

VYSOKÉ  
UČENÍ  
TECHNICKÉ  
V BRNĚ

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

# Robustné PID-regulátory s obmedzeniami

## Robust Constrained PID Control

### Introduction

prof. Ing. Mikuláš Huba, Ph.D.

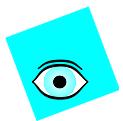
Ing. Peter Ţapák, Ph.D.

Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.



# Overview

- Introduction
- Motivation & Terminology
- Table of fundamental controllers
- Conclusions



# Control Design

- Early/Heuristic/PID Control/Anti Windup control
- Minimum Time/Fuel control
- Linear Pole Assignment/Optimal Control (LQ, LQG,...),  
Predictive Control
- Adaptive Control, Nonlinear Control  
(differential/algebraic/GS,...)
- Robust Control (IMC,...), Fuzzy & Neural Control
- Constrained Control (ellipsoid sets, LMI, ...), etc.
- **Just a Limited Integration in Literature**
- **Needs for a unifying approach**



# PID Controller architecture

- O'Dwyer – 7 PI & 47 PID controller structures
- Which one is the “standard” one?
- What is the PID controller?
- Here, controller derived for the plants

$$S(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} e^{-T_d s}$$

- Dead-time compensators for systems with long dead-time are included

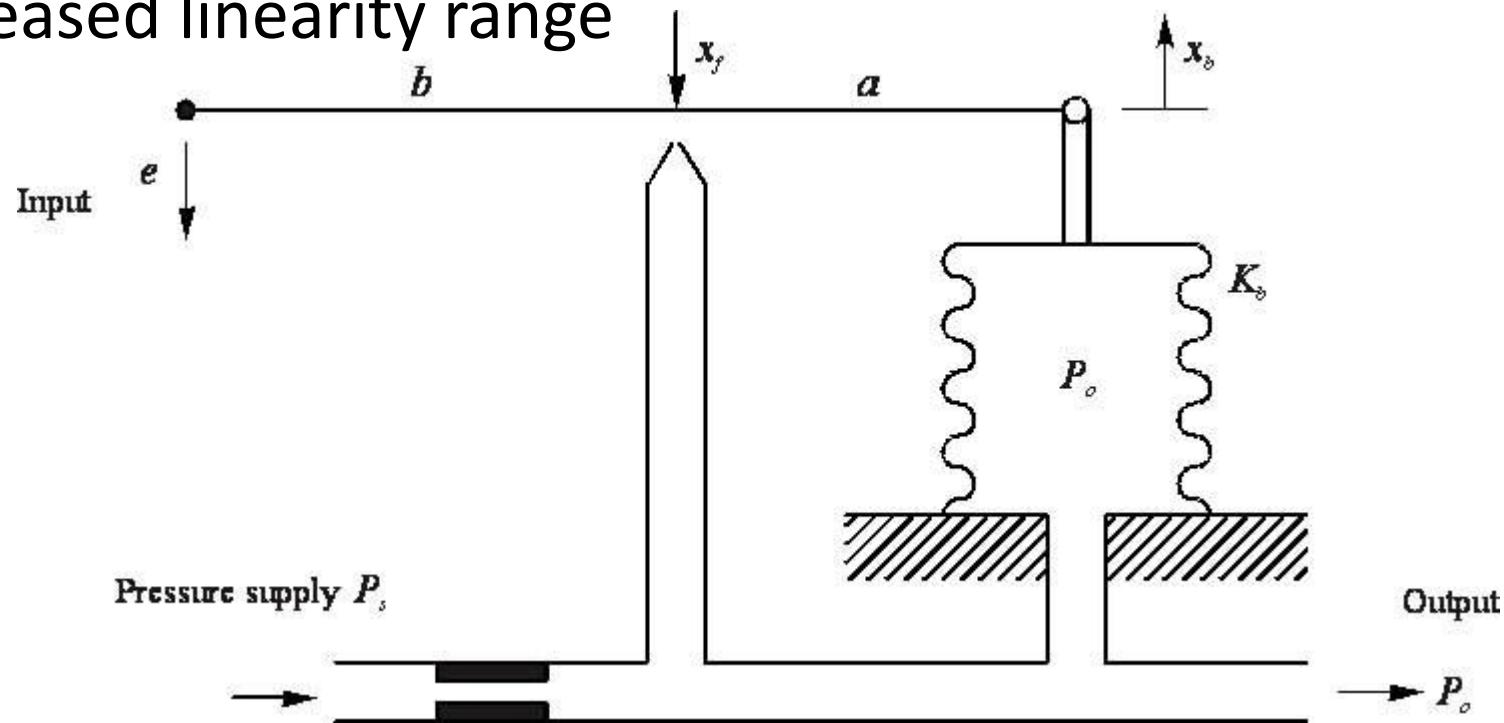


# Pneumatic P-Controller (1918)

Milk pasteurization

Negative **feedback** from the controller output

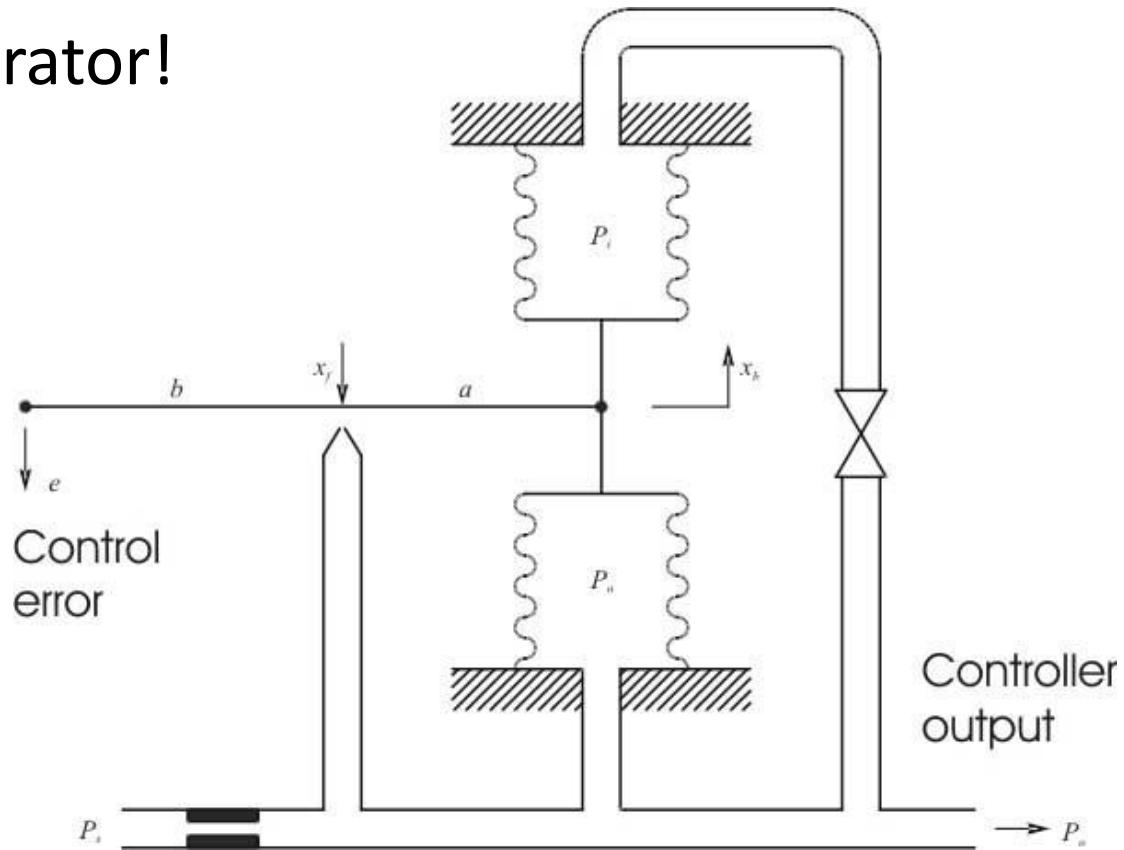
Increased linearity range





# Pneumatic PI Controller (1918)

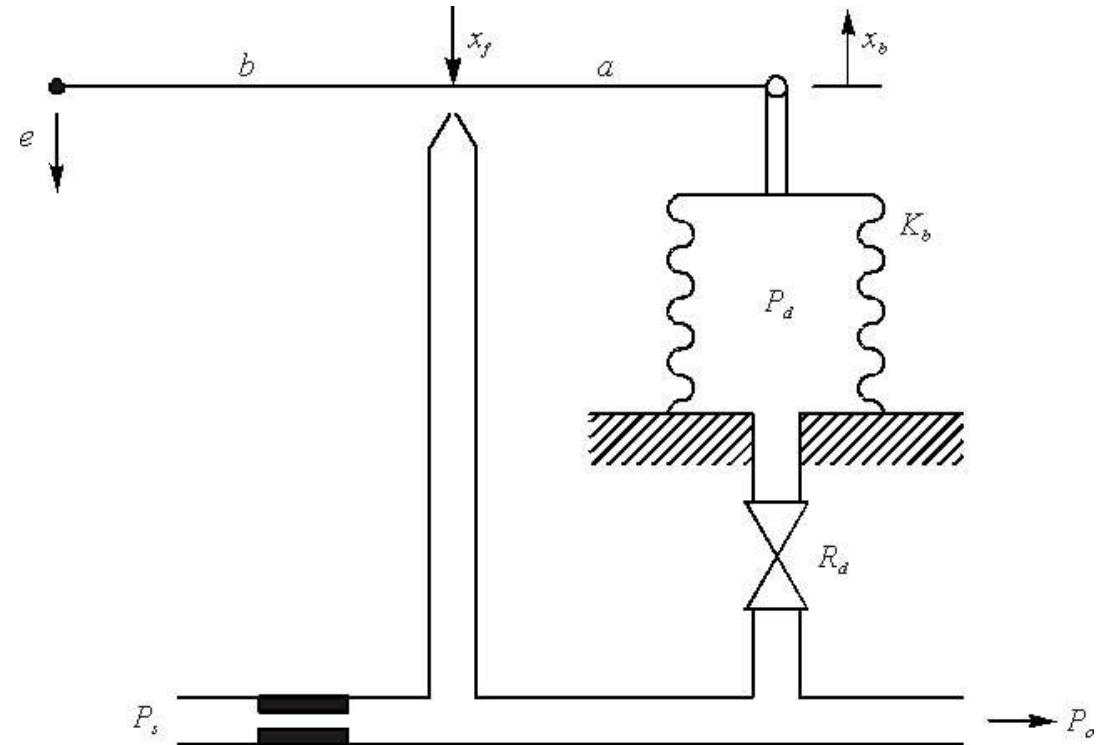
- Integral (I) action called “AUTOMATIC RESET”
- No explicit integrator!
- No windup!





# Pneumatic PD Controller (1935)

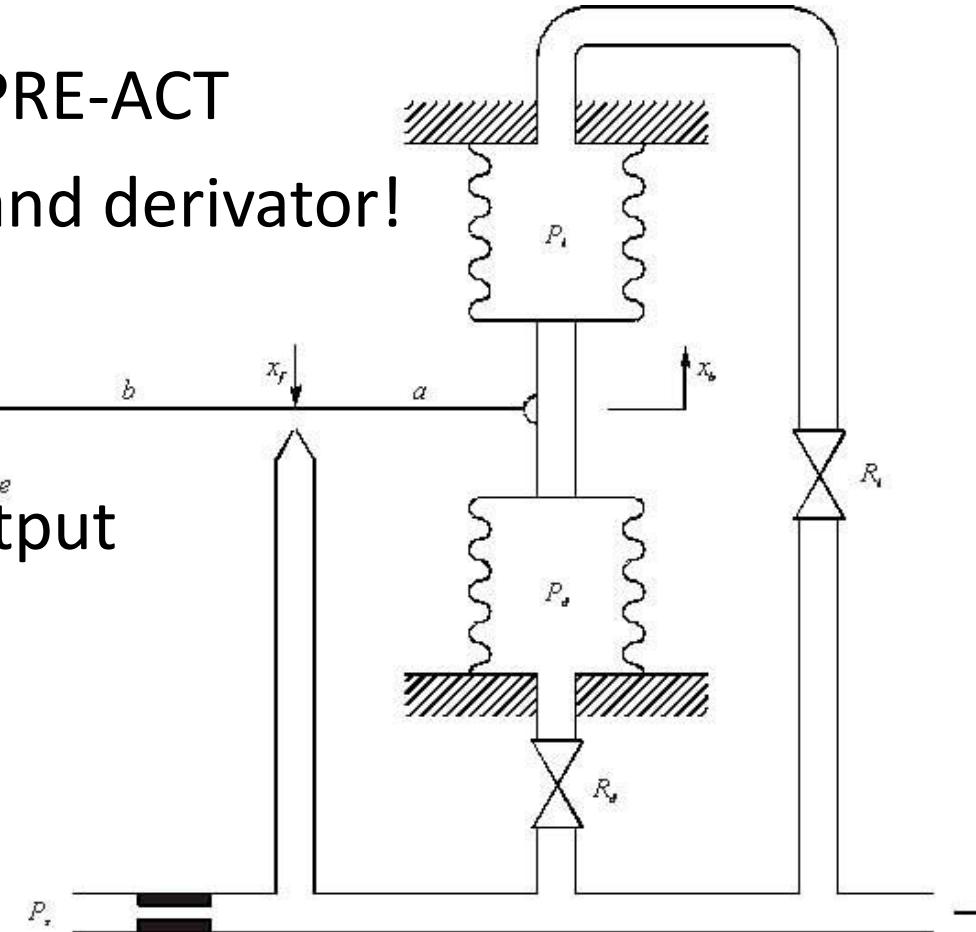
- D action called ‘PRE-ACT’
- No explicit derivator!





# Pneumatic PID Controller (1938)

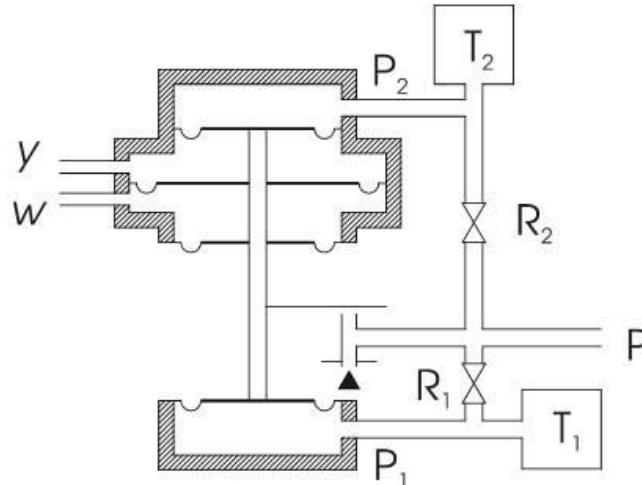
- AUTOMATIC RESET & PRE-ACT
- No explicit integrator and derivator!
- **Modular design**
- based on **feedback**
- from the controller output



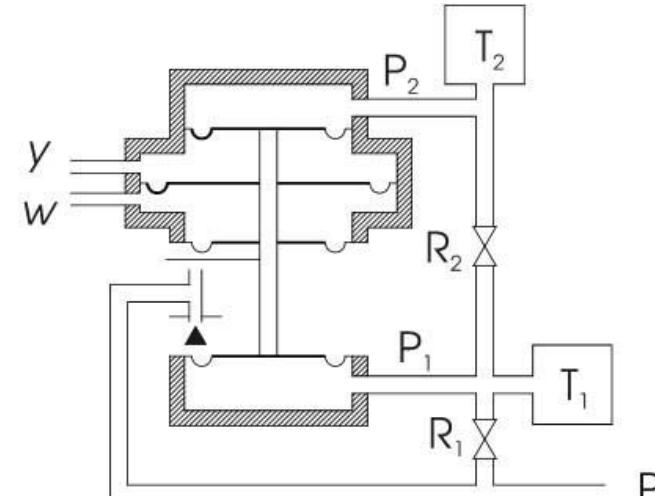


# Pneumatic PID Controller (1938)

- AUTOMATIC RESET & PRE-ACT
- Diaphragm type controllers
- Different realizations:
  - Different range of adjustable parameters
  - Different properties and controller tuning



Parallel feedback



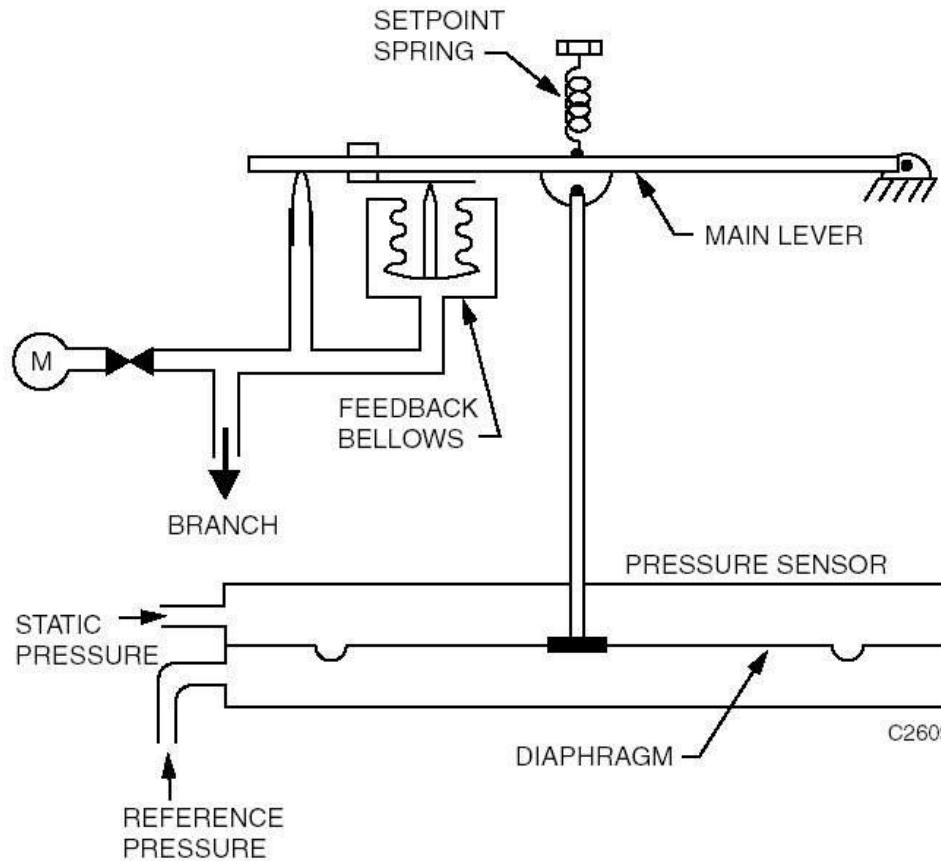
Serial feedback



# Pneumatic PI Controller **HONEYWELL**

<http://customer.honeywell.com/Techlit/Pdf/77-0000s/77-E1100.pdf>

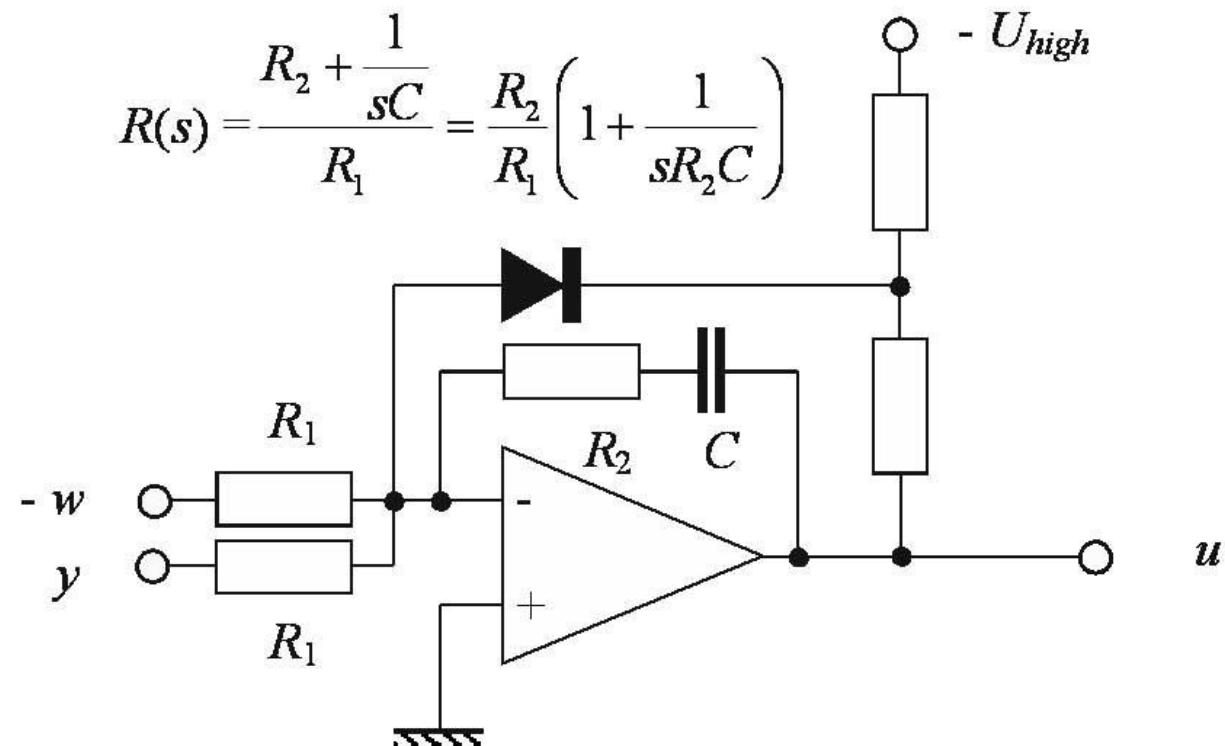
Solution combining  
diaphragm and bellows





# Electronic PI Controller

- No explicit integrator!
- Modular design based on **feedback** from the controller output
- No windup!





# Explicit integrator

- Involved in hydraulic & electromechanical actuators
- Mechanical constraints for integration
- No windup



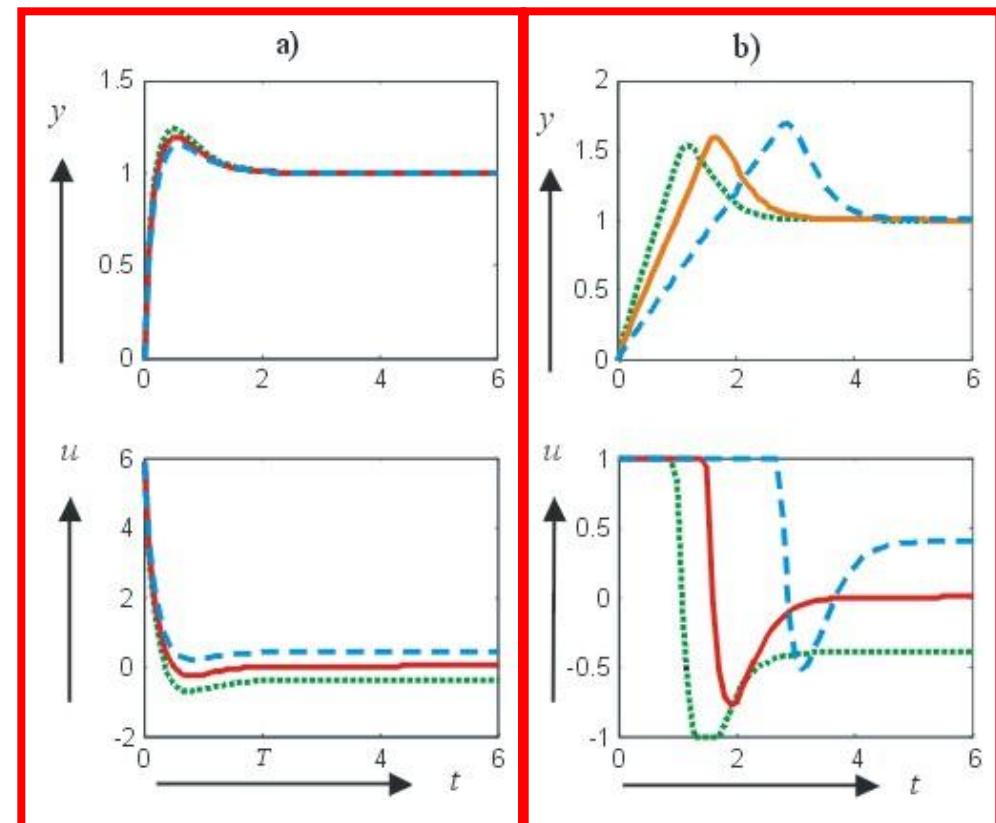


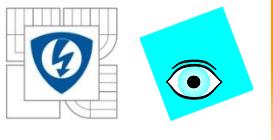
# $\mu$ -Processor Based PID control

- Use of explicit digital integrators and derivators
- No natural control constraints
- **Windup Problem!**

a) Linear Control  
Typical overshooting

b) Constrained Control  
Note increased output overshooting!



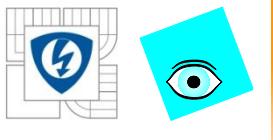


# 1990 – today: Revival of Constrained Control

Henrion, Tarbouriech and Kučera (2001)

Control of linear systems subject to input constraints: a polynomial approach. Automatica 37, 597-604.

- implicit (a posteriori) methods - anti-windup strategies
- explicit (a priori) methods divided into
  - saturation avoidance and
  - saturation allowance methods



# 1990 – today: Revival of Constrained Control

Goodwin (2001), Rojas & Goodwin (2002)

- **cautious** approach = reducing demand on control performance until constraints are avoided (worst case approach  $\leftrightarrow$  explicit saturation avoidance)
- **evolutionary** linear design + compensation of constraints  $\leftrightarrow$  implicit one - anti-windup strategies
- **tactile** approach  $\leftrightarrow$  the explicit allowance one - constraints included in control design from beginning - potentially improved performance, often at the expense of **increased complexity**.



# Anti-Windup (Anti-Reset-Windup)

- The most frequent approach
- No generally accepted windup definition!!!
- No homogenous dynamics for large and small steps
- **More complex structures with increased number of parameters**



# Saturation Allowance-Tactile Approach

- Gain scheduled controllers, invariant sets, piecewise linear (affine) control, LMI, MPC
- Ellipsoidal Technique, Polyhedral Sets, Piecewise Quadratic Estimates of Attraction
- Robust Constrained PID Control – attractive alternative for simple problems



# Robust Constrained PID Control

## Considering

- properties of classes of solutions,
- limited information about the process,
- process properties changing with time or with the operating point
- constraints put on the process input, state and output and on the rate of their changes,
- variety of requirements of practice,
- development of our knowledge.





# Robust Constrained PID Control

Learning by doing supported by:

- MS PowerPoint and Java presentations,
- e-books, Flash and Java animations,
- simulation programmes Matlab/Simulink,
- programmes in Maple V,
- assignments to be solved,
- quizzes for self-testing with feedback,
- manuals for plant models and real-time experiments,
- all integrated in LMS Moodle,
- + printed textbook/workbook





# Robust Constrained PID Control

- Fundamental Solutions
- Dynamical Classes of Control
- Table of fundamental controllers (P, PD, I, PI, PID & Predictive P, PD, I, PI and PID controllers)
- Performance requirements
- Robust controller tuning - performance portrait





# Control Strategies

Close relations to:

- Minimum Time Control
- Pole Assignment Control
- State Space Approach (Disturbance Observer)
- IMC (Internal Model Control),
- Dead Time Compensators



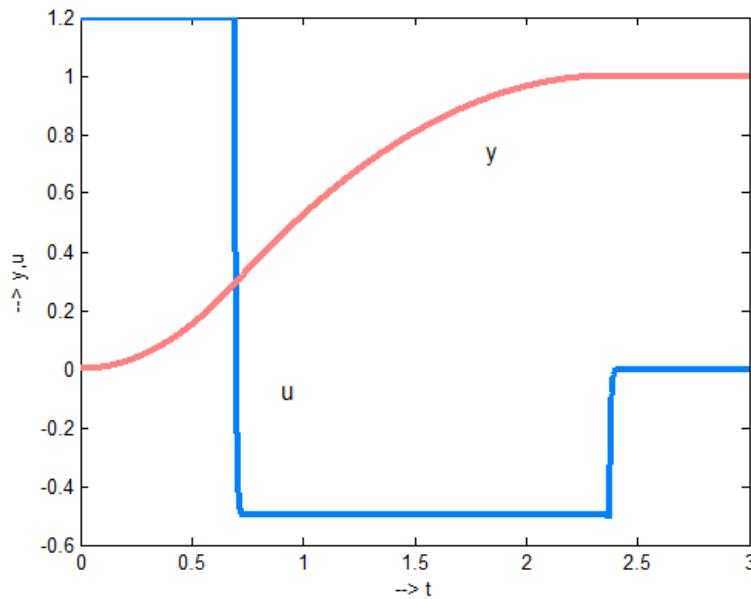
# Control Strategies

- All integrated within:
- Robust Constrained Pole Assignment Control
- Basic building block:
- Fundamental controller (solution)



# Minimum Time Control

- Bringing system from an initial state to the required one by constrained control  $U_1 \leq u \leq U_2$  in the minimum possible time



Double  
integrator



# Minimum Time Control

- Less attention usually devoted to **constrained rate** of control changes and **systems with zeros**
- Ad Hoc (**fixed**) **ideal** solution (nominal dynamics, limited possibility to respect parasitic phenomena)
- **High sensitivity** to noise, unmodelled dynamics, parameters fluctuations - relay chattering etc.
- Not solved **steady states**



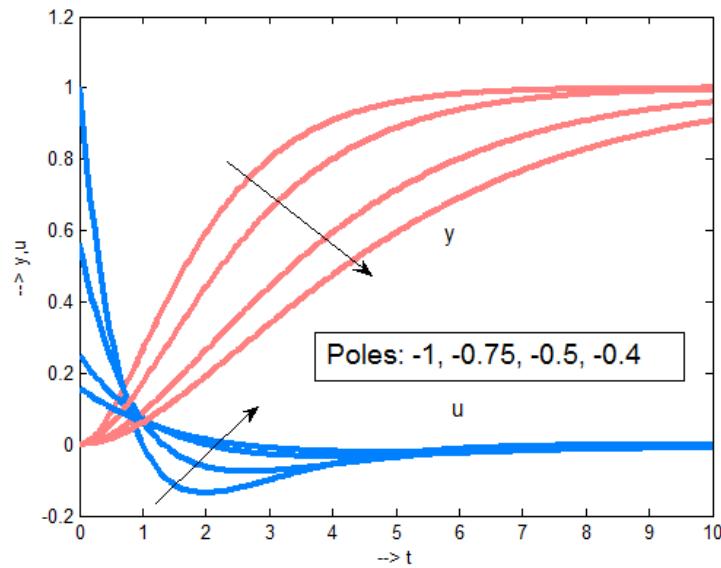
# Minimum Time Control

- Usually not directly applicable in process control
- Theoretical value - giving limits for achievable control dynamics with:
  - no noise,
  - no nonmodelled dynamics,
  - no parameter fluctuations
- Focused on ideal control with no uncertainty



# Linear Pole Assignment Control

- **Continuum of solutions** expressed in terms of poles & polynomials
- Known effect of poles shifting

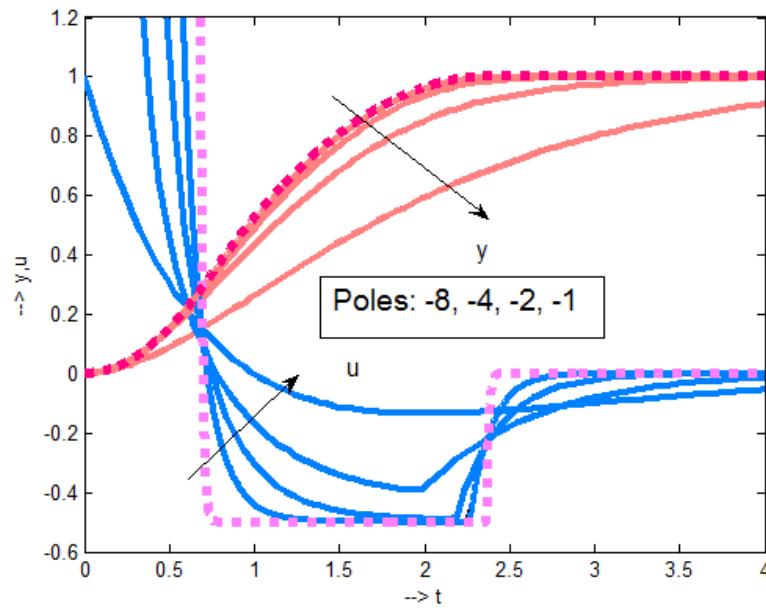


Double  
integrator



# Constrained Pole Assignment Control

- Bringing system from an initial state to the required one by **constrained control**  $U_1 \leq u \leq U_2$  & possible rate constraints
- Continuum of solutions** expressed in terms of poles & polynomials – MTC with additional constraints

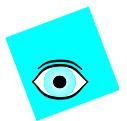


Double  
integrator



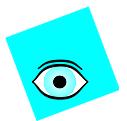
# Linear Pole Assignment Control

- Experimental - “trial and error approach” (noise, constraints, nonmodelled dynamics, parameter fluctuations etc.)
- Not able to respond to the basic question “slow or fast ” poles?
- RCPIDC: Offering poles depending on measurement noise, substantial time delays, parameter fluctuations, control signal constraints.



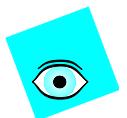
# Fundamental solutions in RCPIDC

- Parameterized solutions with dynamics scaled by the closed loop poles
- Dominant dynamics - monotonic step responses at the plant output with input dynamics ranging from
  - relay minimum time control up to
  - linear pole assignment control
- MTC:
  - n-rectangular pulses of control at the plant input
- RCPAC:
  - dynamical classes with nP control signal



# Dynamical Classes of Control

- Physics in the control
- Felbaum's theorem about  $n$ -intervals of optimal control
- Neglected under linear design  $\Rightarrow$  inflation of optimal controller tuning
- Energy accumulation – integrators – nP control
- Time delays – at least one accumulating element with stabilizing feedback
- Time delays – required output may also be achieved by a constant input
- Dead time – infinite number of stable accumulating elements

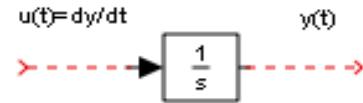


# Dynamical Classes of Control

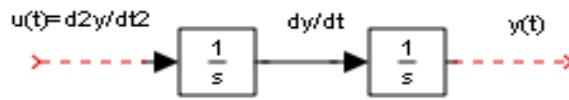
- To achieve **monotonic (OP, MO) output** of the chain of  $n$ -integrators we need  $n$  pulses of continuous control (**nP control**) at its input

OP (MO) signal  $y(t)$

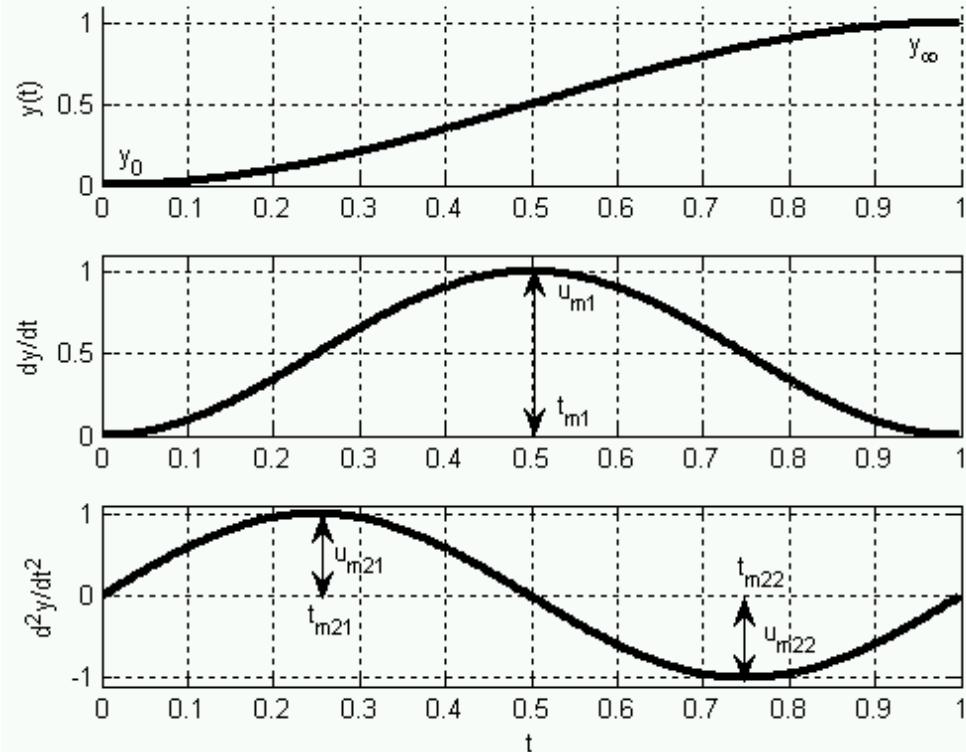
1P signal  $u(t)$



2P signal  $u(t)$



Input and Output Pairs of Single and Double Integrator





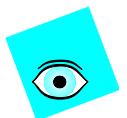
# Dynamical Classes of Control

- Dynamical class of control DC $n$  – all solutions and control structures with MO output and nP input
- Traditional PID control design – dominated by the DC0 and DC1 (robustness, sensitivity to noise, complexity, windup problems)
- DC2 – approached e.g. by the Model Predictive Control (MPC)
- For stable plants, the Dynamical Class of control may be decreased up to 0
- Index of the DC cannot be decreased below the number of unstable poles



# Dynamical Classes of Control

- Table of fundamental controllers
- Similarity with the Mendelejev table
- Similar properties of classes of controllers
- Controllers classified:
  - With respect to the DC of control
  - With respect to the plant approximation
- Each controller may exist in a form allowing discontinuity of control at the origin and in a filtered version with a fully continuous control



# Table of Fundamental Solutions

## Dynamical Class 0

Dynamic class	I-action	Dominant dynamics								
		$K$	$Ke^{-T_d}$	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1} + \frac{K_{s2}}{s+a_2}$	$\left[ \frac{K_{s1}}{s+a_1} + \frac{K_{s2}}{s+a_2} \right] e^{-T_d s}$	$\frac{K_s}{s^2 + a_1 s + a_0}$	$\frac{K_s e^{-T_d s}}{s^2 + a_1 s + a_0}$	
0	N	FF	FF	FF	FF	FF	FF	FF	FF	
	Y	I	Prl	PI	PrPI	PID	PrPID	PID	PrPID	
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD	

## Explicit - Saturation Avoidance Approach

	Y	-	-	-	-	-	-	-	PID	PrPID
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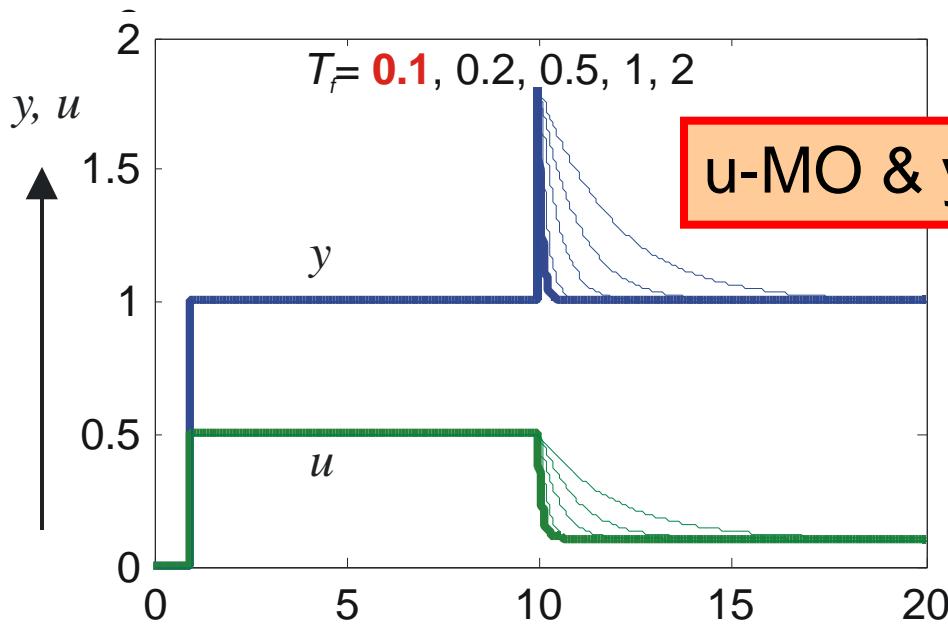
FF = static feedforward control

Pr = predictive (dead time) controller



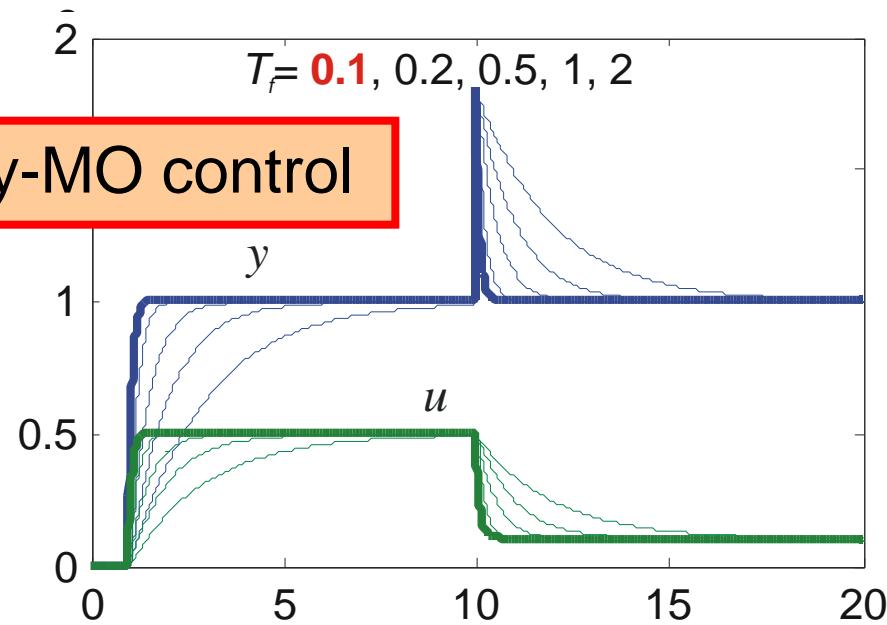
# Fundamental Solutions DC0

Continuous for  $t > 0$



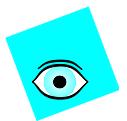
Basic Controllers

Continuous for  $\forall t$



Filtred Controllers

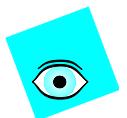
Parametrization by the closed loop time constant ( $T_f = -1/\alpha$ ,  $\alpha$  - closed loop pole)



# Fundamental Solutions DC1

		Dominant dynamics								
Dynamic class	I-action	$K$	$Ke^{-T_d}$	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1} + \frac{K_{s2}}{s+a_2}$	$\left[ \frac{K_{s1}}{s+a_1} + \frac{K_{s2}}{s+a_2} \right] e^{-T_d s}$	$\frac{K_s}{s^2 + a_1 s + a_0}$	$\frac{K_s e^{-T_d s}}{s^2 + a_1 s + a_0}$	
0	N	FF	FF	FF	FF	FF	FF	FF	FF	
	Y	I	Prl	PI	PrPI	PID	PrPID	PID	PrPID	
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD	
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID	
2	N	-	-	-	-	-	-	PD	PrPD	
	Y	-	-	-	-	-	-	PID	PrPID	

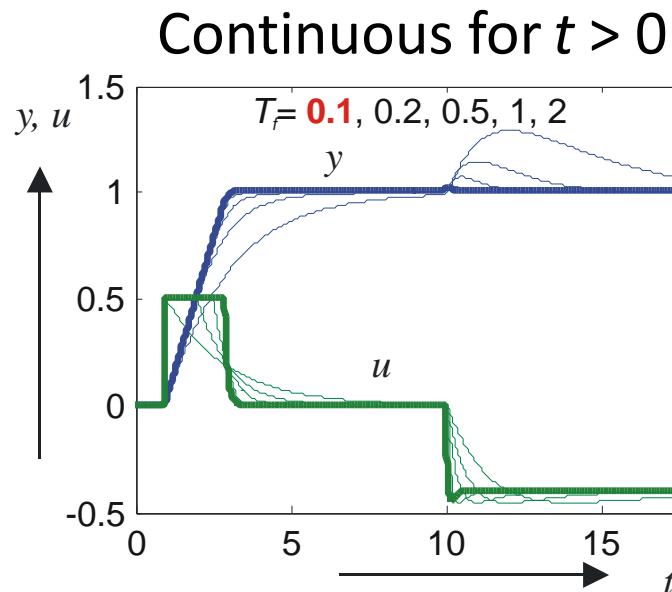
## Explicit - Saturation Allowance Approach



# Fundamental Solutions DC1

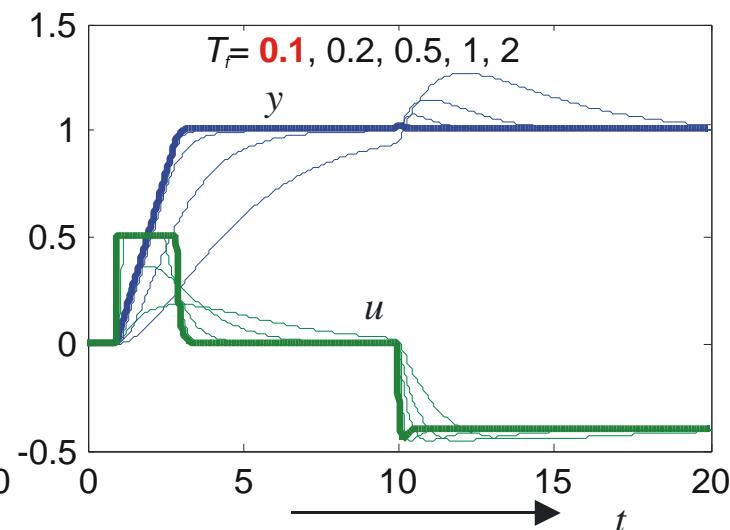
u-1P &  $\gamma$ -MO control

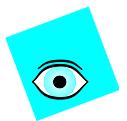
## Basic Controllers



## Filtered Controllers

Continuous for  $\forall t$





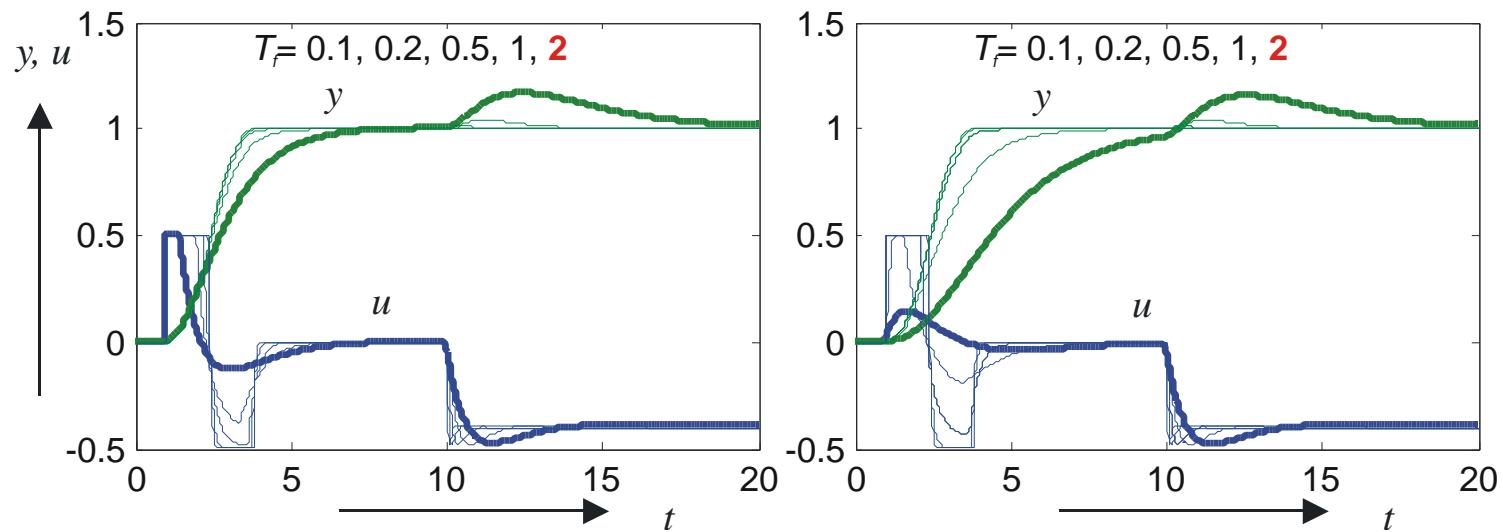
# Fundamental Solutions DC2

		Dominant dynamics							
Dynamic class	I-action	$K$	$Ke^{-T_d}$	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1} + \frac{K_{s2}}{s+a_2}$	$\left[ \frac{K_{s1}}{s+a_1} + \frac{K_{s2}}{s+a_2} \right] e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	Prl	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	PD	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

## Explicit - Saturation Allowance Approach

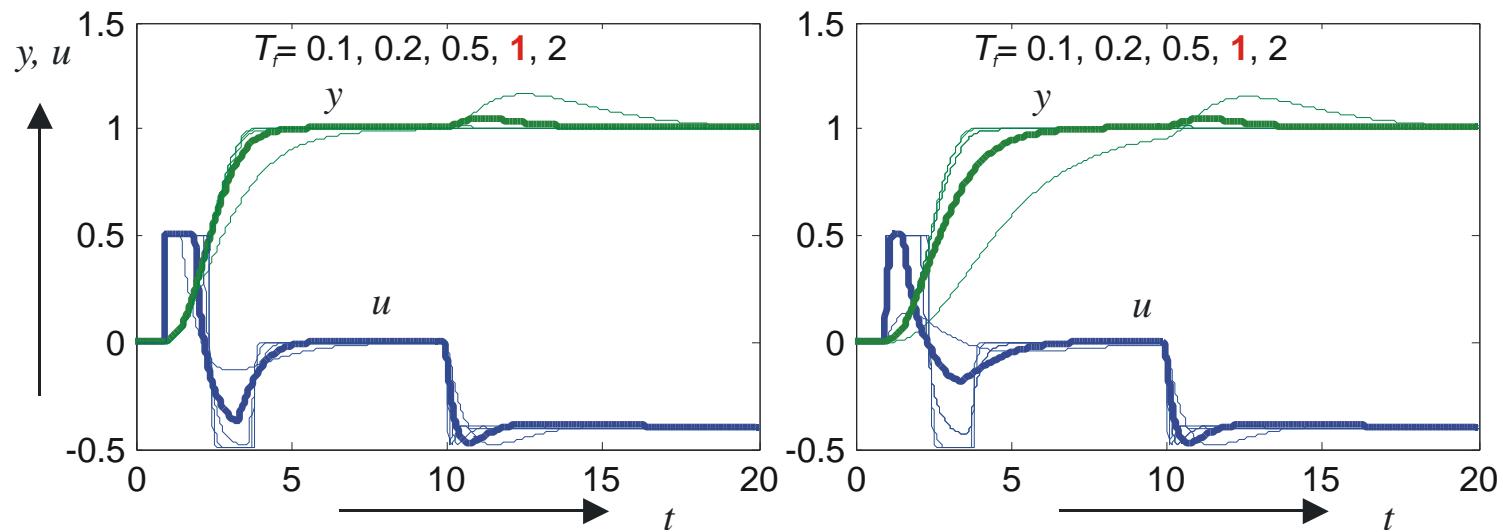


# Fundamental Solutions DC2



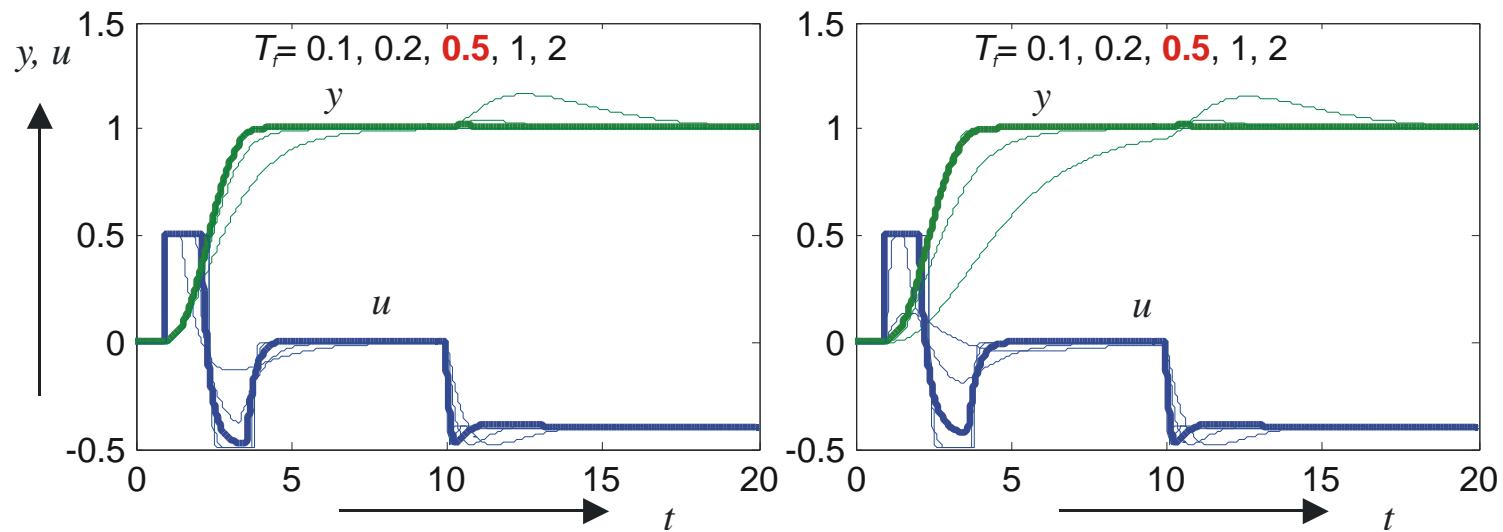


# Fundamental Solutions DC2



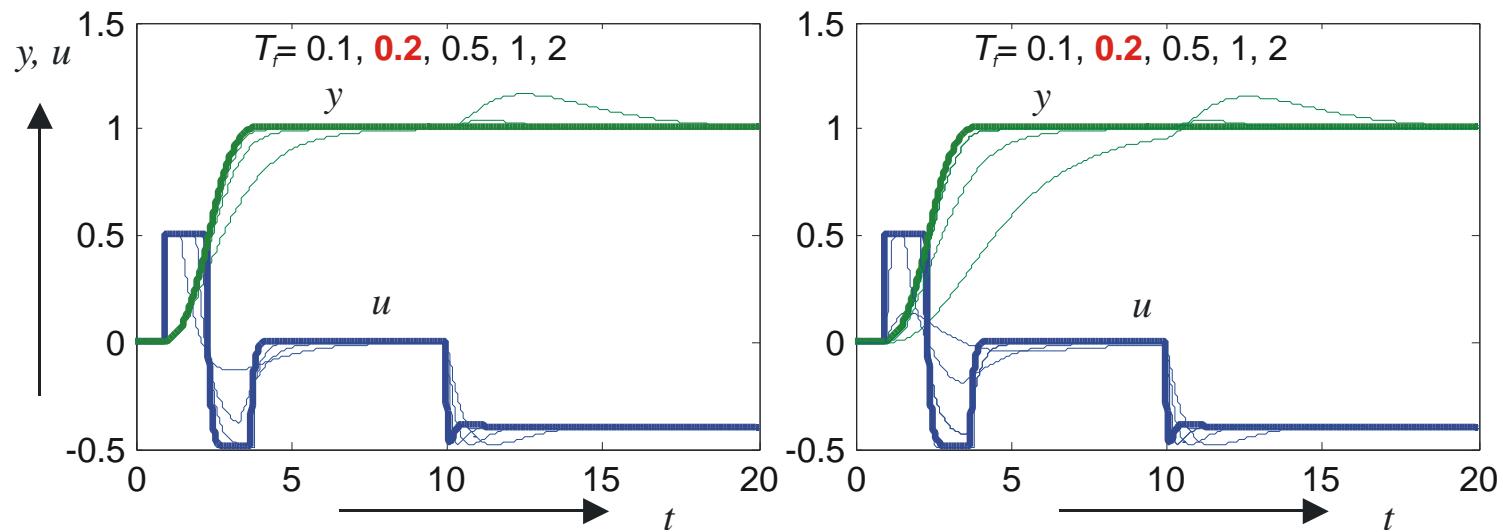


# Fundamental Solutions DC2





# Fundamental Solutions DC2

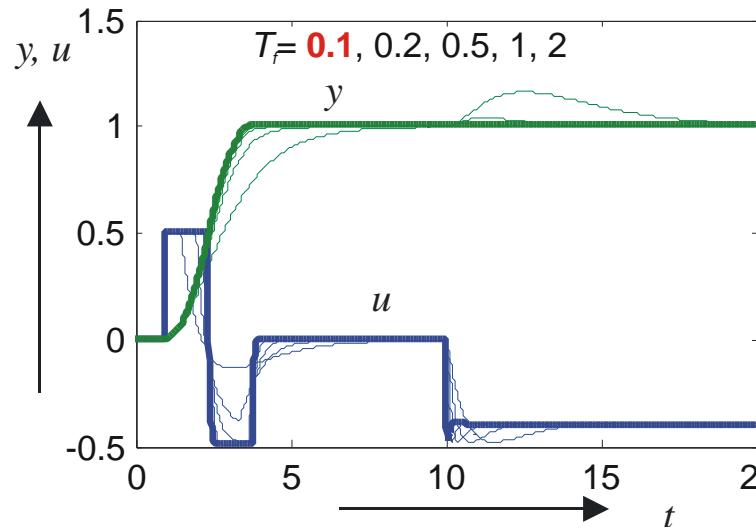




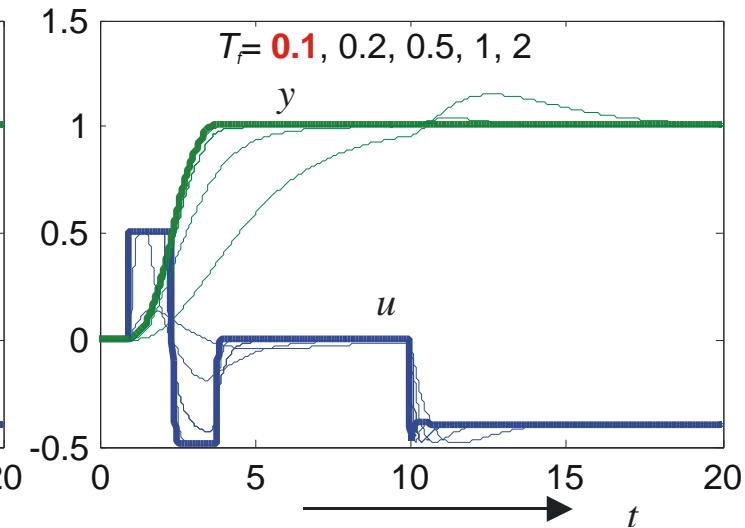
# Fundamental Solutions DC2

u-2P &  $\gamma$ -MO control

## Basic Controllers



## Filtred Controllers



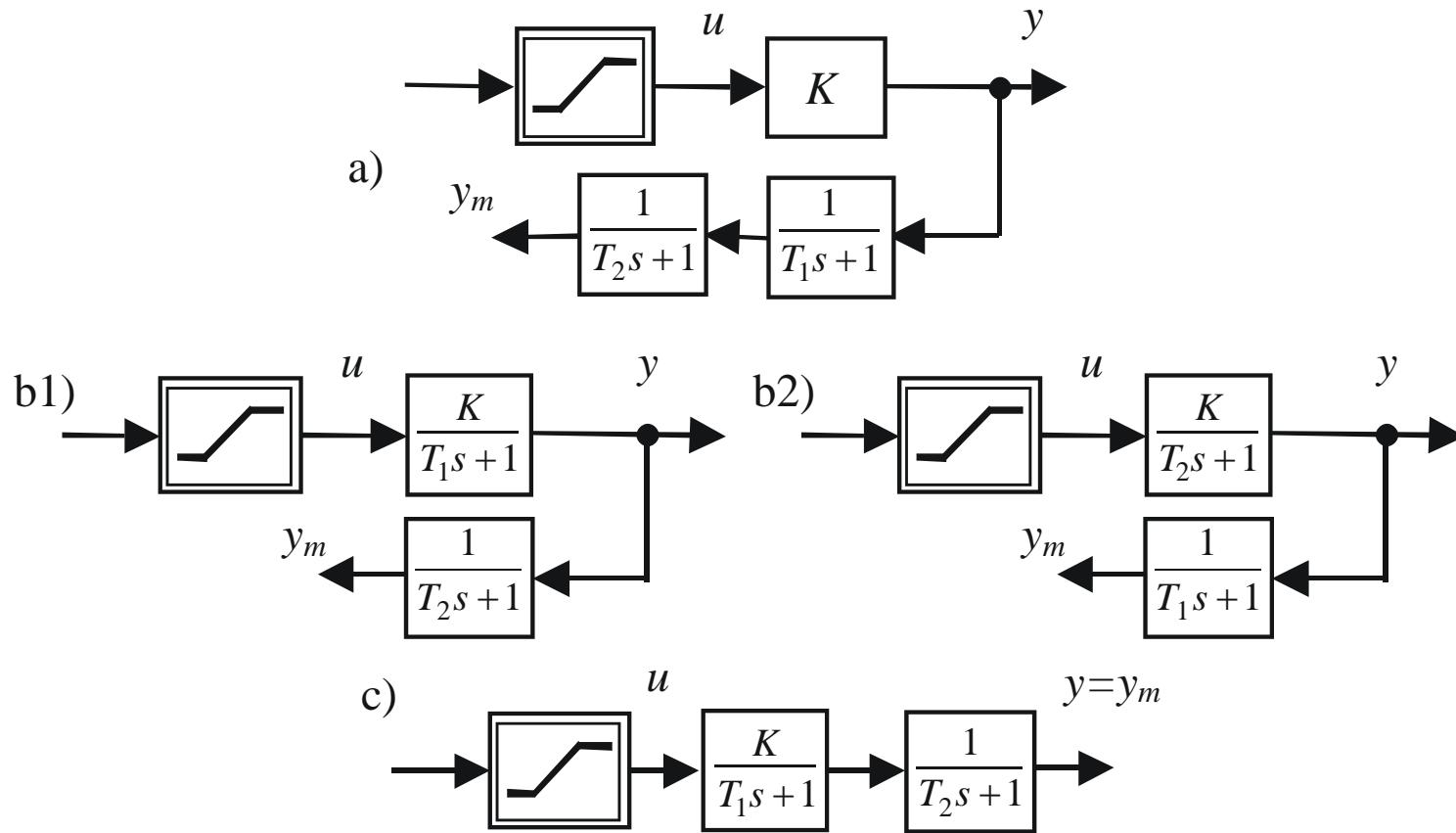


- **2** Dynamical Classes of PI Controller
- **3** Dynamical Classes of PID Controller
- **4** Basic Situations in PID Tuning!

## Loop Dynamics Distribution



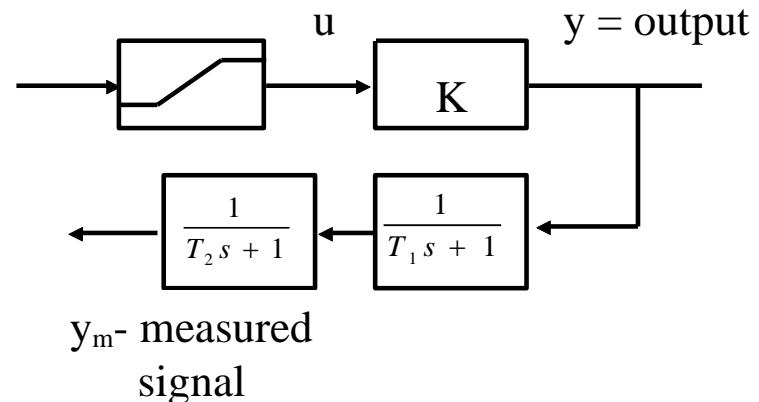
# Loop Dynamics Distribution





# DC0: Dominant Feedback Dynamics

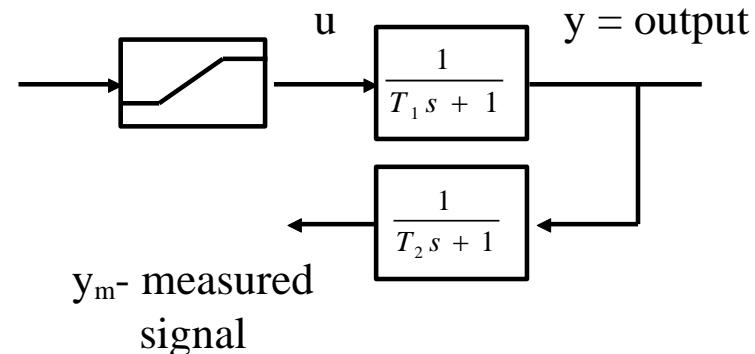
- $y$ -MO and  $u$ -MO control transients
- Optimal control corresponding to a reference step has a step character
- Linear design can be used
- Minimal Noise Sensitivity





# DC1:1st Order Relative Degree

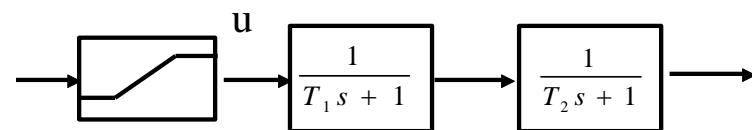
- $y$ -MO and  $u$ -1P control transients
- Modified Anti Windup approach can be used
- Several structures denoted as  $\text{PI}_1$ , or  $\text{PID}_1$
- Higher noise sensitivity



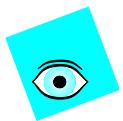


# 2nd Order Relative Degree

- $y$ -MO and  $u$ -2P control transients
- Two intervals of control
- Many Anti Windup approaches fail in the case of unstable (marginally stable) plant dynamics
- Several structures denoted as  $PD_2$ , or  $PID_2$
- Highest noise sensitivity



$y = y_m$   
output =  
measured  
signal



# Conclusions

- To deal effectively with the control constraints, each Dynamical Class of control requires special control structures, despite to the fact that the loop may be approximated by the same model
- Conclusion - for the 1<sup>st</sup> order loop we need two dynamical classes of the PI control and for the 2<sup>nd</sup> order dominant loop dynamics there exist three DCs of the PID control
- DC index  $n$  is determined by the output relative degree, but for stable plant poles it may be decreased (up to the number of unstable poles)