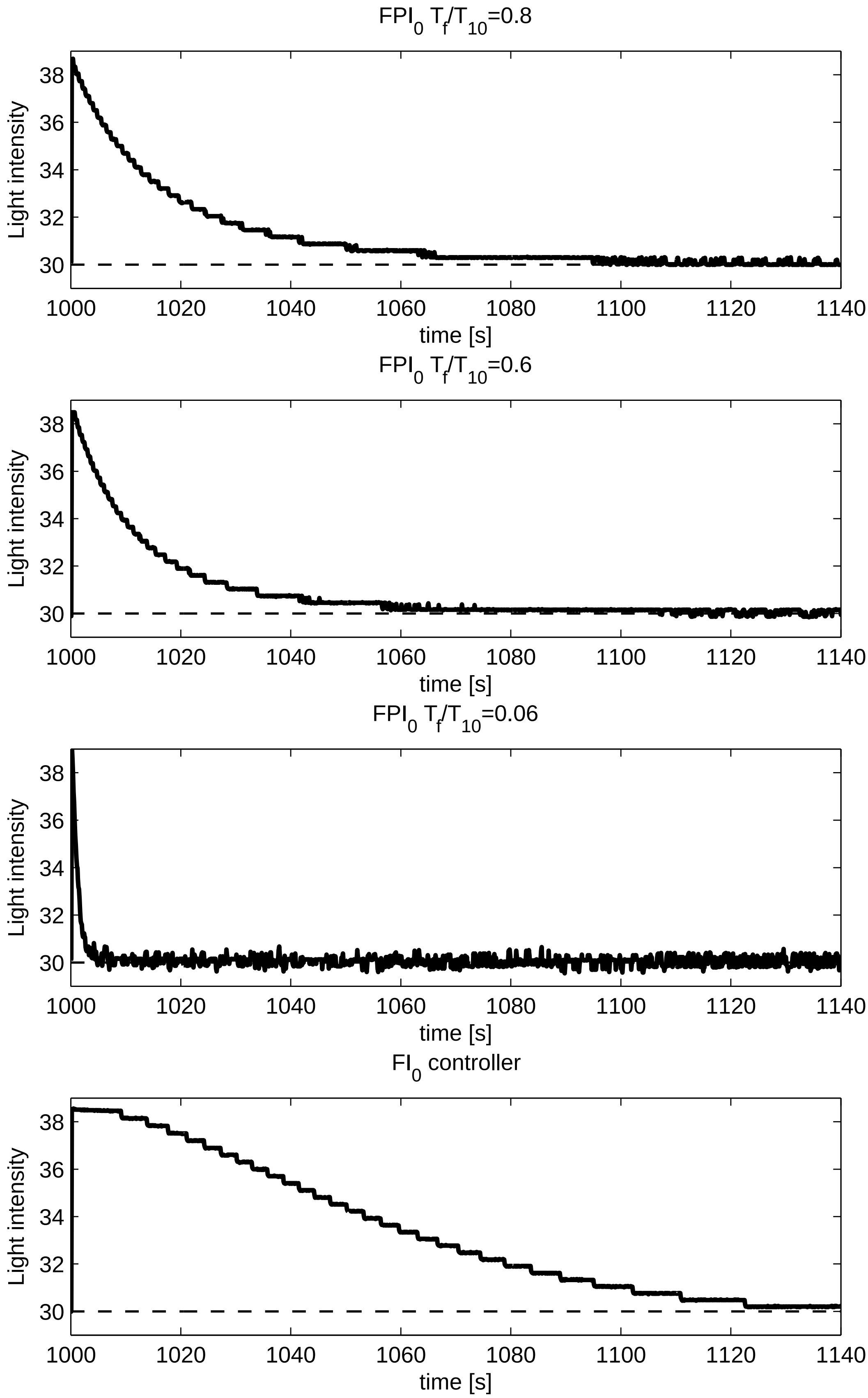

Figure 3.44: Experimental results detail – undelayed system output, disturbance step.

for?

- Analyze the amount of overshooting in the experiments.
- Which controller performed better in disturbance rejection?
- Was there any advantage of using FI_0 over FPI_0 – e.g. in noise sensitivity?

3.2.5 PI_1 controller

Tasks:

- Control the filtered light channel output by PI_1-controller.
- Analyze the control quality for various controller tunings.
- Compare results with control performance of previous controllers.

The PI_1-controller structure and Simulink model are in Figs. 3.45, 3.46. PI_1 may also be used for controlling unstable plant. To cover all stable and unstable plants by one transfer function, it is necessary to use the pole-zero form instead of the time-constant one and to express the plant as

$$G(s) = \frac{K_s}{s+a} \mathrm{e}^{-T_d s} \tag{3.23}$$

For robust controller tuning, the performance portrait method could again be used. For simple nominal tuning use the following rules based on the notion of the so called equivalent poles α_e of the proportional controller and $\alpha_{e,I}$ of the controller with disturbance compensation (I-action):

$$\begin{aligned} \alpha_e &= -(1+aT_d)^2/(4T_d) && (3.24) \\ \alpha_{eI} &= \alpha_e/1.3 && (3.25) \\ P &= -(\alpha_{eI}+a)/K_s && (3.26) \\ T_f &= \mathrm{e}^1 T_d && (3.27) \end{aligned}$$

Plant parameters from FOPTD approximation (3.18) can be used, whereby $K_s = K/T$ and $a = 1/T$. The experimental results for the plant used in this chapter are in Figs. 3.47, 3.48, 3.49, 3.49.

Questions:

- Which process gain used for controller tuning gave better control quality?
- What was the most obvious difference in control quality compared to DC0 controllers?

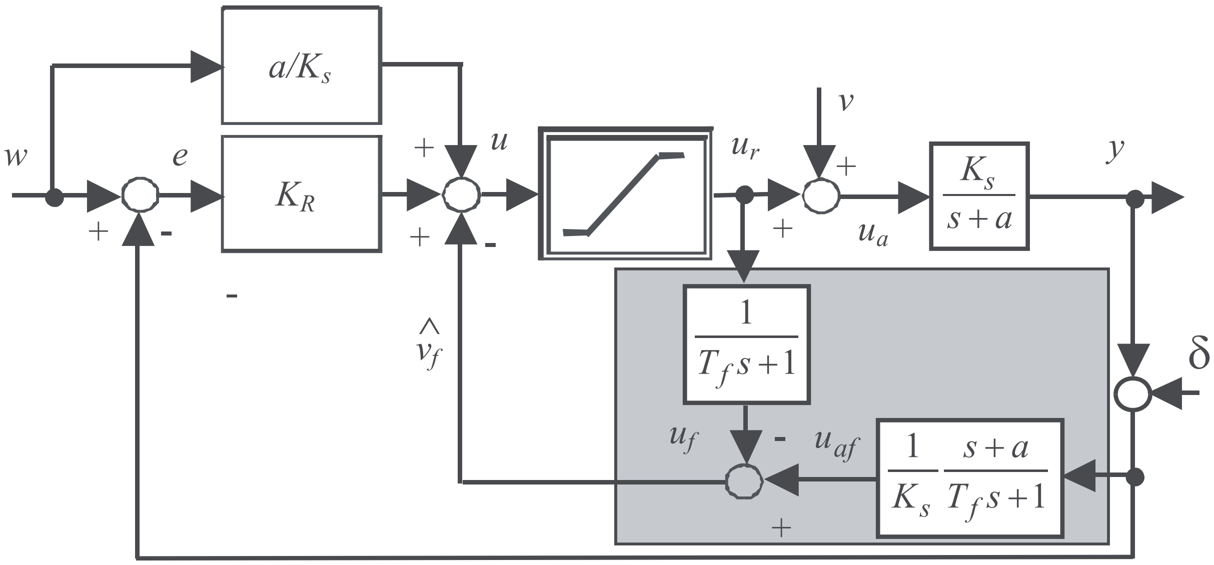

Figure 3.45: PI_1-controller

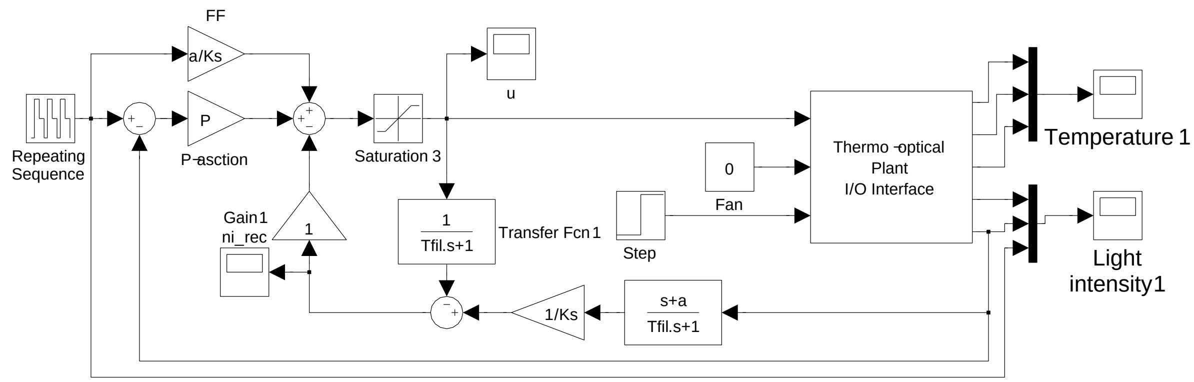

Figure 3.46: PI_1-controller, Simulink model

- What was the most obvious difference in control signal shape compared to DC0 controllers?
- Was there any overshooting in the experiments?

3.2.6 Filtered Smith Predictor (FSP)

Tasks:

- Control the filtered light channel output by FSP.
- Analyze the control quality for various controller tunings.
- Compare results with control performance of PI_1-controller .

The FSP was originally proposed in Normey-Rico et al. (1997) for stable FOPDT processes to improve robustness of the traditional SP. Later, the

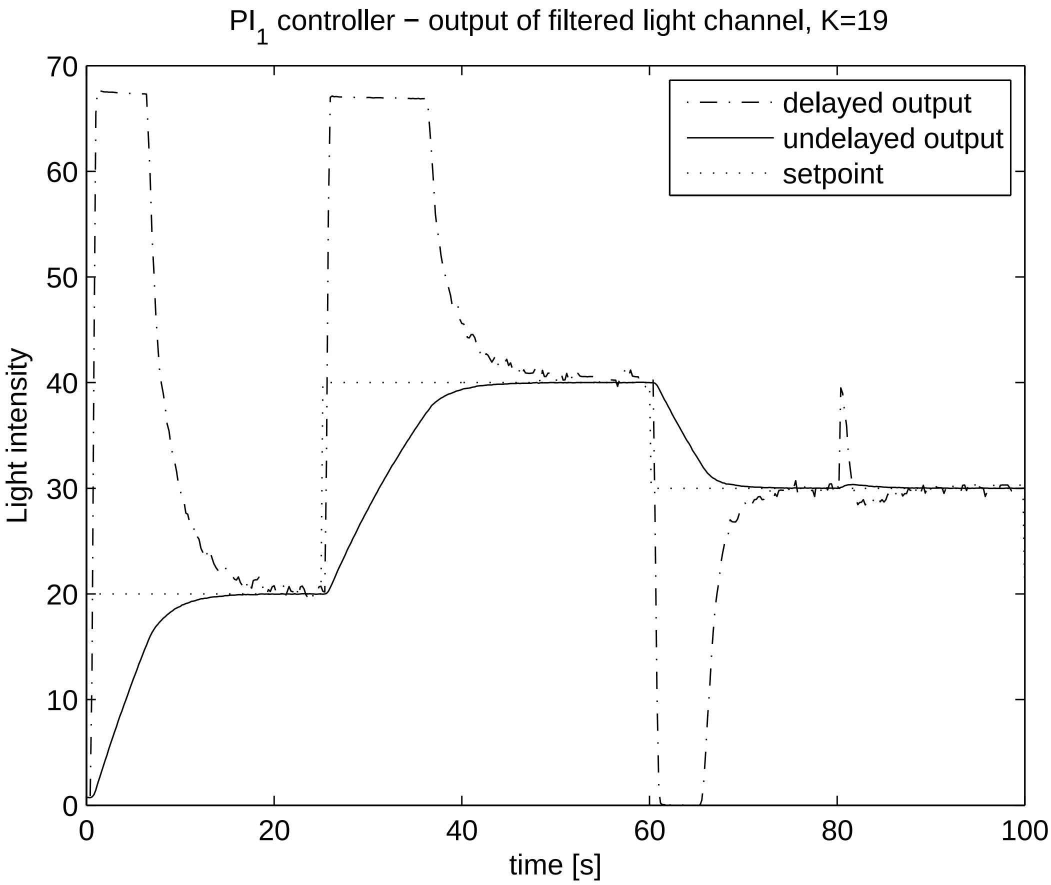

Figure 3.47: PI_1-controller, filtered light channel control for $K = 19$

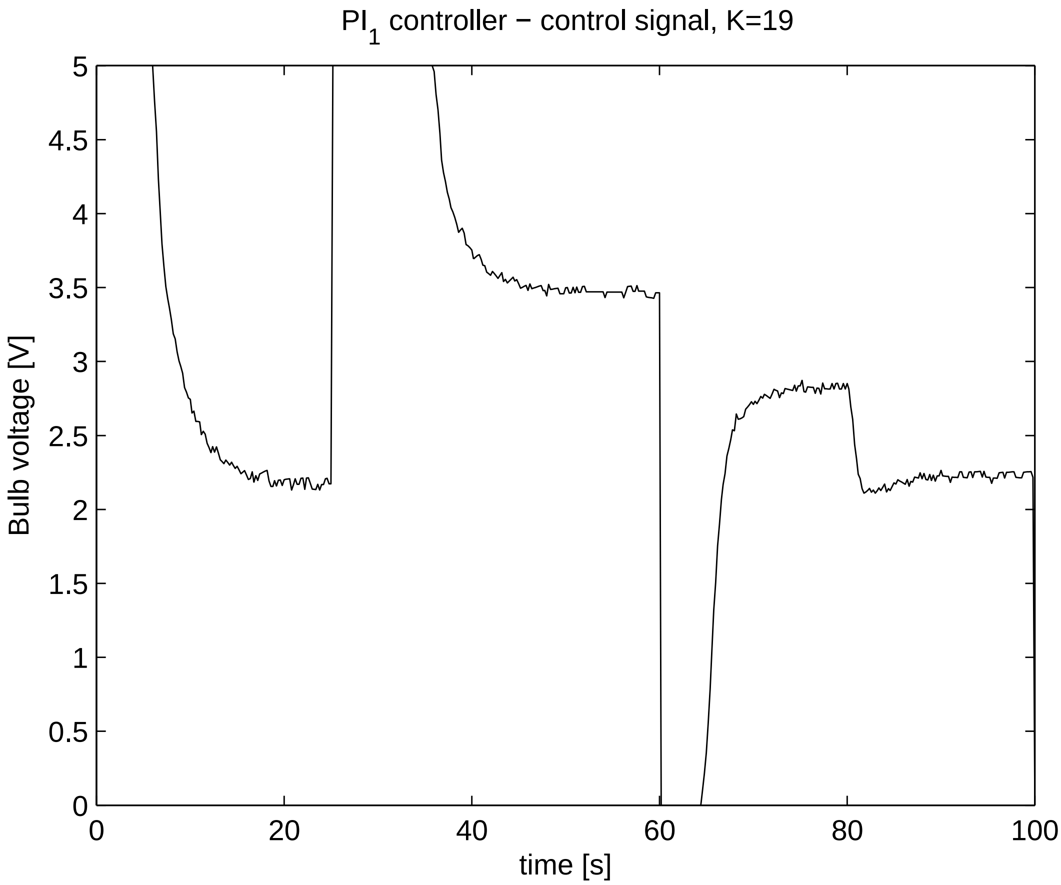

Figure 3.48: PI_1-controller, filtered light channel control for $K = 19$

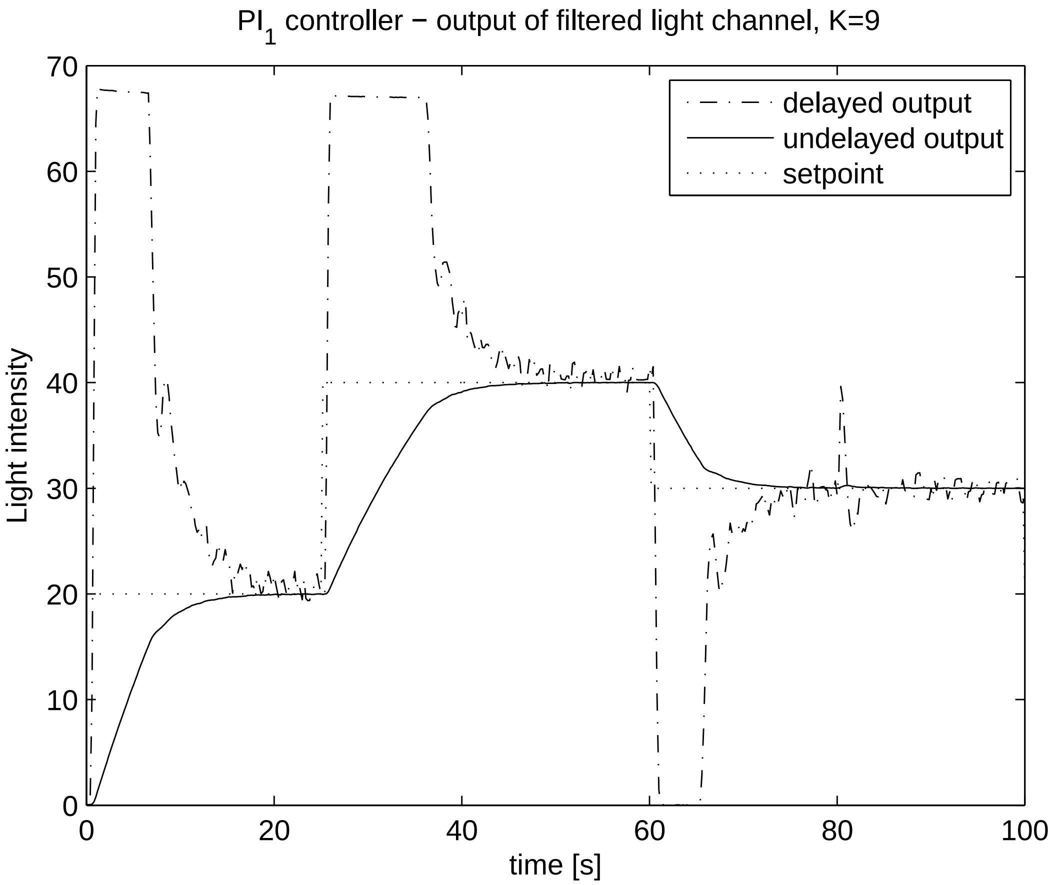

Figure 3.49: PI$_1$-controller, filtered light channel control for $K = 9$

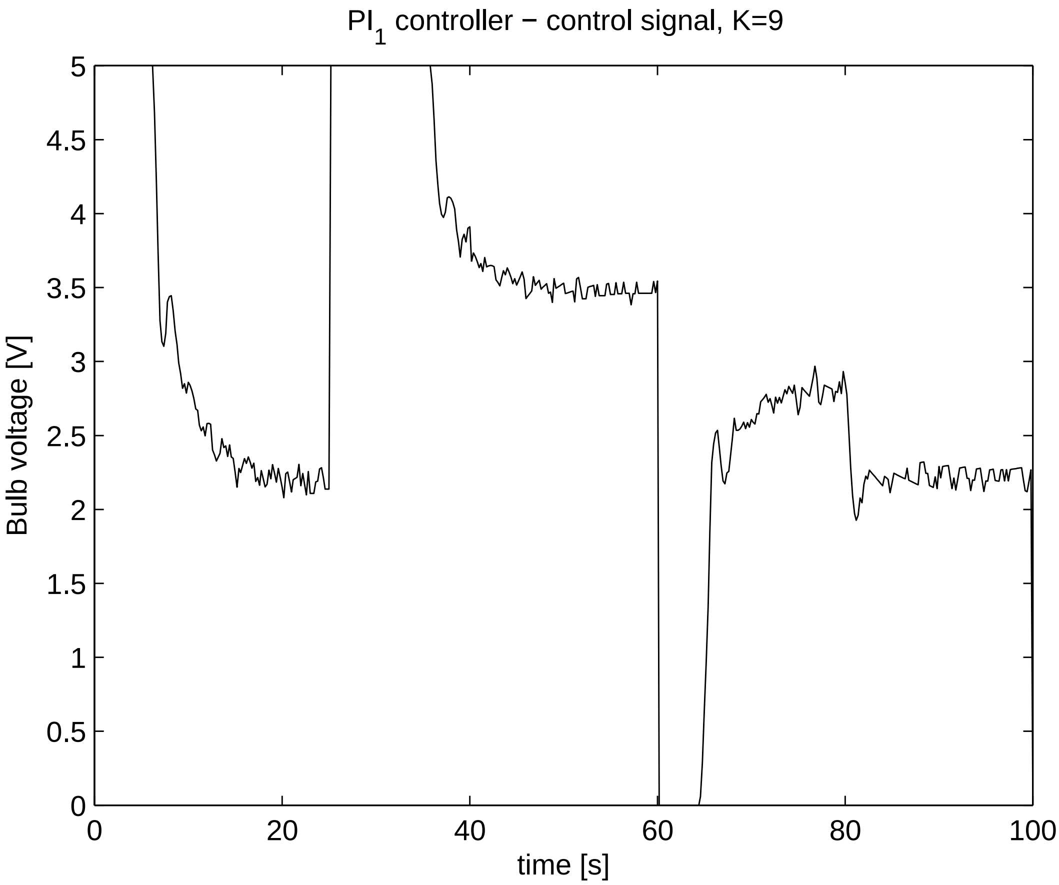

Figure 3.50: PI$_1$-controller, filtered light channel control for $K = 9$

disturbance filter $F_r(s)$ has been also proposed to decouple the reference setpoint and the disturbance response and to stabilize the controller loop in case of unstable and integral plants Normey-Rico and Camacho (2009). It may be interpreted as a structure with the dynamical feedforward control and the reference plant model Aström and Hägglund (2005); Visioli (2006), or the 2DOF IMC structure. The unified approach to designing FSPs for the FOPDT plants introduced in Normey-Rico et al. (2009); Normey-Rico and Camacho (2009) considers compensation of an output disturbance by correction of the reference value, whereby the disturbance is reconstructed by using the PPM. However, despite to the proclaimed unification, it separately presents solutions corresponding to stable, integral and unstable plants.
Use FSP structure from Fig. 3.51 with filter

$$F_{r1}(s) = \frac{1+\beta_{11}s}{1+T_f s} \tag{3.28}$$

Use FOPDT approximation (3.18), whereby $K_s = K/T$ and $a = 1/T$. Choose $K_{s0} = \max(K_s)$, $\theta = \max(T_d)$, $T_{10} = \max T_1$,$a_0 = \min(a)$. Set up the experiments in the same way as in the previous exercise to be able to compare the results. For controller tuning use following rules.
The P-action should be set to

$$K_p = (1/T_r - a_0)/K_{s0} \tag{3.29}$$

Filter parameter β_{11} set to

$$\beta_{11} = T_{10}\left(1-(1-T_f/T_{10})(1-T_r/T_{10})\mathrm{e}^{-\theta/T_{10}}\right) \tag{3.30}$$

Try multiple T_r and T_f settings e.g. $T_r = T_{10}/\{2,4,8,16\}$, $T_f = T_r/\{2,4,8\}$. In this example various T_r settings were used. The filter time constant was set to $T_f = T_r/4$. Experimental results are shown in Figs. 3.52, 3.53.
Questions:

- Was there any overshooting in the experiments?
- How did the increasing of parameter T_r affect control quality?
- Which controller performed better in comparison with PI_1-controller?

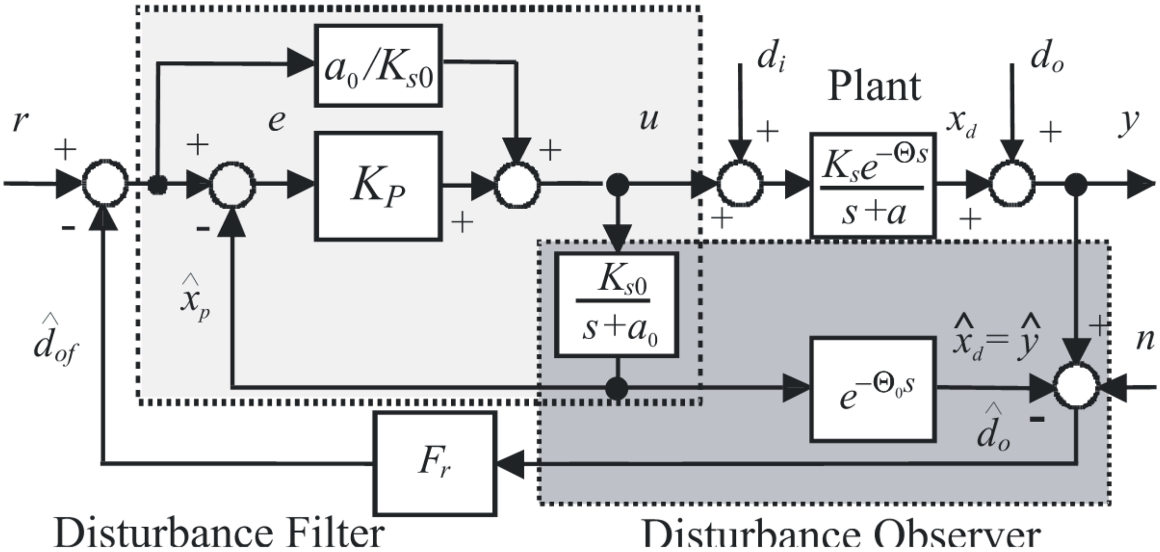

Figure 3.51: Modified P-FSP with the primary loop using 2DOF P-controller with the disturbance filters

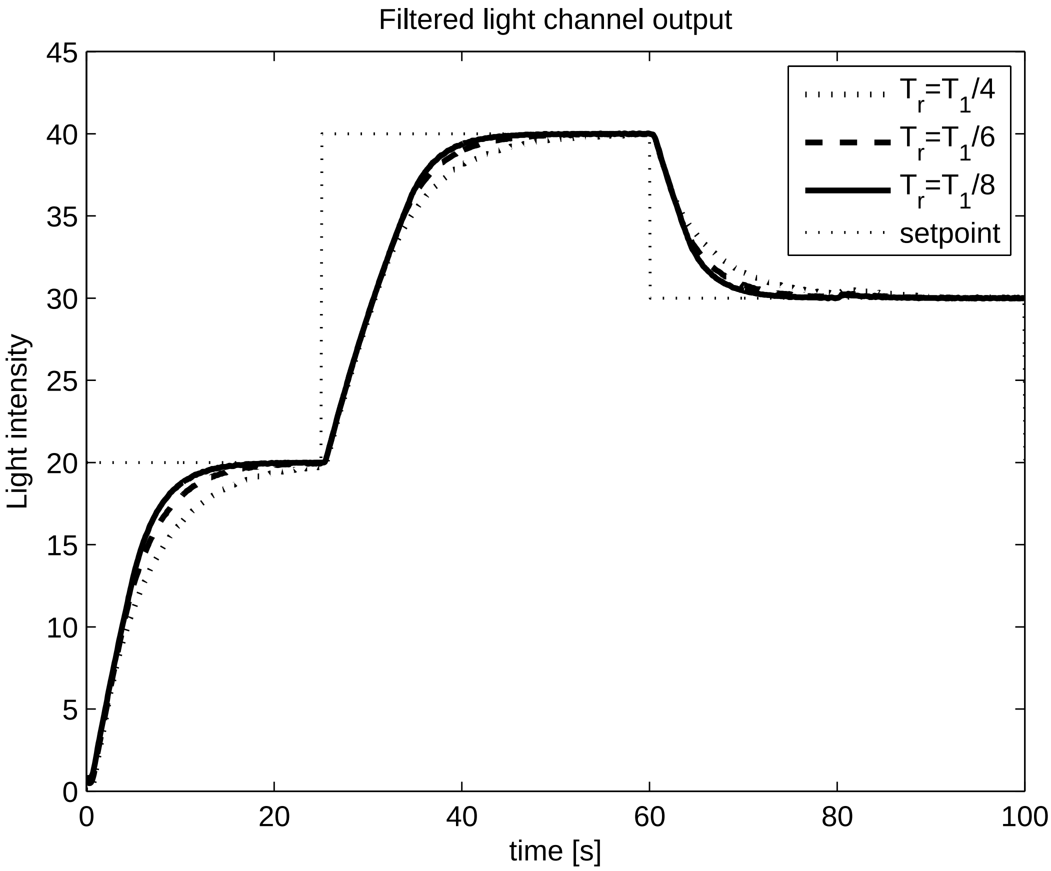

Figure 3.52: FSP for filtered optical channel – system output

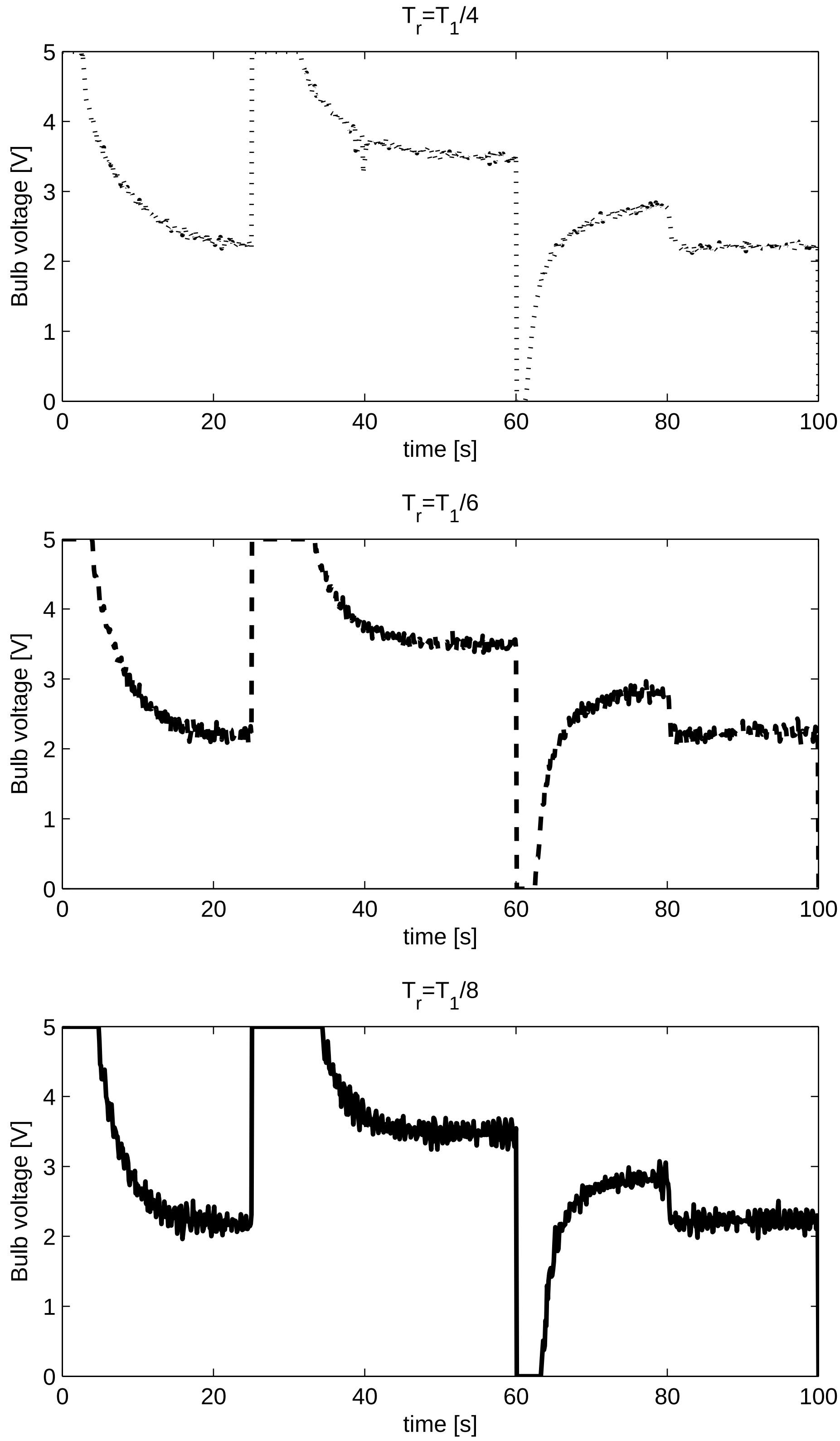

Figure 3.53: FSP for filtered optical channel – control signal

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Chapter 4

Laboratory Model of Coupled Tanks

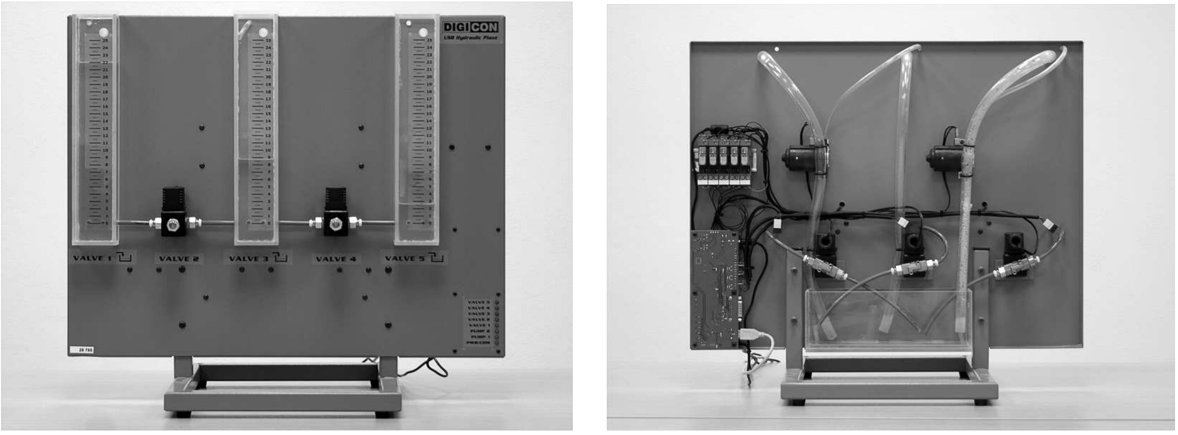
This chapter gives introduction into work with frequently used hydro-system of coupled tanks. The chapter explains analytical modelling for one and two-level systems of coupled tanks, including the experimental identification of system's parameters and sensor's calibration.

4.1 Introduction

To be able to use their knowledge in solving practical tasks, for graduates it is important to develope skills in applying control theory related to linear and non linear systems to real plants control. This may be just partially achieved by computer simulations and the feedback received in such a way is never complete. Therefore, it is important to acquire already during the university studies some practice and experience related competences in designing and testing of controllers on real systems.

Due to the simple manipulation, easily understandable and observable processes and their time constants, one of many systems used at universities frequently as pedagogical instruments in control education the system of coupled tanks has to be mentioned. The theoretical design of controllers included in this publication comprehends two basic configurations: control of one, or of two coupled tanks. In considering coupled tanks, one of possible configurations is the coupling in series with interaction, when the liquid's level in one of the tanks influences the other tank's liquid's level as the liquid may flow between the tanks in both direction. The other two-tank consideration is the cascade coupling with one-way influence, where the tanks are placed one under another. The liquid from the bottom tank can not flow into the upper one, plus the upper tank's inflow impact does not depend on the liquid's level in the lower tank. Due to the available equipment, in the practical part of this chapter we will consider just coupling in series with mutual interaction of tanks.

In order to be able to demonstrate theoretical aims of the control algorithm design and verify real processes by simulation using their models, we need to know the process model and its parameters. Sometimes, the simplest possible model is required, but for some tasks, the most accurate



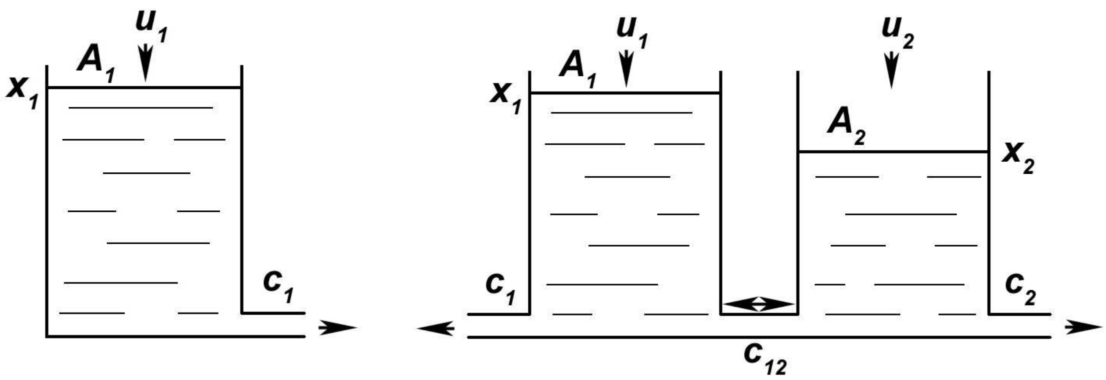
(a) Front view (b) Back view

Figure 4.1: Coupled tanks uDAQ28/3H2. Front and back view.

model possible is wellcommed. In case of inaccurate mathematical model, the accociated controller (if not sufficiently robust) would function only in simulations. That's why we will be dealing from now with the analytical description of the real scheme necessary for construction of the model, with the identification of the proposed model parameters, as well as with confrontation of the model with the real process and with control processes associated with control design based on considered models.

4.2 Coupled Tanks – Hydraulic Plant

Coupled tanks represent one of the most common examples of non-linear system. The task is usually related to basic manipulation required for dealing with the available physical system. System considered in our publication is shown in Fig. 4.1. This hydraulic system is composed of 2 pumps and 3 coupled tanks, each of them with the own drain. It was designed according to Žilka (2007).
The system is easily connectable to the PC via USB interface. There is no need of additional data acquisition cards hence the controlling is performed directly from the Matlab/SIMULINK or Scilab/Scicos environment. By switching the valves between the tanks and between the tanks and the lower reservoir, it enables to create different configurations, e.g. the one-level system, two-levels system with one inflow and one drain, or the multi-input-multi-output (MIMO) system with several inflows and outflows and the mutual tanks' coupling.
At the beginning we will start with the easiest model – we will imagine only one tank where the level of liquid is influenced only by inflow from the pump and drain from the valve. This valve is only two positions (open/close).



(a) One–tank system (b) Coupled Tanks

Figure 4.2: Models of tanks

This kind of connection's model can be seen in Fig. 4.2.
The liquid's level is denoted as x_1, with the liquid's inflow u_1, tank's cross-sections A_1 and the valve's outflow coefficient c_1. This tank model can be described by various differential equations acquired through the application of the law of substance preservation. Thus it applies for the given tank that the change in volume V is defined by the difference between the inflow and outflow. If we know the inflow u_1 and we define the outflow as u_{out} then

$$\begin{aligned} \Delta V &= u_1 - u_{\text{out}} \\ V &= x_1 A_1 \\ A_1 \frac{\mathrm{d}x_1}{\mathrm{d}t} &= u_1 - u_{\text{out}} \end{aligned}$$

To determine u_{out} we can use the Toricelli formula which defines the speed of the liquid outflowing the tank's aperture v_{out}. With the outflow, we have to take into account the co-actor as well μ – for the water 0.63. Plus if we already know the cross-sections the valve's aperture A_{vo}, we can put down the following:

$$\begin{aligned} v_{\text{out}} &= \sqrt{2gx_1} \\ A_1 \frac{\mathrm{d}x_1}{\mathrm{d}t} &= u_1 - \mu A_{\text{vo}} \sqrt{2gx_1} \end{aligned}$$

Now by applying the substitution and defining the flow coefficient of the valve c_1

$$c_1 = \frac{\mu A_{\text{vo}}}{A_1} \sqrt{2g}$$

The resulting differential equation for the one-level system would be

$$\begin{aligned} \dot{x}_1 &= \frac{1}{A_1} u_1 - c_1 \sqrt{x_1} \\ y_1 &= x_1 \end{aligned} \tag{4.1}$$

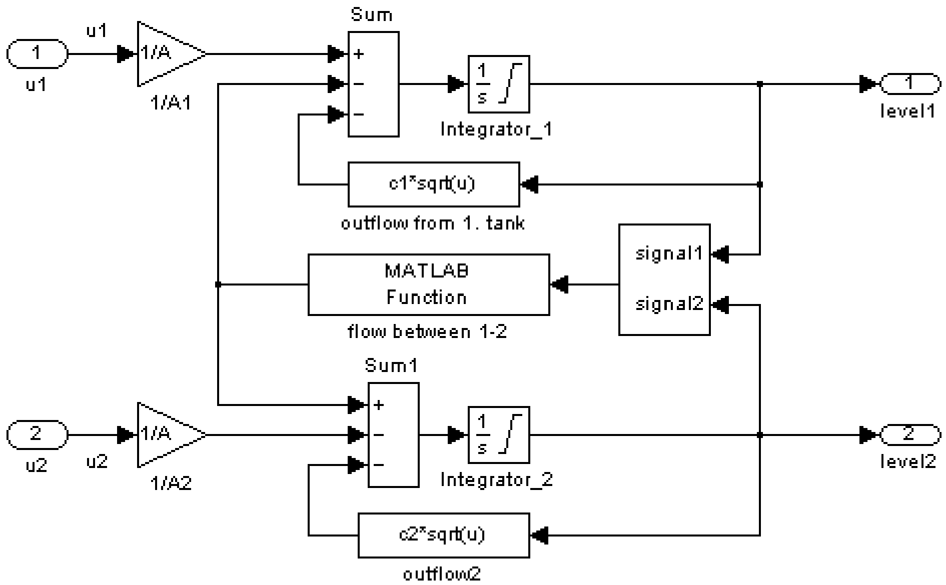


Figure 4.3: Coupled tanks - Simulink model.

In this way we can derive the differential equations for coupled tanks. System is then composed of two mutually interacting tanks with two possible pumps. Each tank would have a separate inflow u_1 a u_2, separate outflow depending on x_1 and x_2 and the flow between the tanks specified by difference of the levels and the constant c_{12}. Such tank model can be then described by differential equations

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{A_1}u_1 - c_{12}\,\mathrm{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|} - c_1\sqrt{x_1} \\
\dot{x}_2 &= \frac{1}{A_2}u_2 + c_{12}\,\mathrm{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|} - c_2\sqrt{x_2} \\
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned} \tag{4.2}$$

The liquid's levels in the first and second tank are denoted as x_1, x_2 and the level surfaces as A_1, A_2. For Simulink model see Fig. 4.3.

4.2.1 Identification

For simulating or controlling the real system it is indispensable to know all parameters of its mathematical model. While the measurement of the level surfaces in tanks does not represent a serious problem, to determine the values of the valve's flow coeficient is usually a bit more difficult. By taking the more detailed view on (4.1) or (4.2) we will realize that we can define at least two approaches for determining parameters c_1, c_2 and c_{12}. One is

based on measurement with constant inflow, the other on experiment with zero inflow – tank's emptying.
In the first approach based on measuring steady state under constant inflow the resulting state equation reduces to (4.1) is

$$\begin{aligned} \frac{1}{A_1}u_1 &= c_1\sqrt{x_1} \\ c_1 &= \frac{u_1}{A_1\sqrt{x_1}} \end{aligned} \tag{4.3}$$

The second approach is based on evaluating transients under zero inflow – i.e. during tanks' emptying. The flow coefficient c_1 of the valve can be easily obtained from (4.1). If the pump is off, the state equation (4.1) reduces to

$$\begin{aligned} \dot{x}_1 &= -c_1\sqrt{x_1} \\ y_1 &= x_1 \end{aligned}$$

In this case, by integrating above equation, c_1 can be get as

$$c_1 = \frac{2\sqrt{x_{1\text{init}}} - 2\sqrt{x_{1\text{final}}}}{\Delta t_1} \tag{4.4}$$

where $x_{1\text{init}}$ represents the initial level of the liquid before opening the valve and $x_{1\text{final}}$ its final level when we finish experiment by closing the valve. Finally, Δt_1 represents the duration of the experiment.
The second way of determination c_1 does not require to know the exact inflow from the pump, which can leads to higher accuracy. This way is also faster. Time required to prepare and perform the measurement takes just about minute.
In both cases it is convenient to perform several measurements at different level heights x_1 and to define the resulting value c_1 as a mean value. It is caused mainly by the fact that the solenoid valves do not have aperture of a simple circular shape, which, together with the relatively low pressure and low level height may lead uncertain and to slightly fluctuating values of the valve constant c_1.
Indetermining the valve constant c_{12} describing interconnection between the tanks we can use similar approach. The difference would only be in the fact that during the experiment the outflow valves from tank 1 and 2 will be closed and before its beginning one tank will be filled to the maximum level and the other one made empty. We stop the measurement just before the complete alignment of both levels. The resulting relation defined from

(4.2) for c_{12} will be

$$c_{12} = \frac{\sqrt{|h_{10} - h_{20}|} - \sqrt{|h_{11} - h_{21}|}}{\Delta t_{12}}$$

With h_{10}, h_{20} as the initial liquid's levels in the first or second tank, h_{11}, h_{21} as the final liquid's levels after finishing the experiment and t_{12} as the duration of the measured interval.

It is important to mention that in case with the tank's outflow orifice not being exactly at the same altitude as the valve's drain orifice (but lower), we have to consider this difference. Then it is to consider that the real liquid's level in the tank producing the resulting pressure at the valve orifice is

$$x_1 = x_{1\mathrm{tank}} + x_{1\mathrm{offset}}$$

where $x_{1\mathrm{tank}}$ represents the hight of the water collumn in the first tank and $x_{1\mathrm{offset}}$ denotes the offset between tank orifice and valve outflow.

The pump identification seems to be easier. It may be based on determining time periods, in which the tank with closed valves will be completely filled under consideration of different pump's input voltage.

$$q_1 = \frac{A_1 x_1}{\Delta t_1}$$

It is vital to choose various working points in whole pump's working scale. Finally, since the pump is a reasonably nonlinear element, we approximate the measured pump characteristic by the curve fitting methods, e.g. with the polynomials of third or fourth degree. The input-output characteristic of the first pump, the inverse input-output characteristic of the pump and the characteristic of the first valve are depicted in Fig. 4.6.

4.2.2 Sensors Calibration

To measure the height of liquid's level in the plant in Fig. 4.1 the pressure sensors are used. These have several advantages: they are easy to change in a case of failure, they don't require any maintenance, they don't corrode and water sediments don't attach to them. On the other hand, the measurement depends on the surrounding pressure, thus it is recommended to perform at least at the beginning of the measurement day, the calibration of these sensors. It consists of the definition of offset and sensor's gain values, what should be done for each liquid's level separately. The dependency is linear. The basic scheme of the conversion of raw signals on liquid's levels in meters can be found in Fig. 4.4. (All the necessary parameters for signal adjustment lay in one array denoted as *calibParams*. Array "pumps"

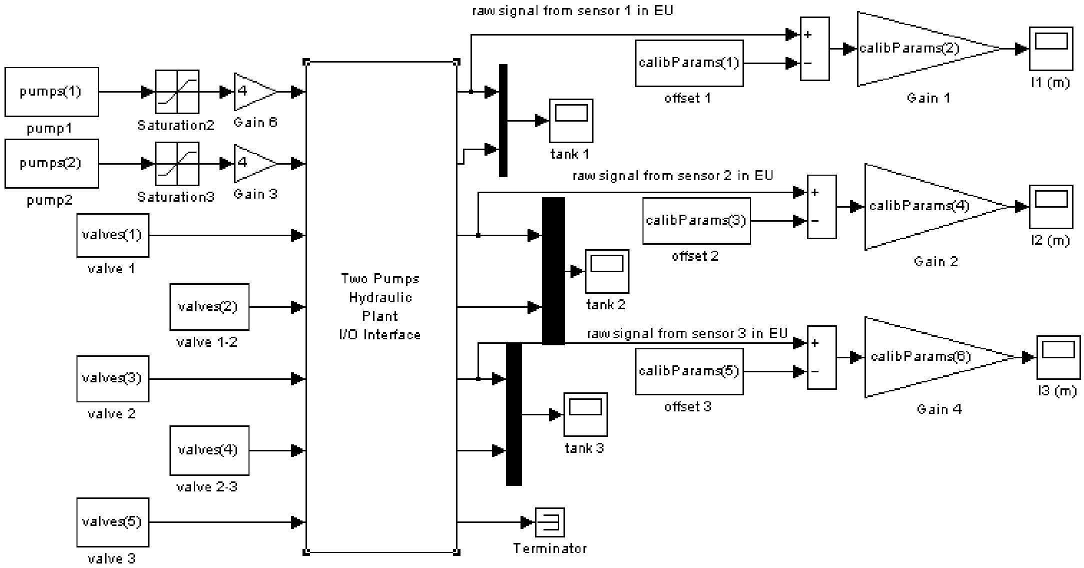


Figure 4.4: Basic diagram for calibration and configuration.

contains the pump's entrances (performance in % from 0% to 100%) and field "valves" contains the valves configuration 0 – close, 1 – open, for each valve.)
Under manual calibration we will partially fill the calibrated tank and empty it subsequently. Raw integer number from each output is sensor offset $x_{1\text{empty}}$. This is specified in engineering units (EU). Afterwards, we fill up the tank to the maximum height of liquid's level and we define the output's value $x_{1\text{empty}}$ in EU. We deduct, once again, the value of sensor's outputs. The conversion to liquid's level in meters is defined by the relation

$$\begin{aligned} sensorGain_1 &= \frac{(x_{1\text{full}} - x_{1\text{empty}})}{\Delta x_1} \\ x_1 &= \frac{x_{1\text{actual}} - x_{1\text{empty}}}{sensorGain_1} \end{aligned} \tag{4.5}$$

Here, $x_{1\text{actual}}$ is the actual value of liquid's level in EU, $x_{1\text{empty}}$ is the output in EU at the zero liquid level and $x_{1\text{full}}$ is the value of the output in EU at maximum liquid level and Δx_1 is the height of the maximum liquid level. For example, if we know that at the zero level $x_{1\text{empty}} = 800\,\text{EU}$, at the maximum level $\Delta x_1 = 0.25\,\text{m}$ and output 3300 EU, for the output 2300 EU it holds:

$$\begin{aligned} sensorGain_1 &= \frac{3300 - 800}{0.25} = 10000\,\text{EU}\,\text{m}^{-1} \\ x_1 &= \frac{2300 - 800}{10000} = 0.15\,\text{m} \end{aligned}$$

After the calibration it is suitable to perform identification of system's

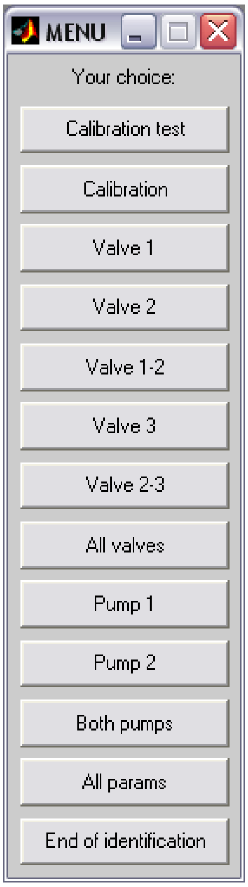


Figure 4.5: Menu of the software module for calibration and identification.

parameters. It consists of the estimation of the flow coefficients of valves and of the pump's parameters according to the above-mentioned process.

4.2.3 Automatic Calibration and Identification

The above mentioned procedure of calibration and identification is obviously time consuming and also not easy to calculate manually. To perform the such long time step-by-step measurements of the pump's characteristics plus to save the measured values subsequently after each measurement and evaluate them afterwards, we have to sacrifice an enormous quantity of time. That's all is the reason why we developed the software package for automatic identification of system's individual parameters and for identification of all valve and pump parameters. The control is very easy – only one click is needed to get the required identification in the menu. Menu's environment can be seen in Fig. 4.5.
As the processes were described in details in the previous chapter, the individual functionalities will be mentioned now just shortly. At the identification of the valve 1,2 or 3, the tank is fill up to the height of approximately

24.5 cm, then we wait till the liquid's level is stabilized and the tank is afterwards emptied to 0.8 cm. (At almost zero level, the valve's behaviour is strictly non-linear and as we are not planning to regulate the level under 1 cm, we don't consider the lowest boundary). The measured values are saved, together with all used constants, into the mat file in the following format:

`valveValvenumber-year-month-day-hour-minute-second.mat`.

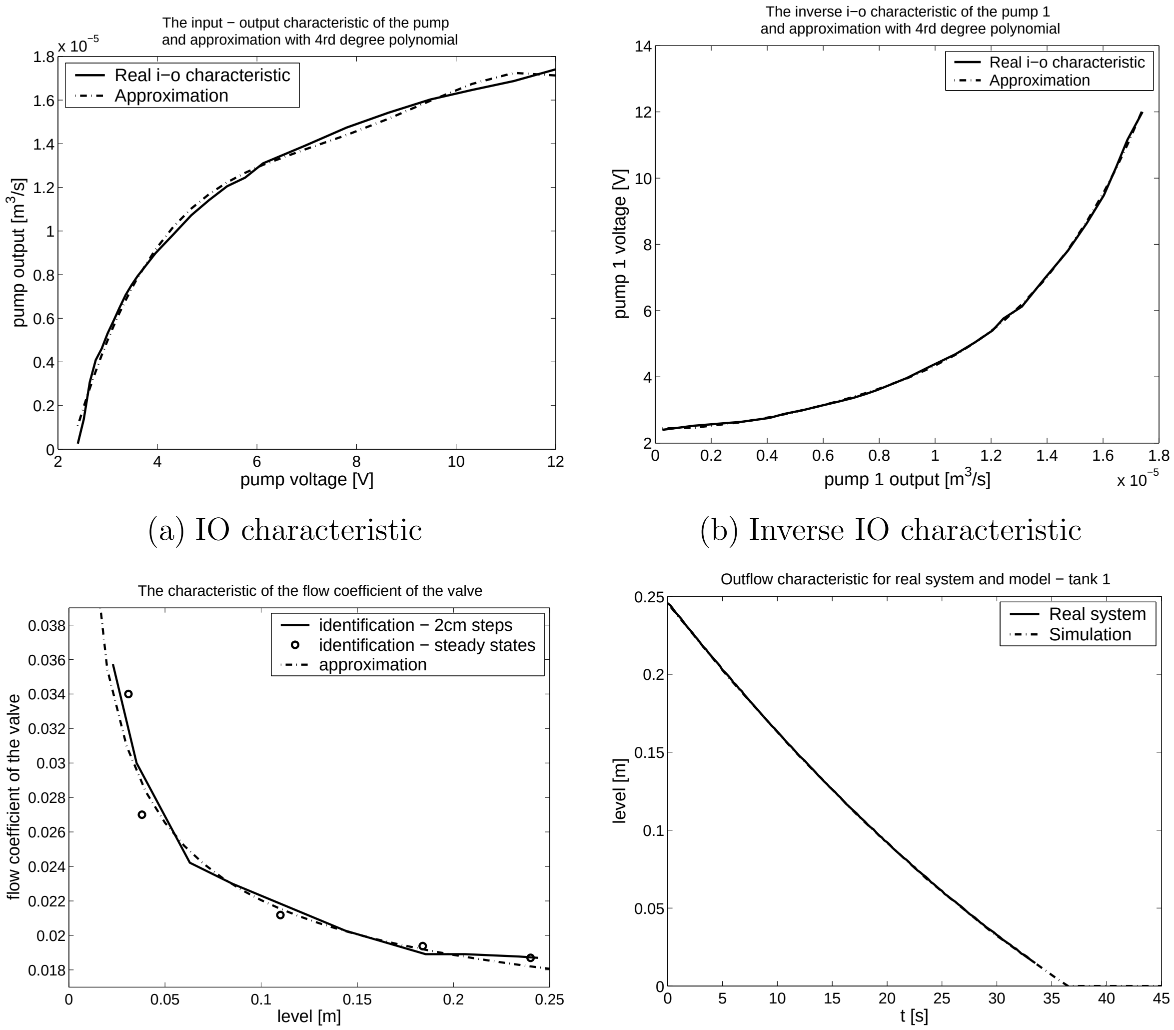
For example: `valve1-2009-12-20-16-26-43.mat` and so on. We don't consider the first and the last measurement sample, from the other values with the step approximately 1.5 cm – we use each sixth sample, define the value c_1 as well as the possible offset value of the valve $offset_1$.

At the end we define the mean value of offset and of flow coefficient and we save those two values together with the time course of emptying into the valve1, valve2 and valve3.mat. We apply them directly to the proposed model, we simulate the emptying and we draw the comparison of simulation and real data into the graph of emptying characteristics such as Fig. 4.6. This procedure allows us to check visually quality of the valve's identification on the graph. The registration to the actual mat file allows us to read variables in the program according to our choice, at the direction of the model or at the work with the real system.

The previous identification may sometimes look better than the new one – that's why the measurements are stored in the mentioned file with date and time – that way we are able to see the history at any time. We apply the similar procedure at the other measurements as well.

Basically in the same way we identify the valve between the tanks. After the measurement is launched, the first tank is filled and the second emptied, the interconecting valve is opened afterwards and the measurement is performed until the leveling of two liquid's levels. Then we perform the second measurement, the second tank is filled up and the first emptied. We identify again the flow coefficient and we use those two measurements to define the mean coefficient c_{12}. We save the measured values, following the above-mentioned procedure, just this time the mat file will be named valve12.mat – for the actual value and valve12day-time.mat for archiving. We'll use the identified coefficient for the simulation and we draw the comparison of model's course and the real into the graph. In case of lesser compliance, we repeat the procedure.

The measurement of pump's characteristics is more time-consuming. In order to achieve the required accuracy, we continue with increasing the pump input by the step 3% of the maximal range 100% up to the moment



(a) IO characteristic

(b) Inverse IO characteristic

(c) Determining c_1 by the first method

(d) Determining c_1 by the second method

Figure 4.6: Measured Input-Output pump characteristic and its approximation by the 4th degree polynomial and inverse input-output pump characteristic and its approximation by the 4th degree polynomial (above). Determination of the valve coefficient c_1 with two approaches – comparing the measurement and approximative data for tank 1 (below).

where the pump starts slowly to draw the liquid. (In our case it will be around 19 – 21 %.) At lower inputs, the pump is not working.
At the beginning of measurement, we empty the tank, and then fill it up by given input up to the fullest state, or during 60 seconds. That's because at the lowest levels of input, the filling up would take too much time. After the last measurement – for full input, we define the maximum inflow of the pump as well. We put the given inflows and power tensions into the field and save them, at the end of experiment, by same way as valves, into the mat files. The first pump's name will obviously start by pump1 and the second one by pump2.
During the individual steps, we include into the measurement the artificial break of 40 s, to prevent the changes in pump's parameters, its heating or to partially isolate the previous measurement from the actual one. We draw the measured IO or inverse IO characteristics into the graph and compare them with the approximations acquired thanks to curve fitting functions.
As the buttons' names indicate, by the above-described procedures we can identify all valves, both pumps and all system's parameters.
To clean the older data, in order to delete the measured and saved values more easily, we can use the script *cleaning* which will preserve only the measured values and will delete all the mat files that contain the date in the name. That's why one has to be very careful when using it. We should use it only if we are sure, that we don't want to archive the older values.
After we get all the required parameters, we only have to load them and have a model that will use them correctly. Pump's subblock is depicted in Fig. 4.7 where function block 'U->Q1' contains:

```
PU1(1)*u(1)^4+PU1(2)*u(1)^3+PU1(3)*u(1)^2+PU1
    (4)*u(1)+PU1(5)
```

This 4 degree polynomial enables linearization of the pump. The input for this function is inflow from controller (in $\mathrm{m^3\,s^{-1}}$). Function convert this input to Volts. The 'Gain4' changes Volts to %, because the pump input in matlab block is in %.
There is also treshold. For too small input (smaller than $10^{-6}\mathrm{m^3\,s^{-1}}$) push to output 0. This feaure respects the fact that the pump can not process smaller inputs.

4.2.4 Some Recommendation for Users

At the end we would like to add some recommendations that may help you in keeping your plant in optimal work.

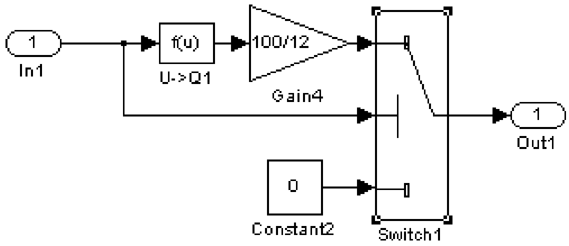


Figure 4.7: Pump implementation in simulink.

Use always distilled water, even in the case you need to add just small amount of it. In this way you may reasonably prolonged intervals between clearings.
For removing sediments from the walls of the tanks use a soft brash appropriate for bottle cleaning. Pump out the water containing such sediments and flush the system several times by clean water. After that you need to pump out also the standard water used for cleaning and again fill the container by the distilled one.
For eliminating the evaporation fill in the water into tanks that are covered and have smaller evaporation area.
After a longer period without use it may happen that some valve does not open or close. In such a case it is necessary to repeat open/close procedure several times (even when it does not function) and then slightly hit the metal screws in the valve centre. This may help to release the valve.
When you know that the system will not be used for a longer time, pump out all water.
When you keep all these rules, the system does not require any maintenance. Only in extreme situations when the system was not used for longer time period without pumping out all water, the sediments may not only cover the walls of containers, but even tubes, pressure sensors and valves. When facing problems with pressure sensor, after emptying the containers by a gentle pull remove the sensor from the tube and clean the tube by a thin wooden tool. When the sediments clog up the valve, it is necessary to unscrew fittings from both sides and to flush the valve under pressure, e.g. by a rubber ball filled by water. As it was, however, mentioned above, in case of regular maintenance and storing such problems will not occur.

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Chapter 5

Tasks for Coupled Tanks Control

This chapter treats basic issues of linear and non-linear controller's design applied to control of hydraulic plant of availabe three-tank-system. Controllers based on approximation of the given non-linear system by the first order linear system by step responses are proposed and compared with controllers based on the analytical linearization of the nonlinear model in a fixed operating point, by the input-output linearization in the actual state (exact linearization) and around a generalized operating point are developed and verified for the one-level and two-level system.

5.1 Introduction

A plant to be controlled may be modelled and identified in many ways. It is e.g. possible to derive mathematical models of the plant that for a given data show nice matching but when used for the controller design, the resulting performance will be pure. In case of inaccurate, or inappropriate mathematical model, the resulting regulators would function only in simulations. That's why we will be dealing from now with the description of the real plant, estimation of its model, identification of its parameters as well as with confrontation of results achieved in control of the plant with the performance required in the controller design.
In the next paragraph, we will show some examples of simple P, PI and PD controllers for different tanks' configurations, from the simplest one tank system, with closed output valves, through one-level systems with one inflow and one outflow up to two-level system linearized by generalized input-output linearization and controlled by the PD_2 controller.

5.2 Basic P and PI controllers

At the beginning, it would be the best to introduce the simplest case of hydraulic system's configuration. In doing so we will consider one-level system which has all the valves closed. The pump feeding the liquid into this tank will represent the actuator. Under control, such system behaves as simple integrator. Considering 4.1 we will define differential equations of such system in the following form:

$$\begin{aligned} \dot{x}_1 &= \frac{1}{A_1}u_1 \\ y_1 &= x_1 \end{aligned} \tag{5.1}$$

Obviously, we are dealing with linear system with the transfer function

$$F_1(s) = \frac{Y_1(s)}{U_1(s)} = \frac{1}{A_1 s}$$

i.e. with integrator having the integral time constant A_1 (gain $1/A_1$). From theory we know that to stabilize such a system (5.1), P-controller with a gain P_1 would be sufficient. The closed loop transfer function will be:

$$G(s) = \frac{P_1 F_1(s)}{1 + P_1 F_1(s)} = \frac{P_1}{A_1 s + P_1}$$

If we want to control this system, we can request it to behave as the first order linear system with the transfer function

$$G(s) = \frac{1}{T_1 s + 1}$$

In such a case, the P-controller's gain results as

$$P_1 = \frac{A_1}{T_1}$$

It is clear that by choosing the time constant T_1, we are able to define the dynamics of the closed loop that might also be expressed by its closed loop pole $\alpha = -1/T_1$. By decreasing this time constant, when the closed loop pole is being shifted to $-\infty$, we can speed up the transients processes. And contrary, by increasing T_1, when the pole is shifted towards the origin, we can slow down them. In increasing the P-controller gain we have to be careful and speed-up the transients only until certain limit.

The limitations are mainly given by factors as nonmodelled dynamics, plant uncertainty, measurement noise and control signal saturation. For shorter time constant T_1, saturation of the control signal typically occur and for $T_1 \to 0$ the control signal tends to rectangular pulse of the minimum time control (Fig. 5.1). P-controller for integrator plant represents solution of the dynamical class 0.

If we increase the amplification of controller too much, the sensitivity of the circuit to the measurement and quantisation noise will increase and this can lead to system's destabilization, increased wear of active elements

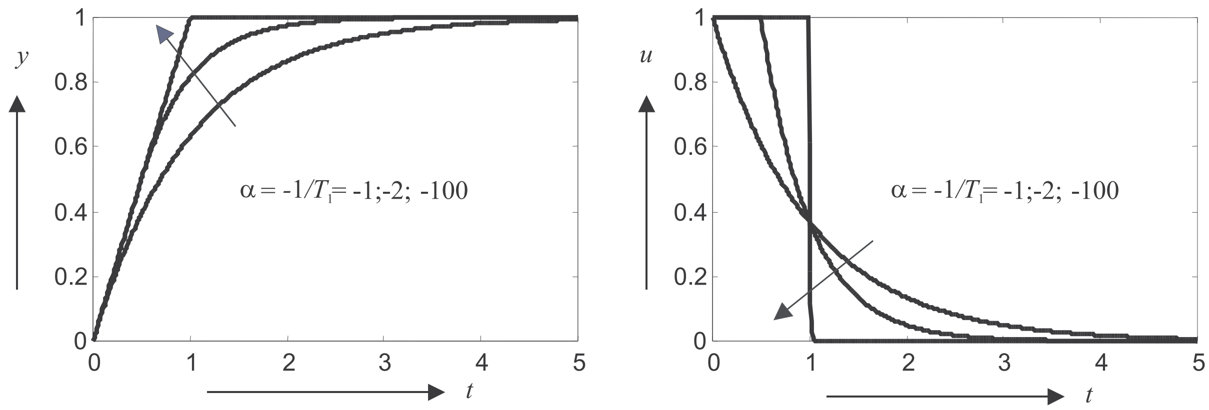


Figure 5.1: Impact of the P controller tuning on the tranient responses

or damage – in this case, by switching the pump on and off over a too long period to maximum or zero voltage.
On the other hand, too small controller gain can slow down the transient processes, or, in an extreme case, the pump will even not start to move. In order to get some insight, which controller gain may be applied without leading to parasitic oscillations, it is necessary to approximate the non-modelled loop dynamics that can be characterized by equivalent dead time T_d. This can be identified as the time between a control signal step and beginning of the reaction of the measured output. Then, it is recommended not to use shorter time constants ("faster" poles) as those given by

$$T_1 = \mathrm{e}^1 T_d; \alpha = -1/\left(\mathrm{e}^1 T_d\right) \tag{5.2}$$

For the given hydraulic system, notice that the pump is reacting only to positive values of the input voltage. So, its control is only possible in one way – to increase the liquid's level in tank. That's why it is necessary to empty the tank before the experiment, or after it.

5.2.1 PI-controller

Now, let's consider situation with opened outflow valve characterized with the flow coefficient c_1. In this case the one-level system will be in the form 4.1. This system does no longer behave as pure integrator. Let's try to control it, at the beginning, with P-controller, designed in the previous part. By examining few operating point, we will realize that P-controller can't control this system without the permanent control error. It is no surprise, if we take into account that the opened outflow requires permanent inflow, but the zero control error leads to zero inflow. With zero control error generates the P-controller zero action, but considering the fact that

the liquid outflows from the tank, it decreases the liquid's level below the desired values. To compensate the outgoing liquid we have to use more advanced controllers.
The easiest way to compensate the outgoing water is to add parallel signal to the output of the controller. Such constant signal set usually manually was originally called as reset (offset). Later, this was replaced by *automatic reset* denoted today as the *integral action* and produced directly by the controller. By this modification we will obtain the PI-controller. The goal of the integral action is to integrate the control error values multiplicated by a suitable constant what will continuously increase the controller output – until the complete removal of the control error. The transfer function of such PI-controller is:

$$R(s) = P + \frac{I}{s}$$

where P represents the gain of the proportional action of the controller and I is the gain of the integral action.
An easy experimental way to set up the PI-controller's parameters may be based on measured step-responses of the system. The step-responses of the system contain information about the basic dynamic attributes of the system and this is why it enables us to set up the controller's parameters better than by the frequently used *trial and error* method. At the same time, we have to realize that the step-response method was developed for the responses of linear system to a unit step. Our system (4.1) is, however, a nonlinear one. We can't thus talk about the system's step-responses for the whole scope of liquid's levels. The obtained information will depend on the choice of concrete operating point, in which the step will be measured. For the PI-controller design having the best attributes in the most ample areas, it may seem to be useful to choose the operating point approximately in the middle of the working area.
In the surroundings of chosen operating point we will approximate our non-linear system (4.1) by the first order linear model. This will be obtained by the step-response method from step responses measured firstly by setting up by appropriate input pump voltage the liquid's level approximatelly to the the middle of the tank hight and here to stabilize it for some time, optimally few centimeters under the middle. When the liquid's level will be stabilized, we will switch the pump input to higher voltage by approximately $1V$ step and wait again for the liquid's level to stabilize (it should stabilize few centimeters over the middle). From the measured step-response we will find out by constructing tangent at the origin the approximate value of time constant T_{1_n} and the value of the approximative

plant gain $K_1 = \Delta y/\Delta u$.
So, in a close surroundings of the operating point corresponding to the input voltage about $u_1 = 4V$ the system may be approximated by linear transfer function

$$F_1(s) = \frac{K_1}{T_{1_n}s + 1}$$

Then, in the previous paragraph mentioned P-controller design may now be used as well. When the open loop transfer function was in the case of the P-controller design given as $1/T_1 s$ the closed loop had the transfer $1/(T_1 s + 1)$. Now, we will design controller to yield the same open loop transfer function as previously, i.e. $1/T_1 s$, what yields

$$R_1(s) = \frac{1}{T_1 s} \cdot \frac{1}{F_1(s)} = \frac{T_{1_n}s + 1}{K_1 T_1 s} = \frac{T_{1_n}}{K_1 T_1} + \frac{1}{K_1 T_1 s}$$

Thus the P and I actions will be

$$\begin{aligned} P_1 &= \frac{T_{1_n}}{K_1 T_1} \\ I_1 &= \frac{1}{K_1 T_1} \end{aligned}$$

After calculating and testing this tuning we recommend you to experiment a bit and to track the influence of the changes of P_1 and I_1 on the control performance. You may observe that the achieved dynamics is differnt from the one possible pulse at saturation that was typical for the P controller design. This soultion represents basic part of the PI controller of the dynamical class 0 (it should yet be completed by a prefilter with the time constant T_{1n} – then the control signal after a setpoint step exponentially increases monotonically to its new steady-state value). Attempts to achieve faster dynamics may be connected with strong overshooting of the output signal.

5.2.2 PI$_1$ controller

Improved dynamics for larger setpoint steps without producing windup effect is possible to achieve by the so called PI$_1$ controller that typically has one interval of the control signal at the saturation limit. This controller is based on reconstruction of the equivalent input or output disturbances by appropriate observer using inverse or parallel plant model. Since the hydraulic plant (4.1) is stable, inversion of the plant dynamics may be directly derived also for the nonlinear model as

$$\widehat{u}_1 = A_1 \left[\dot{y}_1 + c_1\sqrt{y_1}\right] \tag{5.3}$$

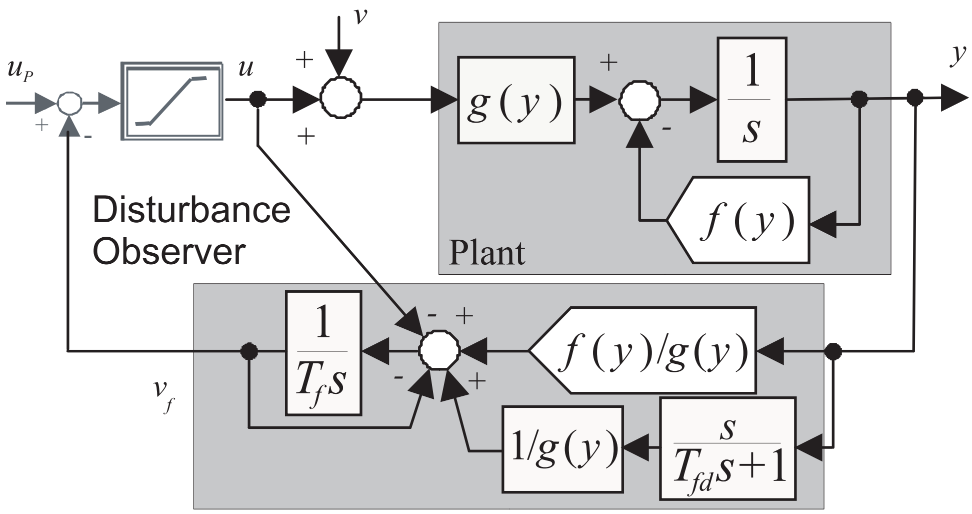


Figure 5.2: Rejection of the input disturbances v by correction of the P controller output in the nonlinear PI_1 controller for system $\mathrm{d}y/\mathrm{d}t = g(y)(u+v) - f(y)$

The input disturbance v may then be reconstructed by comparing the reconstructed plant input $\widehat{u}_1$ and the controller output according to

$$\widehat{v} = \frac{\widehat{u}_1 - u}{1 + T_f s} \tag{5.4}$$

and then compensated at the P-controller output u_P. Such a compensation may be treated as a special case of nonlinear disturbance observer in Fig. 5.2. Besides of the P-controller gain, the new tuning parameter T_f may be used to increase system robustness against uncertainties and non-modelled dynamics and to filter measurement noise.

You may test your integral controller by filling to different levels also the second tank and then opening or closing the inteconnecting valve among the tanks.

The controller structure in Simulink is depicted in Fig. 5.4. Control program, which work with results of automatic calibration and indentification is listed below.

5.3 Linearization around a fixed operating point

Analytical linearization around a fixed operating point shows one of the simplest ways of designing controller for the one level system. Linearized model may be determined directly from the system's equation (4.1). For a chosen working point and its closest environment, the non-linear system

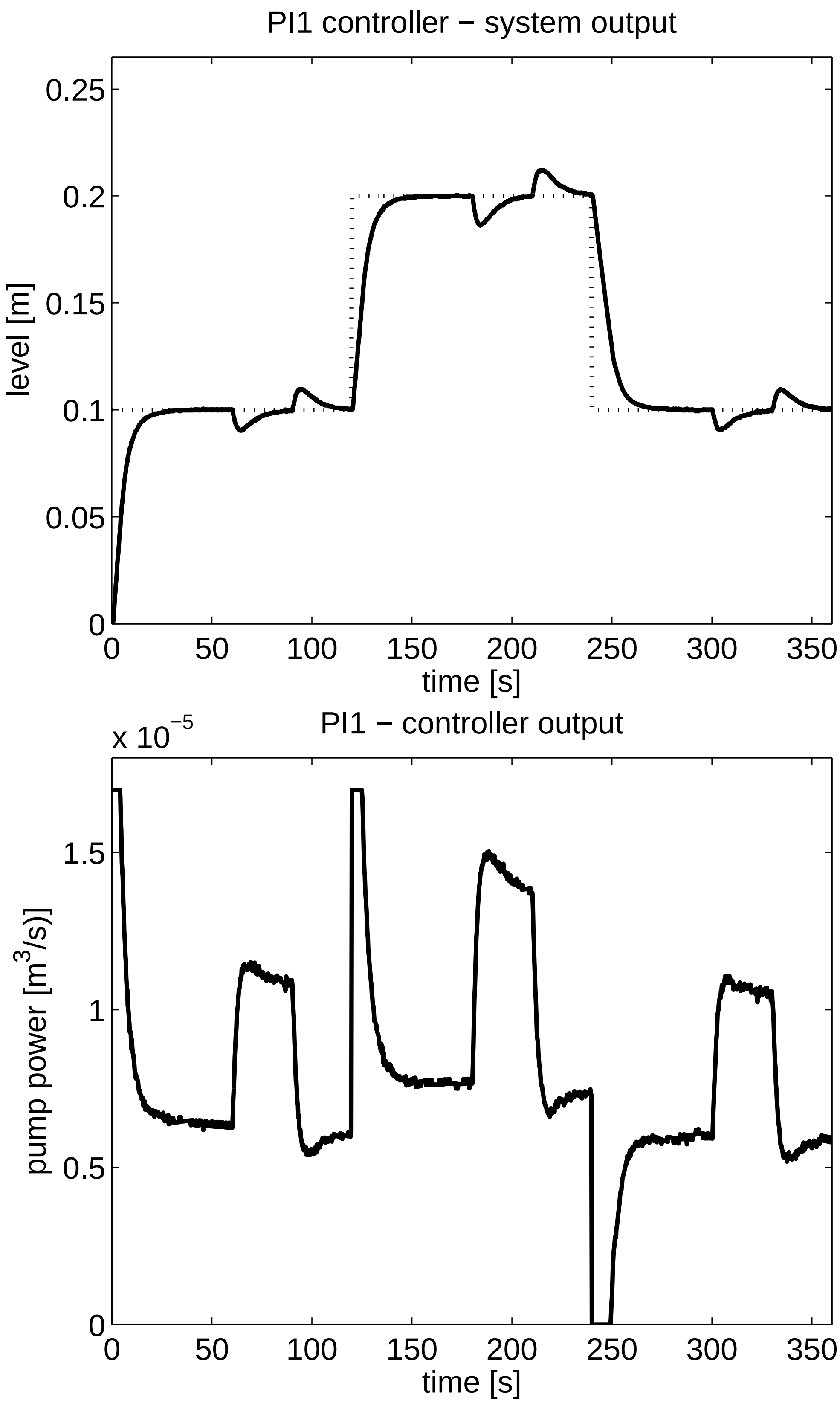


Figure 5.3: Typical setpoint and disturbance steps in the loop with PI_1 controller contain one step at saturation after setpoint steps

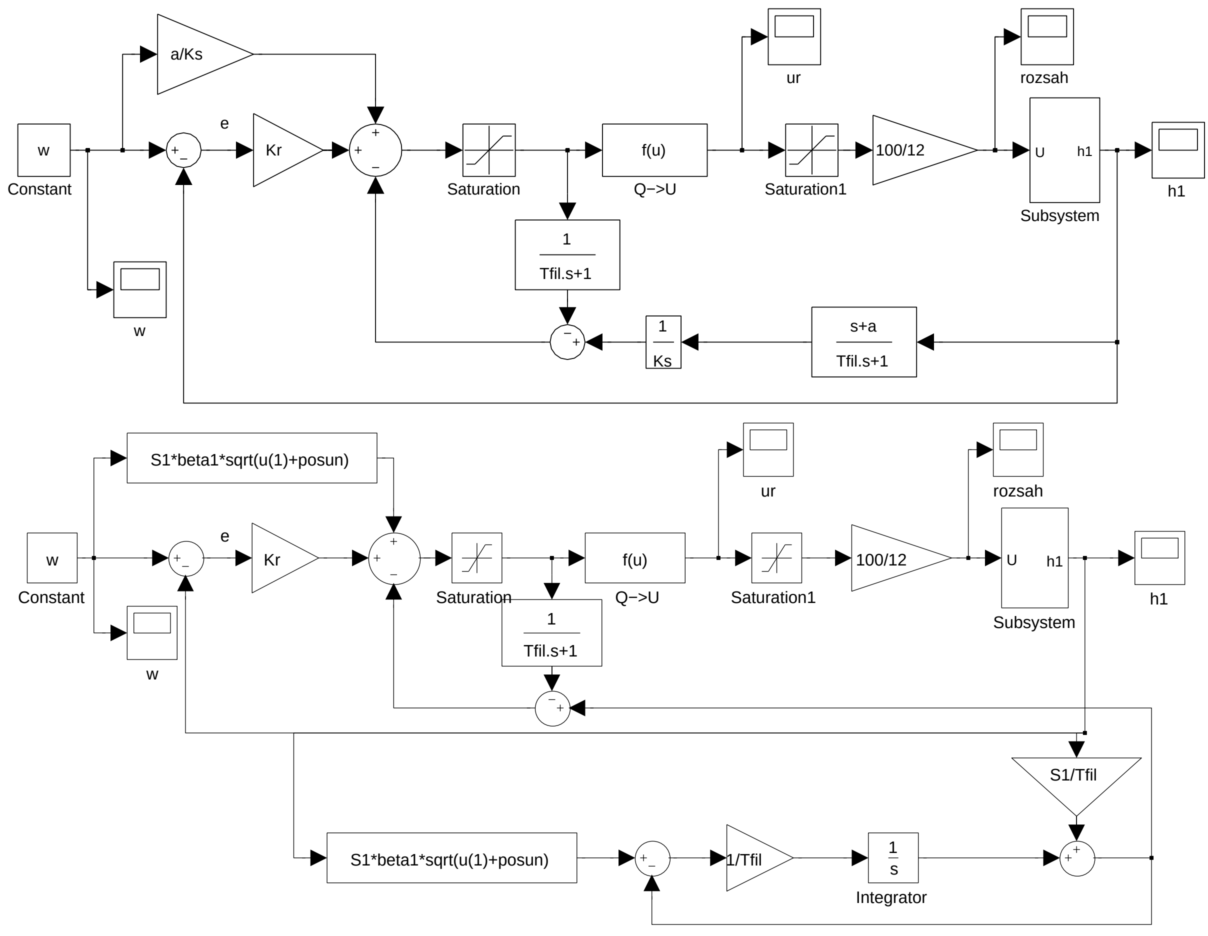


Figure 5.4: Control structure for linear and nonliner PI1 implemented in Matlab / Simulink

will behave as linear and therefore we can use this as the basic possibility for deriving linear controllers, e.g. the PI one.
While making this linearization we talk about approximation of the non-linear system

$$\begin{aligned}\dot{x}_1 &= f(x_1, u_1)\\ y_1 &= g(x_1)\end{aligned}$$

by a linear system

$$\begin{aligned}\Delta x_1 &= \vec{A}\Delta x_1 + \vec{B}\Delta u_1\\ \Delta y &= \vec{C}\Delta x_1\end{aligned}$$

where $\Delta x_1 = x_1 - x_{1_0}$, $\Delta u_1 = u_1 - u_{1_0}$, $\Delta y_1 = y_1 - y_{1_0}$ and x_{1_0}, u_{1_0}, y_{1_0} is our fixed operating point. It is to stress that the linearization is considered around a steady state, state, input and output are replaced by deviations from it as: Δx_1, Δu_1 and Δy_1 and the matrices A, B and C are in general given as:

$$\vec{A} = \left(\frac{\partial f}{\partial x_1}\right)_{x_1=x_{1_0}, u_1=u_{1_0}}, \vec{B} = \left(\frac{\partial f}{\partial u_1}\right)_{x_1=x_{1_0}, u_1=u_{1_0}}, \vec{C} = \left(\frac{\partial g}{\partial x_1}\right)_{x_1=x_{1_0}, u_1=u_{1_0}}$$

In case on one level hydraulic system (4.1)we get by such a linearization

$$\begin{aligned}\Delta\dot{x}_1 &= \frac{1}{A_1}\Delta u_1 - \frac{c_1}{2\sqrt{x_{1_0}}}\Delta x_1\\ \Delta y_1 &= \Delta x_1\end{aligned} \tag{5.5}$$

We would like to point that (5.5) is not valid for $x_{1_0} = 0$. Taking into account that our system (5.5) is linear, we can consider its velocity transfer function

$$F_1(s) = \frac{\Delta Y_1(s)}{\Delta U_1(s)} = \frac{\frac{1}{A_1}}{s + \frac{c_1}{2\sqrt{x_{1_0}}}}$$

Now for this system we propose PI-controller (for the first order linear system). Transfer function of the controller should again be made in such a way to yield an open transfer function $1/T_1 s$ what may e.g. be achieved by choosing

$$R_1(s) = \frac{1}{T_1 s}\cdot\frac{1}{F_1(s)} = \frac{s + \frac{c_1}{2\sqrt{x_{1_0}}}}{\frac{1}{A}T_1 s} = \frac{A_1}{T_1} + \frac{A_1 c_1}{2T_1\sqrt{x_{1_0}}s}$$

where

$$\begin{aligned}P_1 &= \frac{A_1}{T_1}\\ I_1 &= \frac{A_1 c_1}{2T_1\sqrt{x_{1_0}}}\end{aligned}$$

are gains of P_1 a I_1 action of the controller. Note that the time constant T_1 can be easily obtained from step-response of the system. We should not omit the fact, that by designing the controller, we proceed from the system (5.5), which works with deviations from steady state, not with the absolute varaibles. That's why the controller is using them as well, i.e. it holds

$$R_1(s) = \frac{\Delta U_1(s)}{\Delta E_1(s)}$$

where $\Delta U_1(s)$ and $\Delta E_1(s)$ are Laplace transforms of the deviations of the controller output, $\Delta u_1 = u_1 - u_{1_0}$ and of the control error, $\Delta e_1 = e_1 - e_{1_0}$, from steady states u_{1_0} a e_{1_0}. Of course, for control error in steady state is $e_{1_0} = 0$. In this case $\Delta e_1 = e_1$, but at the controller output one still has to consider deviation signal Δu_1. The system (4.1), which we are in fact controlling, has as its input directly u. So, we need to arrange the controller output by adding the stabilized steady-state value u_{1_0} and thus obtaining directly u_1

$$u_1 = \Delta u_1 + u_{1_0}$$

The value u_{1_0} represents the stabilized value of the pump's voltage, at which the liquid's level or liquid's volume in the tank has the stabilized value x_{1_0}. The numerical value u_{1_0} is, off course, needed at the implementation of the controller. We'll define it based on system's (4.1). In a steady-state all time derivatives are zero, thus $\dot{x}_1 = 0$. After substituting into (4.1) we'll obtain u_{1_0} defined as

$$u_{1_0} = A_1 c_1 \sqrt{x_{1_0}}$$

5.4 Exact Feedback Linearization

In this task, we would like to control the liquid's level for different operating points. By appropriate gain scheduling it is possible to have various operating points and the associated controllers and then to switch between them, but as more practical it seems to schedule the controller parameters continuously. This is usually done by the input-output feedback linearization. In general, it is supposed a non-linear system in the following form:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x) \end{aligned}$$

where the state $x \in \vec{R}^n$, the input $u \in \vec{R}^m$, the output $y \in \vec{R}^p$. Find, if possible, a static state feedback

$$u = \varphi(v, x)$$

in such a way that the system would be from point of view of new input v and output y linear. For the solution to exist, it has to be given that the system's relative degree r_i is natural number. The system's relative degree r_i is a natural number that defines the number of output's derivations y_i we have to perform in order to obtain the direct dependence on the output u. In case of non-existence of such number, we say that the output has the infinite relative degree.

$$r_i = \min\{k \in N; \frac{\partial y_i^{(k)}}{\partial u} \neq 0\}$$

More information can be found for example in Conte et al (2007).
For system (4.1) we may say

$$\dot{y}_1 = \frac{1}{A_1} u_1 - c_1 \sqrt{x_1} \tag{5.6}$$

that it has the relative degree 1. But also the system (4.2) has the relative degree 1 because

$$\begin{aligned} \dot{y}_1 &= \frac{1}{A_1} u_1 - c_{12} \operatorname{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|} - c_1 \sqrt{x_1} \\ \dot{y}_2 &= \frac{1}{A_2} u_2 + c_{12} \operatorname{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|} - c_2 \sqrt{x_2} \end{aligned}$$

the basic condition is satisfied. The problem of the static-state feedback linearization is solvable if

$$\operatorname{rank} \frac{\partial (y_1^{(r_1)}, ..., (y_p^{(r_p)})}{\partial u} = p$$

where r_i are the relative degrees of outputs y_i, for $i = 1, ..., p$. Static-state feedbacks can be found by solving equation

$$y_i^{(r_i)} = v_i$$

The proof, technical details and additional references can be found in Conte et al (2007).
For the system (4.1), the static-state feedback that solves the linearization problem can be found by solving the equation

$$\dot{y}_1 = v_1$$

for u_1 that is

$$u_1 = A_1 v_1 + A_1 c_1 \sqrt{x_1} \tag{5.7}$$

When we apply this, the feedback we get from the original system (4.1) linear system $\dot{y}_1 = v_1$ with transfer function:

$$F_1(s) = \frac{1}{s}$$

Now it is enough to design controller which will provide the required loop behavior. It will be necessary for it to have opened loop transfer function $\frac{1}{T_1 s}$. When we talk about simple first degree integrator, in this case it is enough use simple P-controller. Its gain will be

$$P_1 = \frac{1}{T_1}$$

For the two-level system (4.2) we will do the same. By requiring

$$\begin{aligned} \dot{y}_1 &= v_1 \\ \dot{y}_2 &= v_2 \end{aligned}$$

the solution will be given equations for u_1 and u_2

$$\begin{aligned} u_1 &= A_1 v1 + A_1 c_{12} \operatorname{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|} + A_1 c_1 \sqrt{x_1} \\ u_2 &= A_2 v_2 - A_2 c_{12} \operatorname{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|} + A_2 c_2 \sqrt{x_2} \end{aligned}$$

Similarly to previous case, after application of these feedbacks we obtain two independent (decoupled) systems $\dot{y}_1 = v_1$, $\dot{y}_2 = v_2$ having the transfer functions

$$F_1(s) = \frac{1}{s}$$
$$F_2(s) = \frac{1}{s}$$

We can continue again in the same way as with the one level system. As we now have two independent liear systems, for each level we can design separate P-controller.

$$P_1 = \frac{1}{T_1}$$
$$P_2 = \frac{1}{T_2}$$

We would like to recall as well the special case of two tanks connection. It differs from (4.1) in the way that the outflow from the first tank is at zero

– valve 1 is closed, the second pump is turned off and the liquid's height in the second tank is controller by controlling the inflow into the first tank. We can write down the system equations as it follows:

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{A_1}u_1 - c_{12}\sqrt{x_1 - x_2} \\
\dot{x}_2 &= c_{12}\sqrt{x_1 - x_2} - c_2\sqrt{x_2} \\
y &= x_2
\end{aligned} \tag{5.8}$$

The above-mentioned system has the relative degree $r = 2$. That's why we have to proceed, when looking for the feedback, from the equation:

$$\ddot{y} = v_1$$

so, u_1 can be defined as

$$u_1 = \frac{2A_1}{c_{12}}v_1\sqrt{x_1 - x_2} + A_1c_{12}\sqrt{x_1 - x_2} - 2A_1c_2\sqrt{x_2} + A_1c_2\frac{x_1}{\sqrt{x_2}}$$

Different from the previous examples, by application of this feedback we obtain from the original system (5.8), linear system $\ddot{y} = v_1$ with transfer function

$$F(s) = \frac{1}{s^2}$$

Thus, we would have to apply a different approach to continuous gain-scheduling of the controller paramters based on (5.9), whereby the new input v_1 should be computed in a way respecting all basic performance limitations – nonmodelled dynamics, model uncertainty, meausrement and quantization noise and the contorl signal constraints.

5.5 PD$_2$ controller

In reality, the control signal is always constrained, what can be expressed as

$$u_r \in [U_1, U_2]; \quad U_1 < 0 < U_2 \tag{5.9}$$

So, despite to the fact that the control signal is usually generated by linear controllers, the input to the plant u_r is modified by a nonlinear function that can be denoted as the *saturation* function

$$u_r(t) = \text{sat}\left\{u(t)\right\}; \qquad \text{sat}\left\{u\right\} = \begin{cases} U_2 & u > U_2 > 0 \\ u & U_1 \le u \le U_2 \\ U_1 & u < U_1 < 0 \end{cases} \tag{5.10}$$

In the classical period of control (up to late 60s in the 20th century), the constraints in the control were treated by a huge amount of papers. Then, in the subsequent decades in the main stream of the control theory the problem of constraints practically disappeared. Just a limited number of authors dealing with the anti-windup design continued to investigate this important feature. Today, the problem of constraints is again in the focus of the research activities and it is hardly possible to give here just a brief overview of different approaches.
For the sake of brevity, we will show here briefly one possible approach to controlling the coupled tanks combining constrained pole assignment control Huba and Bisták (1999); Huba (1999); Huba et al (1999); Huba (2001, 2005, 2006, 2010, 2011a,c) with extended exact linearization method that is based on transformation of the existing nonlinear system by (5.9) into the double integrator one and in controlling such double integrator system by the PD_2 controller that fully considers the existing control saturation limits.
Let us start with considering the double integrator system

$$\frac{\mathrm{d}^2\bar{y}(t)}{\mathrm{d}t^2} = K_s\bar{u}\,(t) \tag{5.11}$$

For $y = \bar{y} - w$; w being the reference input, $\mathbf{x}(t) = [y(t), d(t)]$; $d = \mathrm{d}y/\mathrm{d}t$ the system state and

$$u = K_s\bar{u}; \quad U_i = K_s\bar{U}_i; \quad i = 1, 2 \tag{5.12}$$

being the normalized control, it can be described in the state space as

$$\frac{\mathrm{d}\vec{x}(t)}{\mathrm{d}t} = \vec{A}x(t) + \vec{b}u(t); \quad \vec{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad \vec{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5.13}$$

The linear pole assignment PD-controller is given as

$$u = \mathbf{r}^t\mathbf{x} \ ; \ \ \mathbf{r}^t = [r_0; r_1] \ ; \ \bar{u} = u/K_s \tag{5.14}$$

The PD_2 controller is composed from the linear pole assignment controller used for the relatively low velocities and given as

$$u = \frac{\vec{a}^t\left[\alpha_2 I - \vec{A}\right]}{\vec{a}^t\vec{b}}\vec{x} \tag{5.15}$$

$$r_0 = -\alpha_1\alpha_2 \ ; \ \ r_1 = \alpha_1 + \alpha_2 \tag{5.16}$$

For $d \in \left[d_0^2,\ d_0^1\right]$ the corresponding constrained pole assignment controller is again given by (5.15), (5.16) and with limiter (5.10). For higher velocities

d the nonlinear algorithm is used that moves the representative point in the state space towards the so called Reference Braking Trajectory and guaranteeing braking with dynamics specified by one of the closed loop poles and without overshooting. The controller for the double integrator is then given as

$$u = \left[1 - \alpha_2 \frac{y - y_b}{d}\right] U_j \tag{5.17}$$

$$\bar{u}_r = \mathrm{sat}\left\{u/K_s\right\} \tag{5.18}$$

$$j = \left(3 + \mathrm{sign}\left(y\right)\right)/2 \tag{5.19}$$

When controlling the nonlinear system (5.9), it is at first necessary to transform by means of inverse equation to (5.9) the existing saturation limits (e.g. 0 and 5 V) into the limits valid for the fictive controller input v_1. Then, using information about the plant state and the new saturation limits considered in (5.19), the fictive control signal v_1 is computed by means of (5.15), or (5.19) and by using (5.9) the real control signal is achieved.

When substituting for x_1, x_2 in the above transformations directly the actual level values, we use standard approach of the exact linearization. Due to the non-modeled dynamics it is, however, better to work with generalized operating points for x_1, x_2 defined somewhere between the actual and the final state as

$$x_i = w_i * m_i + \left(1 - m_i\right) h_i\,; m_i \in \langle 0, 1\rangle\ ;\ i = 1, 2 \tag{5.20}$$

In this way we are using a method that represents combination of the exact linearization and of linearization around fixed operation point.

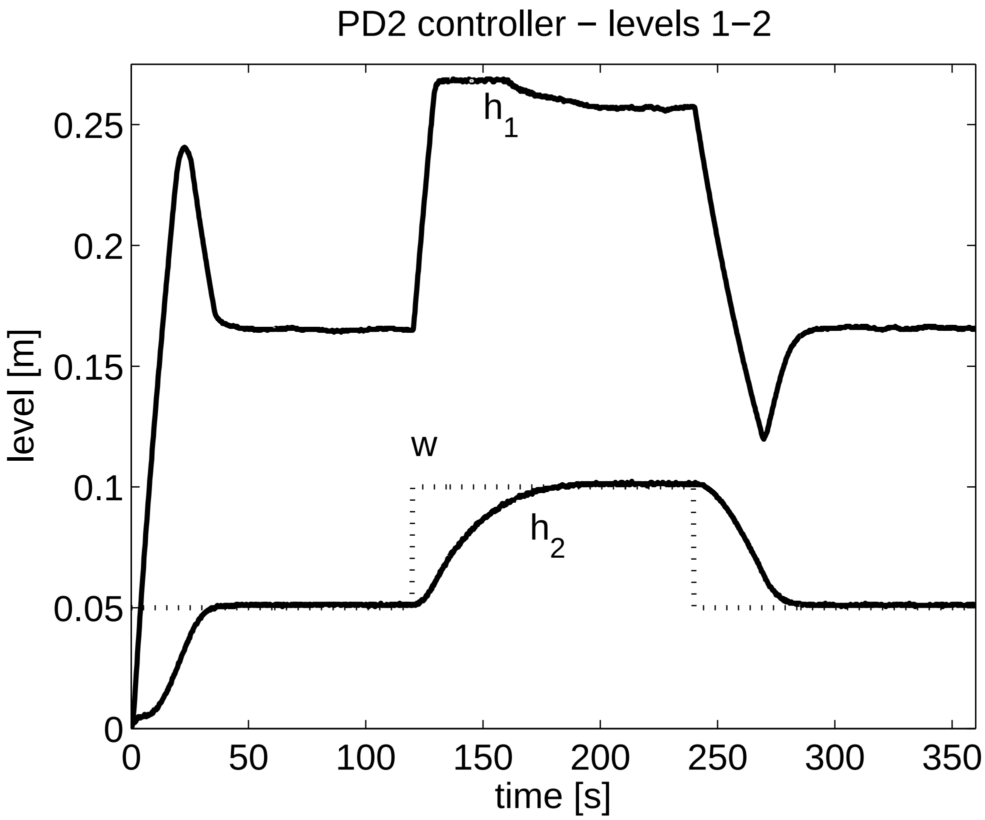
Example of transient responses are in Fig. 5.5 and Fig. 5.6. The traditionally used tuning derived by the conditions of the triple real pole

$$\alpha_{1,2} = -0.321/T_d \tag{5.21}$$

used in older works, the closed loop poles were now chosen according to Huba (2011b) as

$$\alpha_{1,2} = -0.321/T_d \tag{5.22}$$

where $T_d = 0.8s$ is the identified loop dead time, was now confronted with newer one Huba (2011b) derived by the performance portrait method. The dynamics of the control signal changes is obviously from the dynamical class 2. But, due to the plant character the second pulse in the control transients is not so dominant as the first one, what is a typical feature of all stable systems.



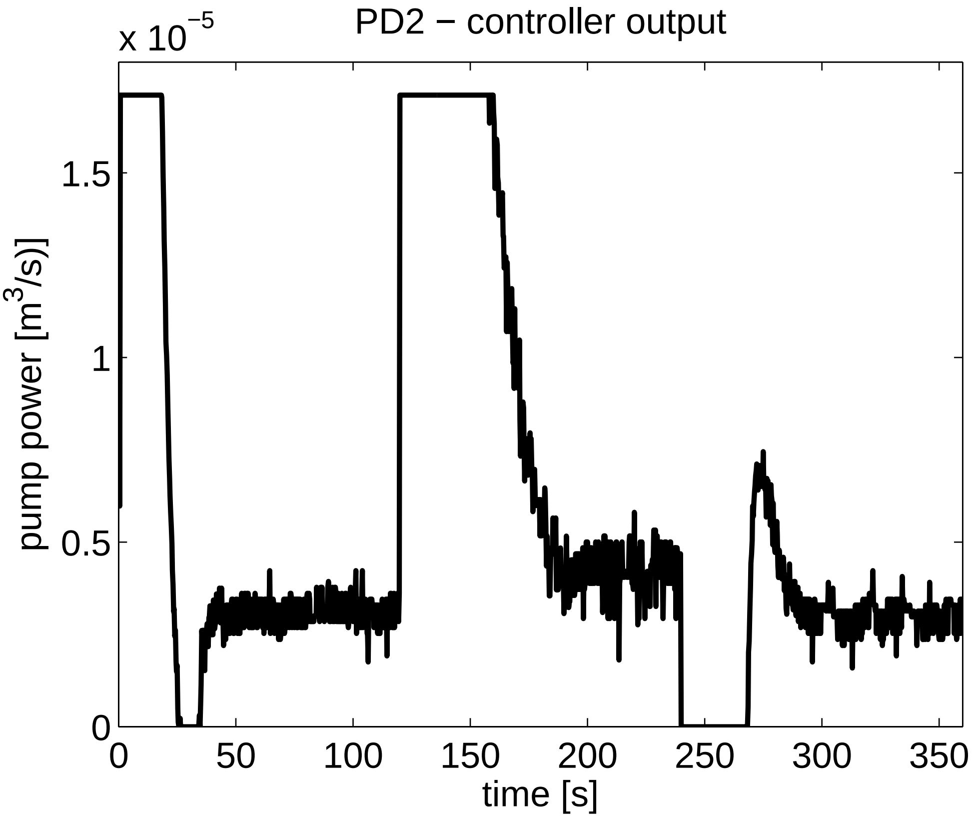


Figure 5.5: Typical setpoint steps in the loop with PD_2 controller with tuning (5.21) may contain up to two step at saturation limits after larger setpoint steps at its output, but for stable systems the 2nd interval at saturation corresponding to braking the transient may disappear or not reach the saturation

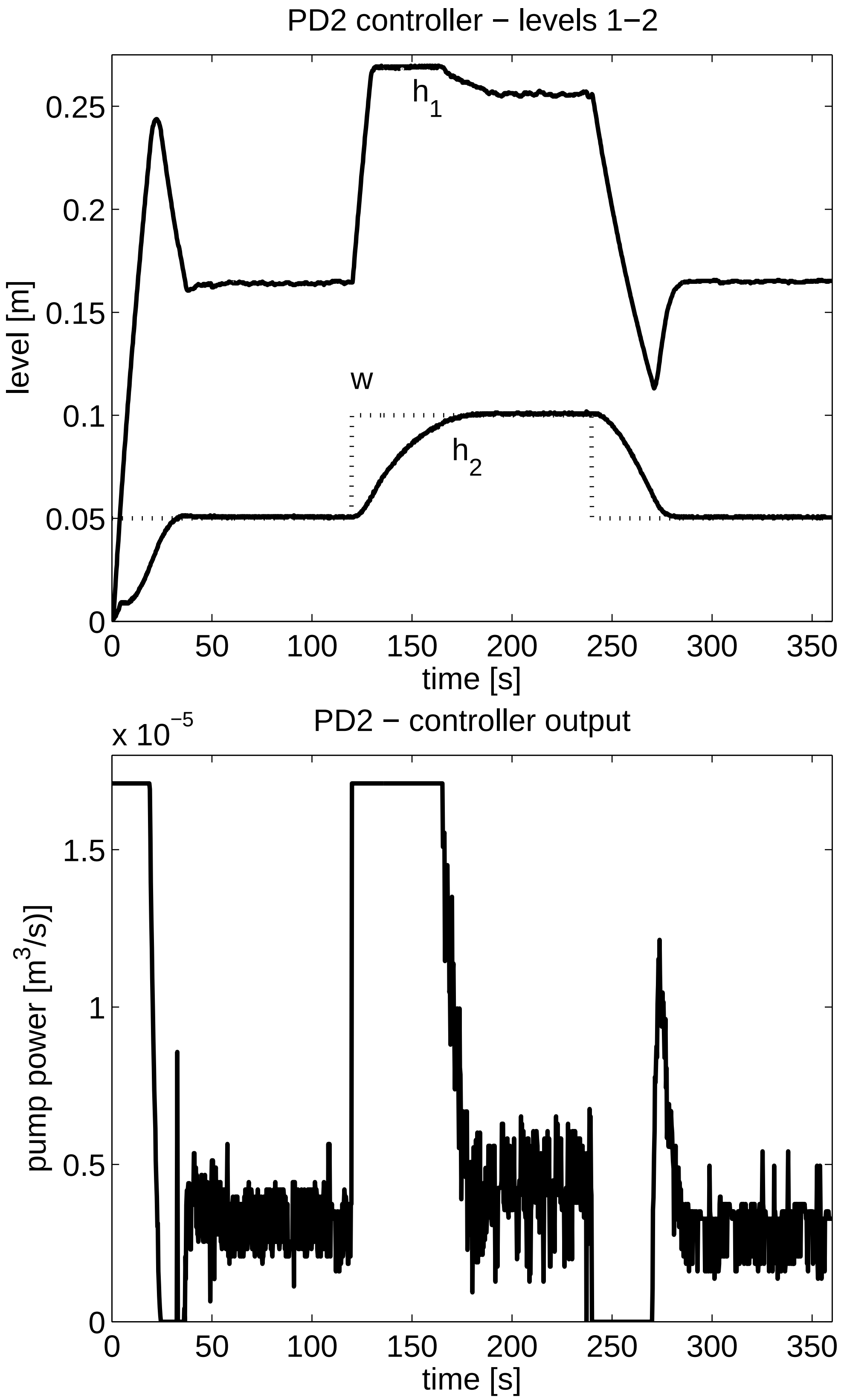


Figure 5.6: Typical setpoint steps in the loop with PD_2 controller with tuning (5.22) derived by the performance portrait method Huba (2011b) gives a bit tighter control and faster dynamics of transient responses than the traditionally used tuning (5.21) what may be e.g. observed by longer period at the maximal level of h_1 in the first tank and yet more by higher noise amplification

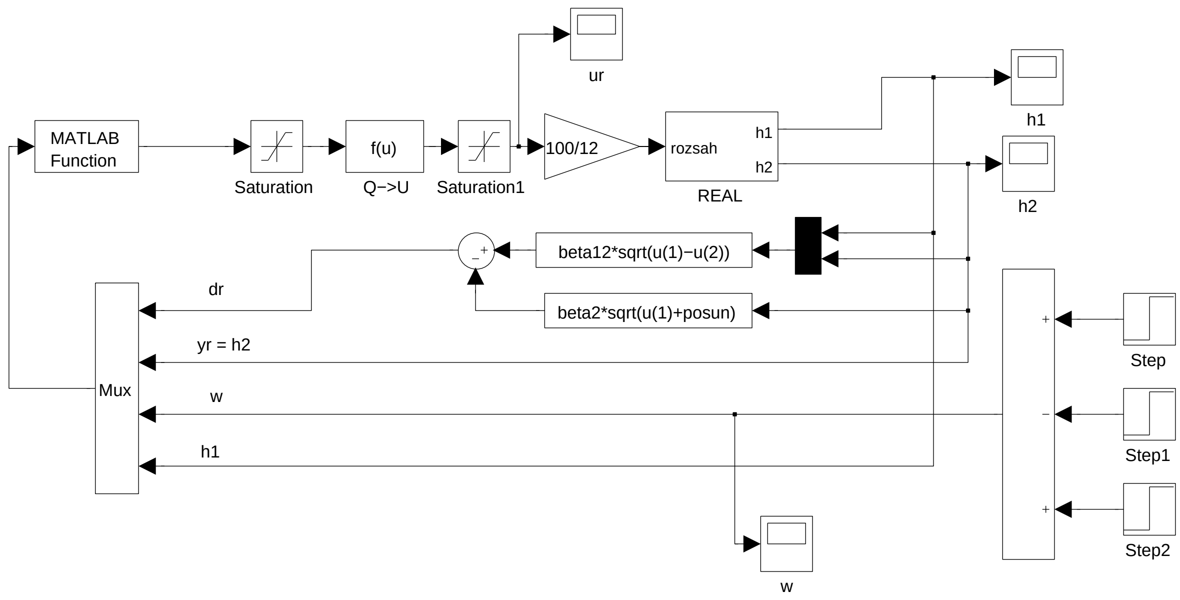


Figure 5.7: Control structure for nonliner PD1 implemented in Matlab / Simulink. Matlab function block call `PD2(u(1),u(2),u(3),u(4))`

The controller structure in Simulink is depicted in Fig. 5.7. Control program which works with results of automatic calibration and indentification is listed below.

Program for Matlab/Simulink

5.6 Conclusion

We have described the basic possibilities of control of available hydraulic system established by different configuration of one or two containers with one or two pumps. It is important to mention, that in case of control of real system with the mentioned controllers we will not obtain the ideally supposed results. Certainly, due to modelling errors we will register some deviations between supposed and real transients. The control performance will depend on the accuracy of identification and robustness of controller. As we will never be able to define the system's parameters with 100% precision, the system wild never behave as the perfect integrator and P-controller will not assure the necessary quality of control. That's why we propose to design the robust controllers having sufficient robustness against influence of disturbances, uncertainties and measurement noise. At the same time, these controllers should also respect the available system's limits. Such proposals for one-level system can be found for example Huba (2003); Pan et al (2005); Halás (2006); Almutairi and Zribi (2006); Žilka et al (2009). Further development of the above-mentioned procedures, as e.g. the disturbance decoupling problem of coupled tanks was studied in Žilka and Halás (2010) and non-interacting control of coupled tanks in Žilka and Halás (2011).

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