INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Robustné PID-regulátory s obmedzeniami
Robust Constrained PID Control
DC0: Robust I_0 and FI_0 controller design

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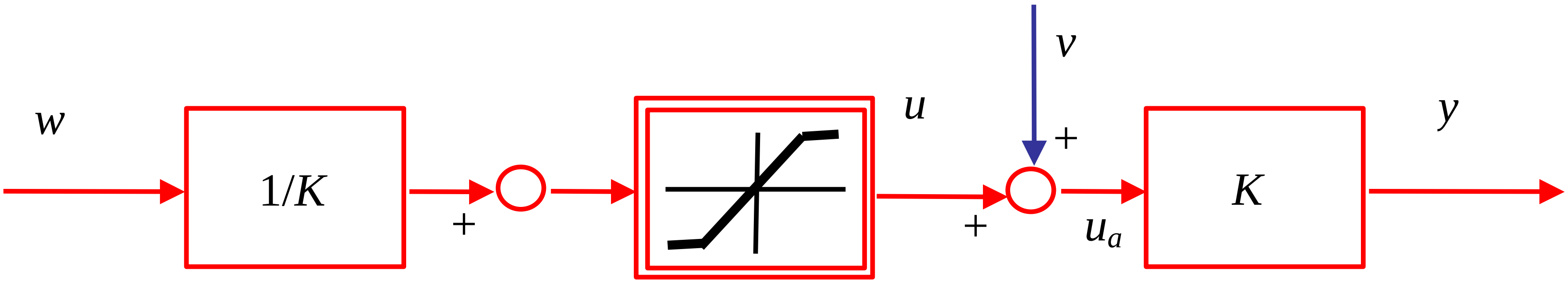
Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.

Contents : I_0 and FI_0 controllers

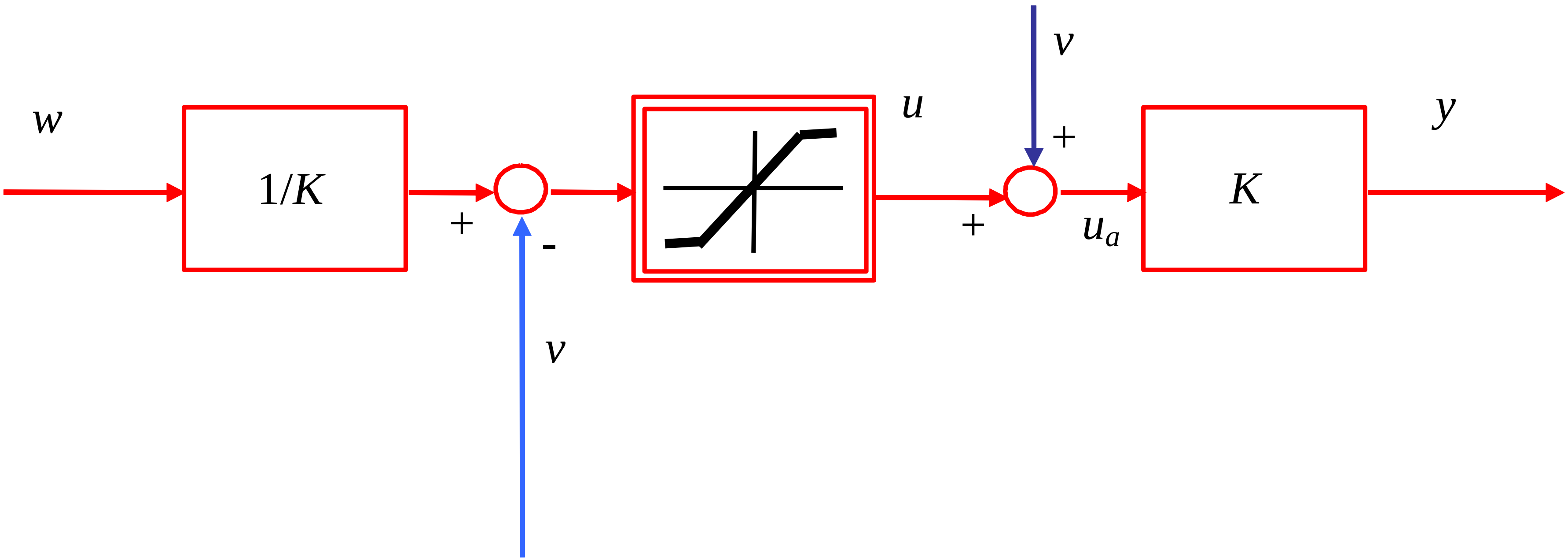
- Introduction
- Static feedforward control, FI_0 and I_0 controllers
- Generic and equivalent structures for compensation of input and output disturbances
- Nonmodelled dynamics
- Performance Portrait and Robustness Characteristics
- History

- Static feedforward control



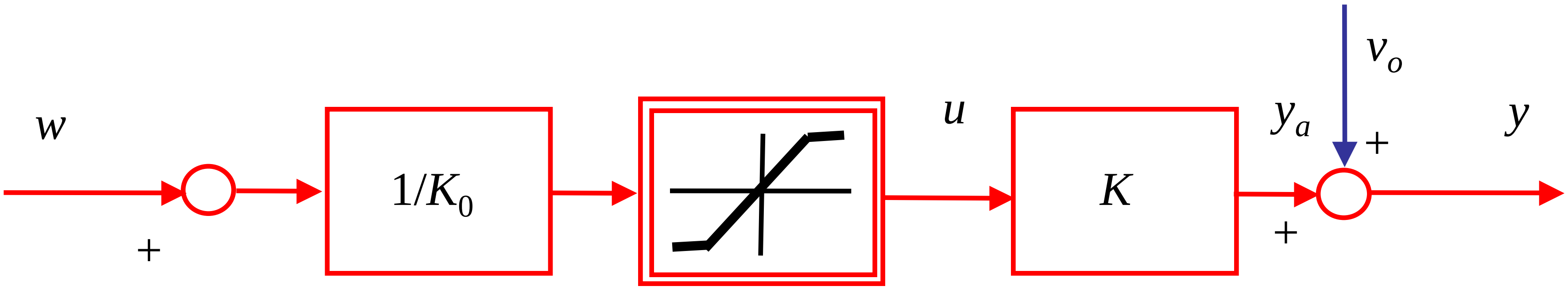
Input Disturbance (ID) Compensation

Static feedforward control with compensation of the input disturbance



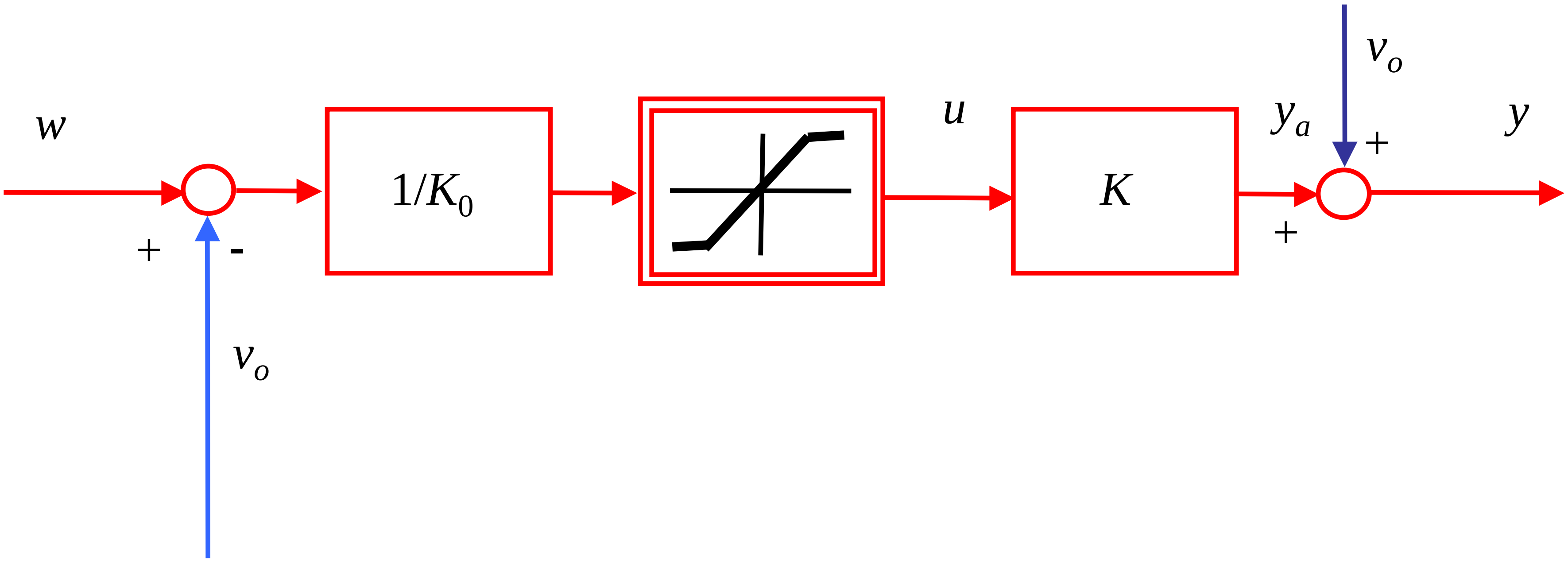
Output Disturbance (OD) Compensation

Static feedforward control



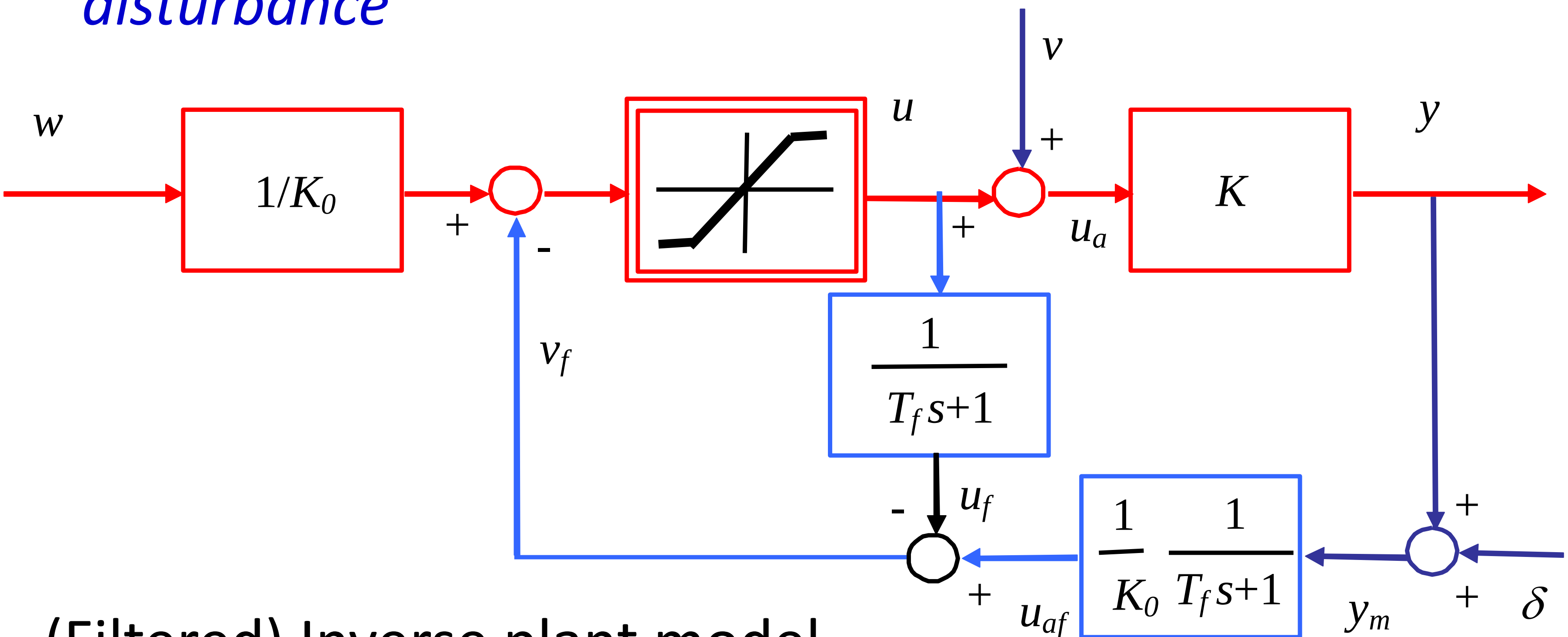
Output Disturbance Compensation

Static feedforward control with compensation of the output disturbance



I_0-ID controller – generic structure

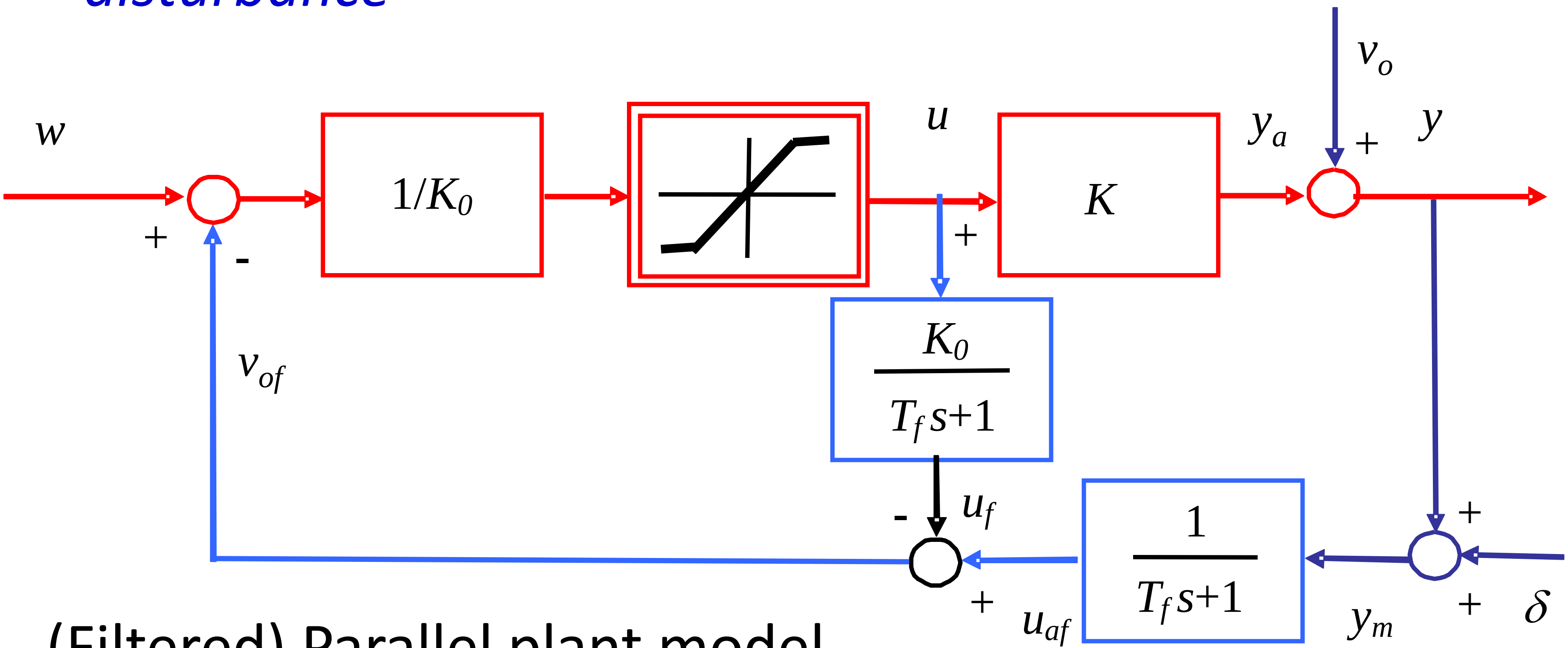
Static feedforward control *with* *compensation of input disturbance*



(Filtered) Inverse plant model

I_0-OD controller – generic structure

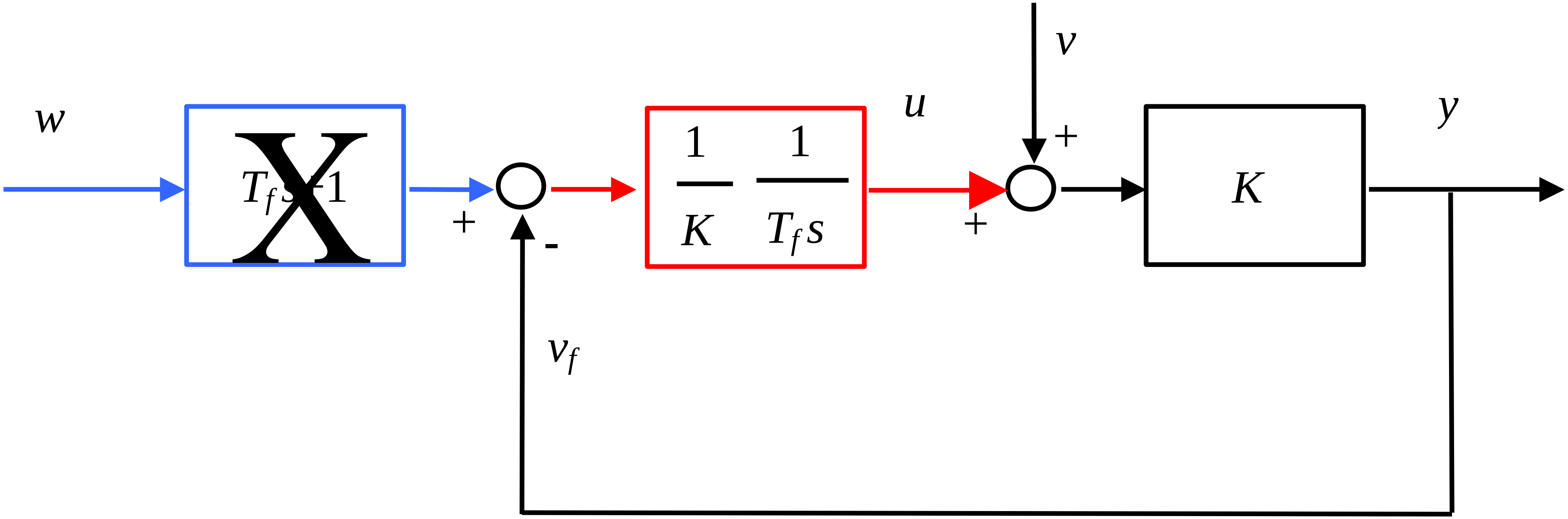
Static feedforward control with compensation of output disturbance



(Filtered) Parallel plant model

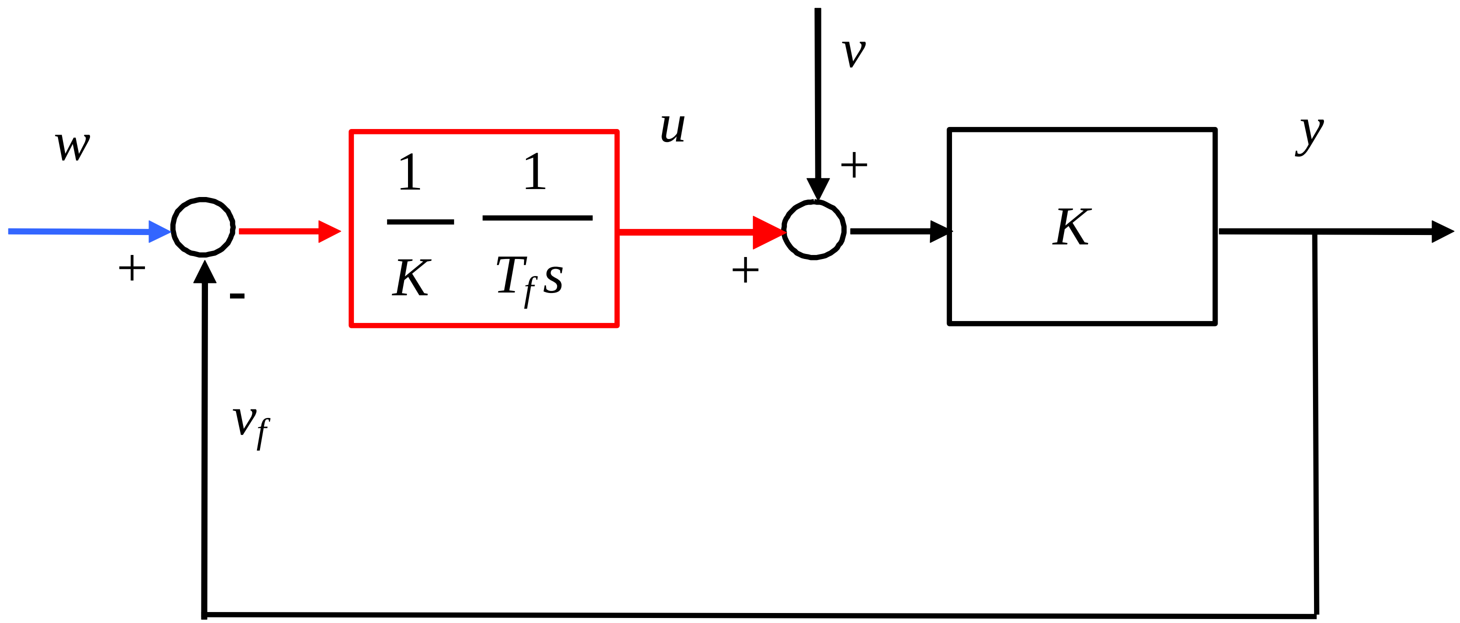
I_0-ID controller - equivalent structure

- explicit integrator
- Omitting prefilter = I-controller = FI_0 – controller



FI_0 controller = I controller

Filtered response continuous for t = 0

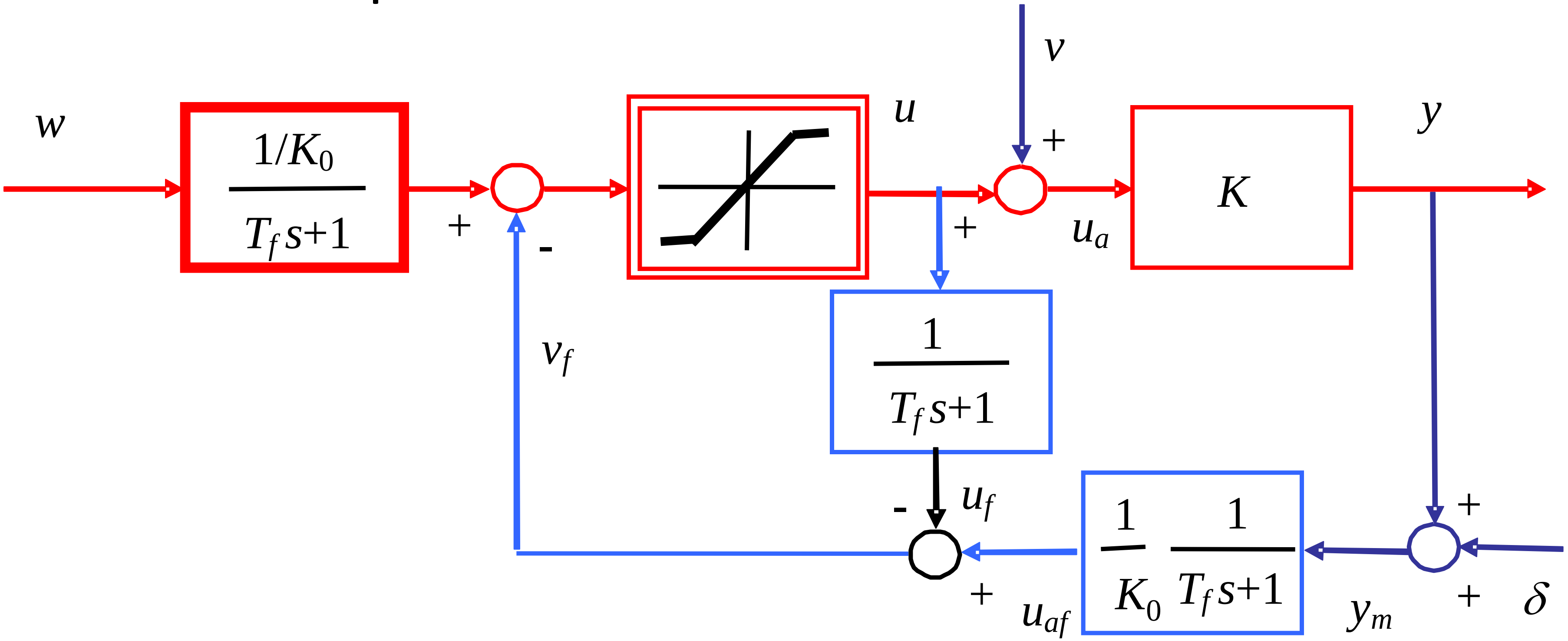


Explicit integrator

Involved in hydraulic & electromechanical actuators – mechanical constraints for integration

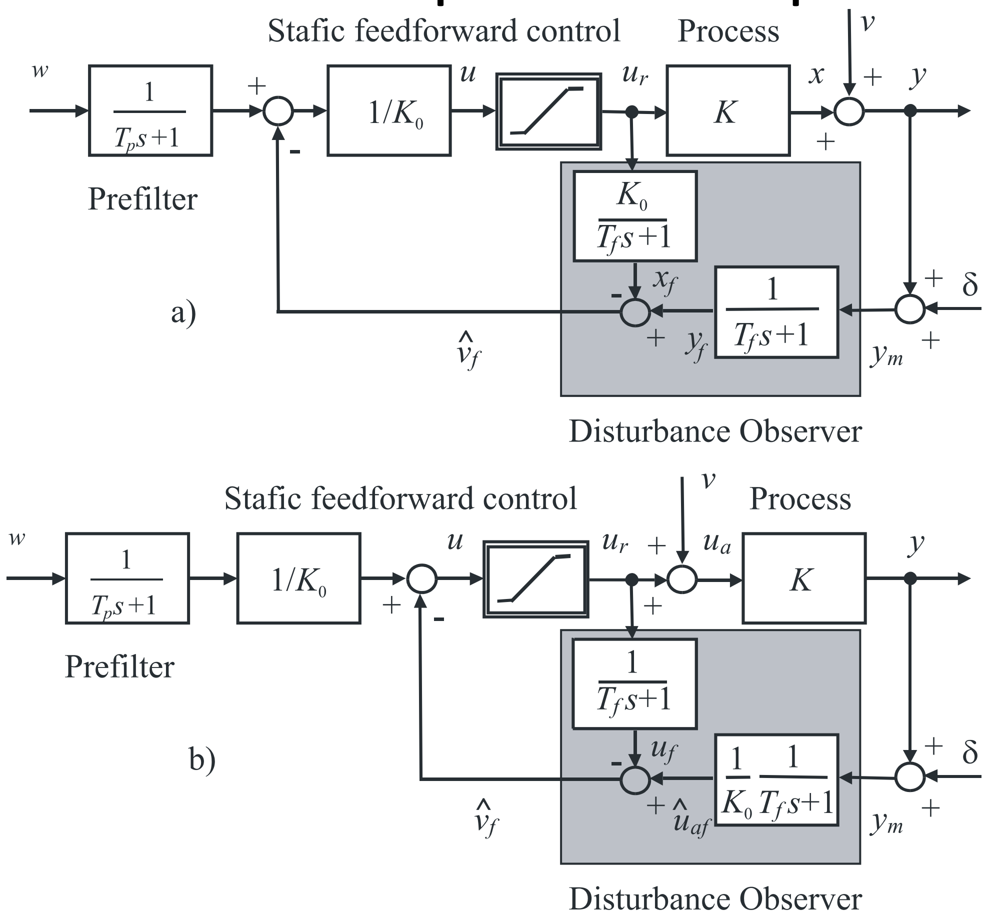
FI_0-ID-controller - generic fundamental structure

Omitting ideal prefilter in the equivalent scheme = Introducing low-pass prefilter into generic scheme = filtered response continuous for t = 0



FI_0-controllers (generic fundamental structures)

Two basic modifications for input and output disturbances



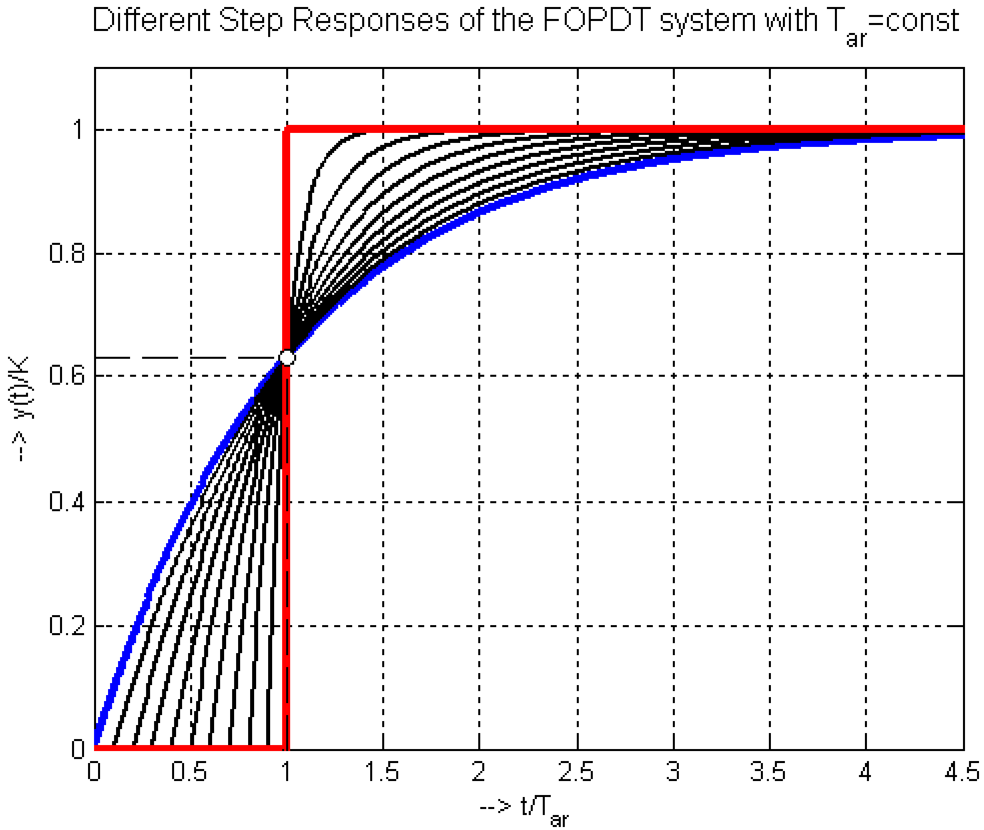
Influence of Nonmodelled Dynamics

Approximation for the non-modeled dynamics
(not considered in deriving controller equation/structure)

$$F_{nd}(s) = \frac{e^{-T_d s}}{1+T_a s} \; ; \; T_{ar} = T_d + T_a$$

Several methods for identification of T_{ar} available

T_{ar} = time, at which the normed step response reaches 63% of the steady state value



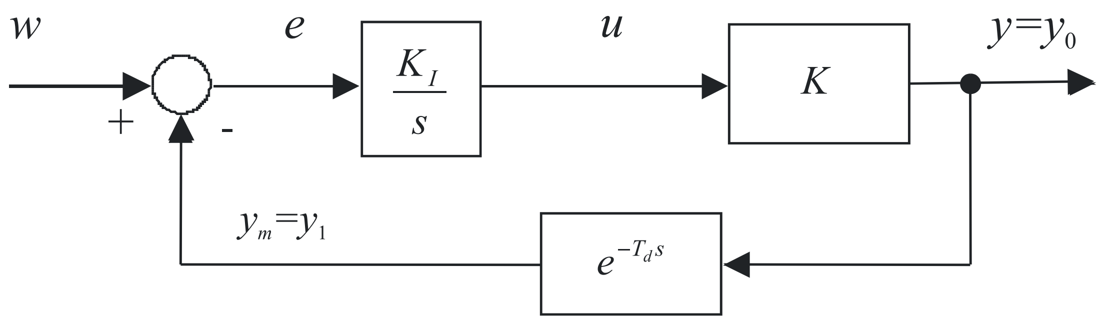
FI_0 controller+dead time T_d

The dominant dynamics (memoryless plant) determines the controller structure

Usability limits and control quality of this structure depend on the nonmodelled dynamics

Dead time Td – frequently used approximation of the nonmodelled dynamics

Dead time influence for the plant outputs y_0 and y_1 may be evaluated analytically or using the performance portrait method



FI_0 controller+dead time T_d

Closed loop transfer functions

$$F_{w0}(s)=\frac{Y_0(s)}{W(s)}=\frac{K_I K e^{T_d s}}{s e^{T_d s}+K_I K}\ ;\ K_I=\frac{1}{K_0 T_f}$$

$$F_{w1}(s)=\frac{Y_1(s)}{W(s)}=\frac{K_I K}{s e^{T_d s}+K_I K}$$

Normalized variables

$$p=T_d s\ ;\ \kappa=\frac{K_0}{K}\ ;\ \Omega=\frac{T_d}{T_f}\ ;$$

Normalized transfer functions

$$F_{w0}(p)=\frac{e^p\Omega/\kappa}{pe^p+\Omega/\kappa}\ ;\ F_{w1}(p)=\frac{\Omega/\kappa}{pe^p+\Omega/\kappa}$$

FI_0 controller+dead time T_d

Charakteristic polynomial

$$A(p) = pe^p + \Omega / \kappa$$

Conditions of the double real pole p_0

$$A(p_0) = 0\ ;\ \dot{A}(p_0) = 0$$

Solution

$$\dot{A}(p) = (p+1)e^p \Rightarrow p_0 = -1$$
$$q = \Omega / \kappa = \exp(-1);\ \tau = 1/q = \exp(e) = 2.71\ldots$$

Optimal (analytical) tuning

$$\Omega = \kappa \exp(-1)\ ;\quad \kappa = K_0 / K\ ;\ \Omega = T_d / T_f$$

FI_0 controller+dead time T_d

Characteristic polynomial

$$A(p) = pe^{p} + \Omega/\kappa$$

Stability border $\quad A(j\omega) = 0$

Stability border conditions

$$A(j\omega) = j\omega e^{j\omega} + \Omega/\kappa$$

$$j\omega\cos\omega - \omega\sin\omega + \Omega/\kappa = 0 \Rightarrow \omega = \pm\pi/2$$

Critical tuning

$$\boxed{q = \Omega/\kappa = \pi/2}$$

Critical pole

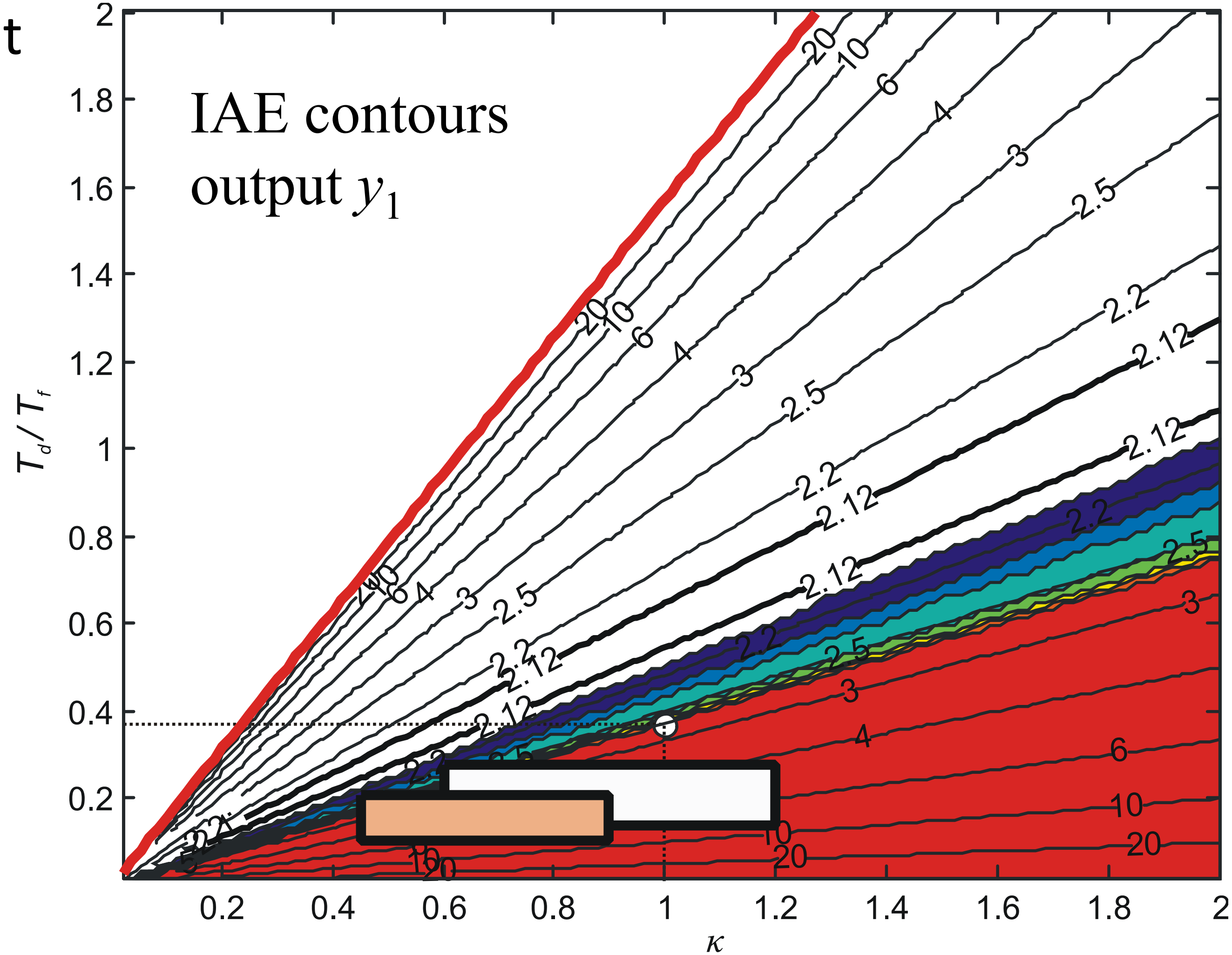
$$\boxed{\alpha_{crit} = -1/T_{f,crit} = -(K_0/K)\pi/(2T_d)}$$

FI_0 controller+dead time T_d

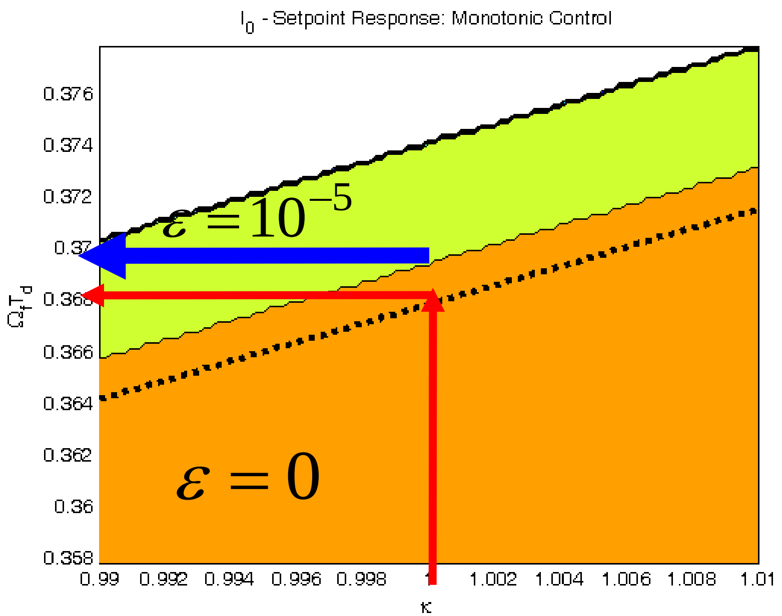
Performance Portrait tolerated deviations of MO & NO ε:

- 0 (red)
- 0.00001
- 0.0001
- 0.001
- 0.01
- 0.02
- 0.05 (darkblue)+

uncertainty boxes with respect to κ and T_d/T_f



FI_0 controller+dead time T_d



Weakened and strict MO
Experimentally determined MO border different from the aperiodicity border

$$\Omega = \kappa \exp(-1)\;;\text{dotted}$$
$$\kappa = K_0 / K\;;\;\; \Omega = T_d / T_f$$

$100\varepsilon_y$ [%]	10	5	4.04	2	1	0.1	0.01	0.001	0	0
$\tau = \kappa / \Omega$	1.724	1.951	2.0	2.162	2.268	2.481	2.571	2.625	2.703	2.718
$q = \Omega / \kappa$	0.580	0.515	0.5	0.465	0.441	0.403	0.389	0.381	0.37	0.368
IAE_0 / T_d	1.105	1.147	1.17	1.240	1.314	1.486	1.571	1.625	1.703	1.718
IAE_1 / T_d	2.105	2.147	2.17	2.240	2.314	2.486	2.571	2.625	2.703	2.718

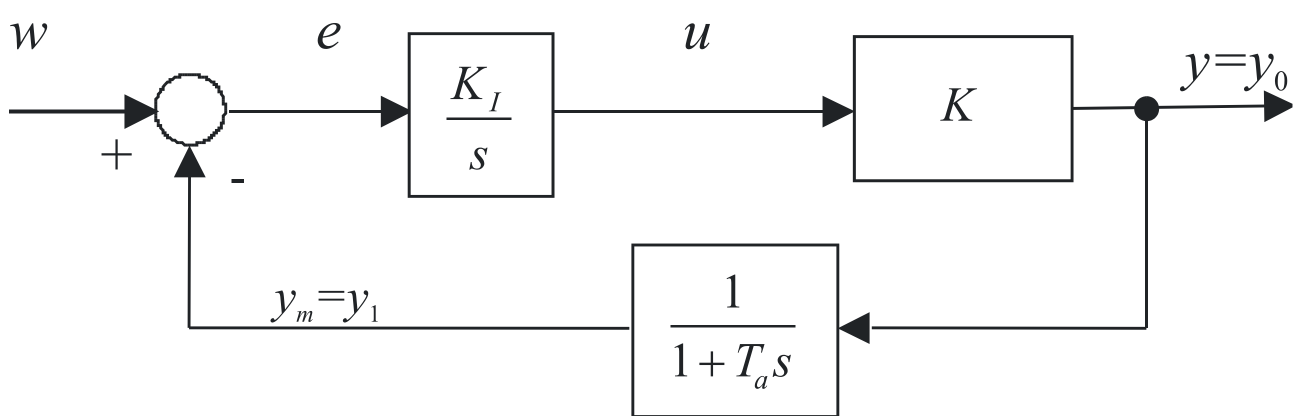
FI_0 controller+time constant T_a

Closed loop transfer functions

$$F_{w0}(s)=\frac{Y_0(s)}{W(s)}=\frac{(1+T_a s)K_I K}{s(1+T_a s)+K_I K}=\frac{(1+T_a s)\Omega_f/\kappa}{s(1+T_a s)+\Omega_f/\kappa}=\frac{B_0(s)}{A(s)};$$

$$F_{w1}(s)=\frac{Y_1(s)}{W(s)}=\frac{K_I K}{s(1+T_a s)+K_I K}=\frac{\Omega_f/\kappa}{s(1+T_a s)+\Omega_f/\kappa}=\frac{B_1(s)}{A(s)};$$

$$\Omega_f=1/T_f;\quad \kappa=K_0/K$$



FI_0 controller+time constant T_a

Closed loop transfer functions

$$F_{w0}(p) = \frac{(1+p)\Omega/\kappa}{p(1+p)+\Omega/\kappa} = \frac{B_0(p)}{A(p)} \ ;$$

$$F_{w1}(p) = \frac{\Omega_f\ /\kappa}{p(1+p)+\Omega/\kappa} = \frac{B_1(p)}{A(p)} \ ;$$

$$\Omega = T_a\ /T_f\ ;\ \ \kappa = K_0\ /\ K$$

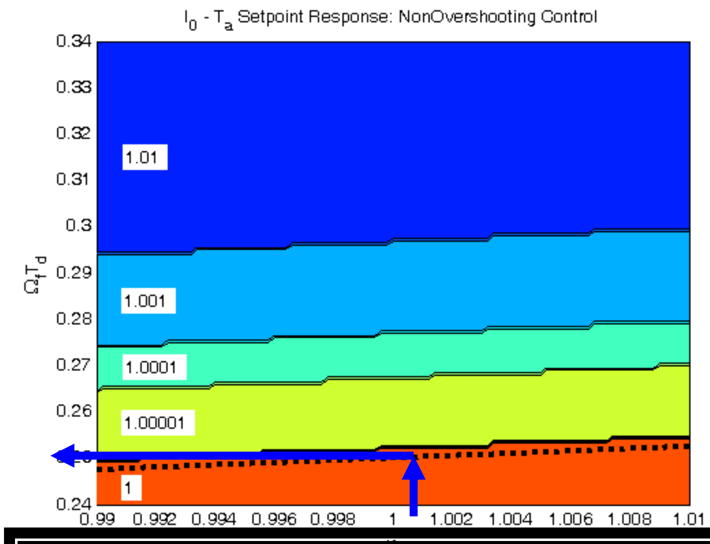
Double real closed loop pole

$$\boxed{\tau = \kappa/\Omega = 4}$$

Critical tuning

$$\boxed{\Omega/\kappa \to 0}$$

FI_0 controller+time constant T_a



Weakened and strict monotonicity
Experimentally determined monotonicity border
different form the aperiodicity border based on the DRDP

$$\kappa = 4\Omega$$

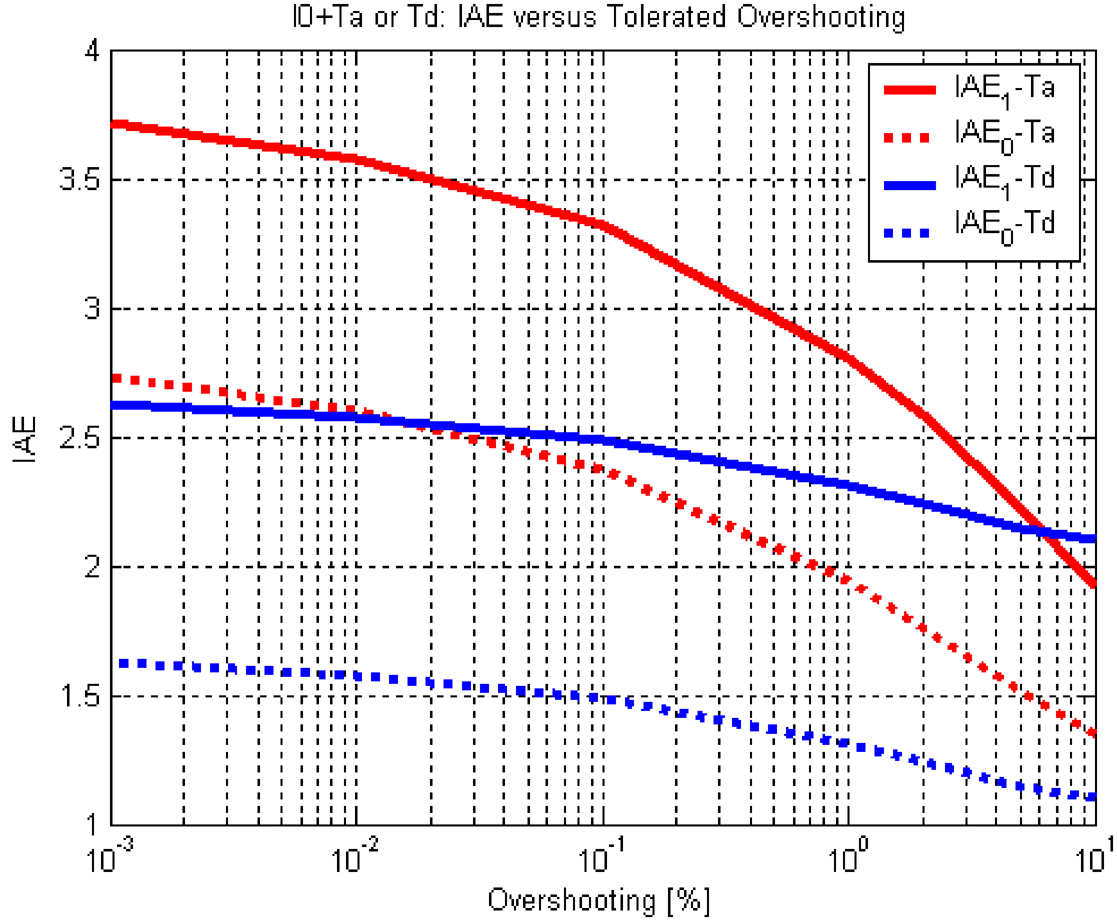
$100\varepsilon_y$ [%]	10	5	2	1	0.1	0.01	0.001	0	**0**
$y_0:\ \tau = \kappa/\Omega$	1.754	2.169	2.604	2.857	3.367	3.597	3.731	3.968	**4**
$q = \Omega/\kappa$	0.570	0.461	0.384	0.350	0.297	0.278	0.268	0.252	**0.25**
IAE_0/T_a	1.348	1.510	1.760	1.941	2.376	2.598	2.732	2.968	**3**
$y_1:\ \tau = \kappa/\Omega$	1.398	1.908	2.433	2.724	3.311	3.571	3.717	3.968	**4**
$q = \Omega/\kappa$	0.715	0.524	0.411	0.367	0.302	0.280	0.269	0.252	**0.25**
IAE_1/T_a	1.921	2.221	2.581	2.806	3.321	3.573	3.717	3.968	**4**

FI_0 controller: comparing loops with dead time T_d or with time constant T_a

IAE versus tolerated overshooting

For T_d it holds $IAE_1 = IAE_0 + 1$

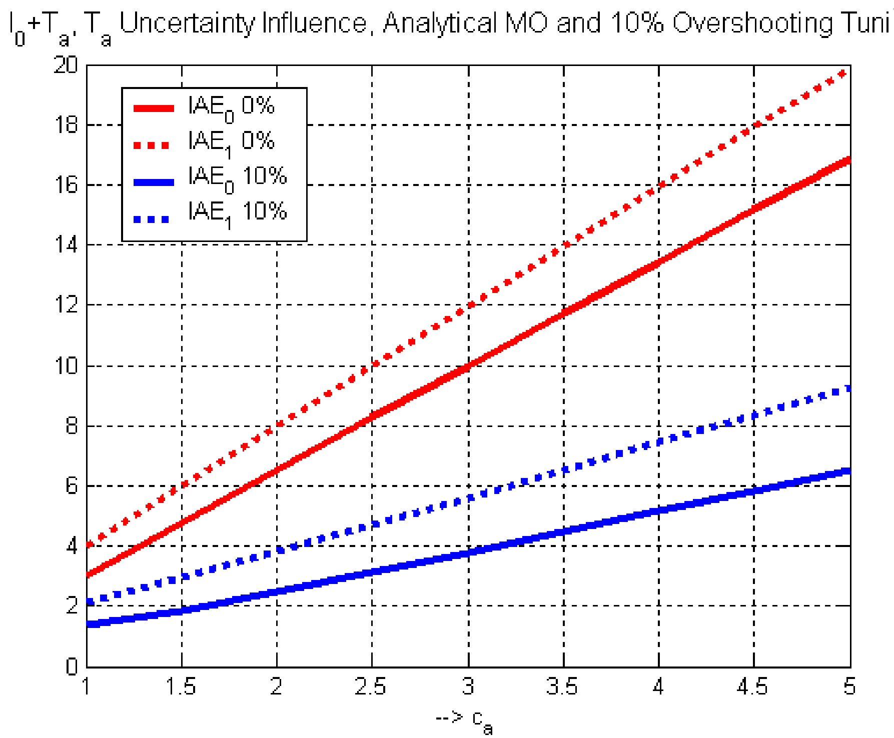
For T_a this holds just for $\varepsilon \rightarrow 0$

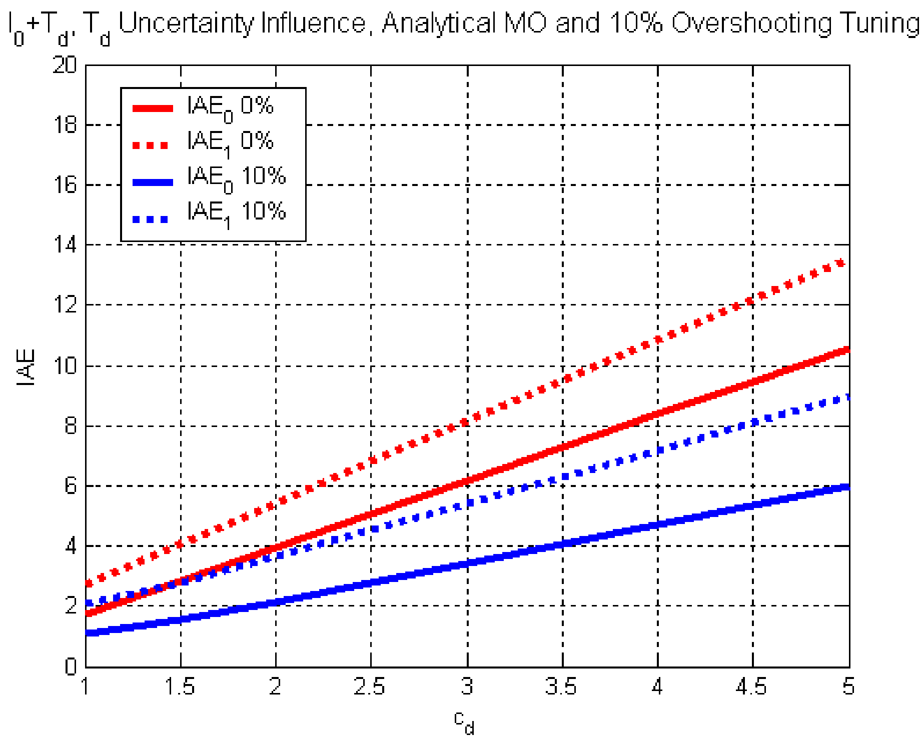


FI_0 controller - robustness characteristics

Influence of uncertainty c_a or c_d *on IAE for different tolerated overshooting*

$$c_a = T_{amax}/T_{amin},\ c_d = T_{dmax}/T_{dmin}$$





History

Aperiodicity border – double real dominant pole

Oldenbourg, R.C. and H. Sartorius: Dynamik selbsttätiger Regelungen. R.Oldenbourg-Verlag, München, 1944

Newer references – simple formula with τ=2 for overshooting 4.04%

Skogestad, S.: Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control* Volume 13, Issue 4, 2003, 291-309.

Oldenbourg a Sartorius, 1944

T_d for overshooting 0 % given as

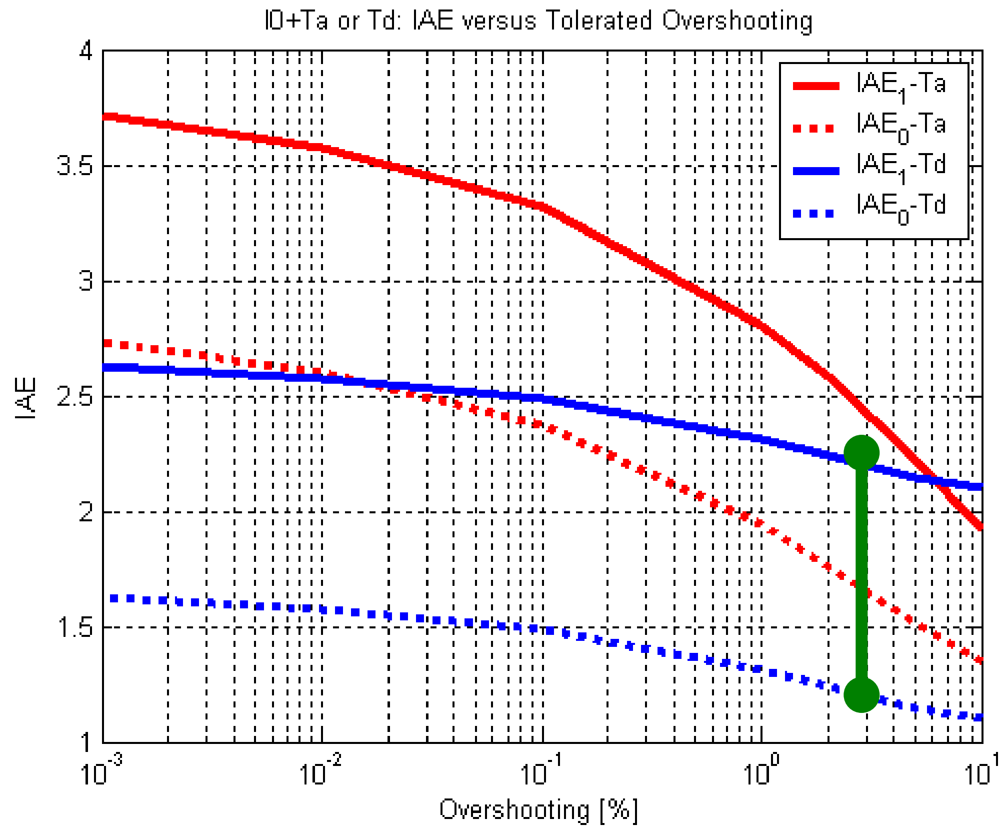
$$\boxed{\Omega = \kappa / e}$$

Skogestad, 2003

T_d for overshooting 4.04% given as

$$\boxed{\Omega = \kappa / 2}$$

$-\infty$



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I_0 controller+dead time T_d

Characteristic polynomial & Critical tuning remain the same as for the FI_0 controller

Differences with respect to FI_0 control are in the:

Performance Portrait

Achievable dynamics and

Optimal tuning rules

I_0 controller+dead time T_d

Closed loop transfer functions for normalized variables

$$p = T_d s \; ; \; \kappa = \frac{K_0}{K} \; ; \; \Omega = \frac{T_d}{T_f} \; ;$$

$$F_{w0}(p) = \frac{Y_0(p)}{W(p)} = (1 + p/\Omega)\frac{e^p \Omega/\kappa}{pe^p + \Omega/\kappa}$$

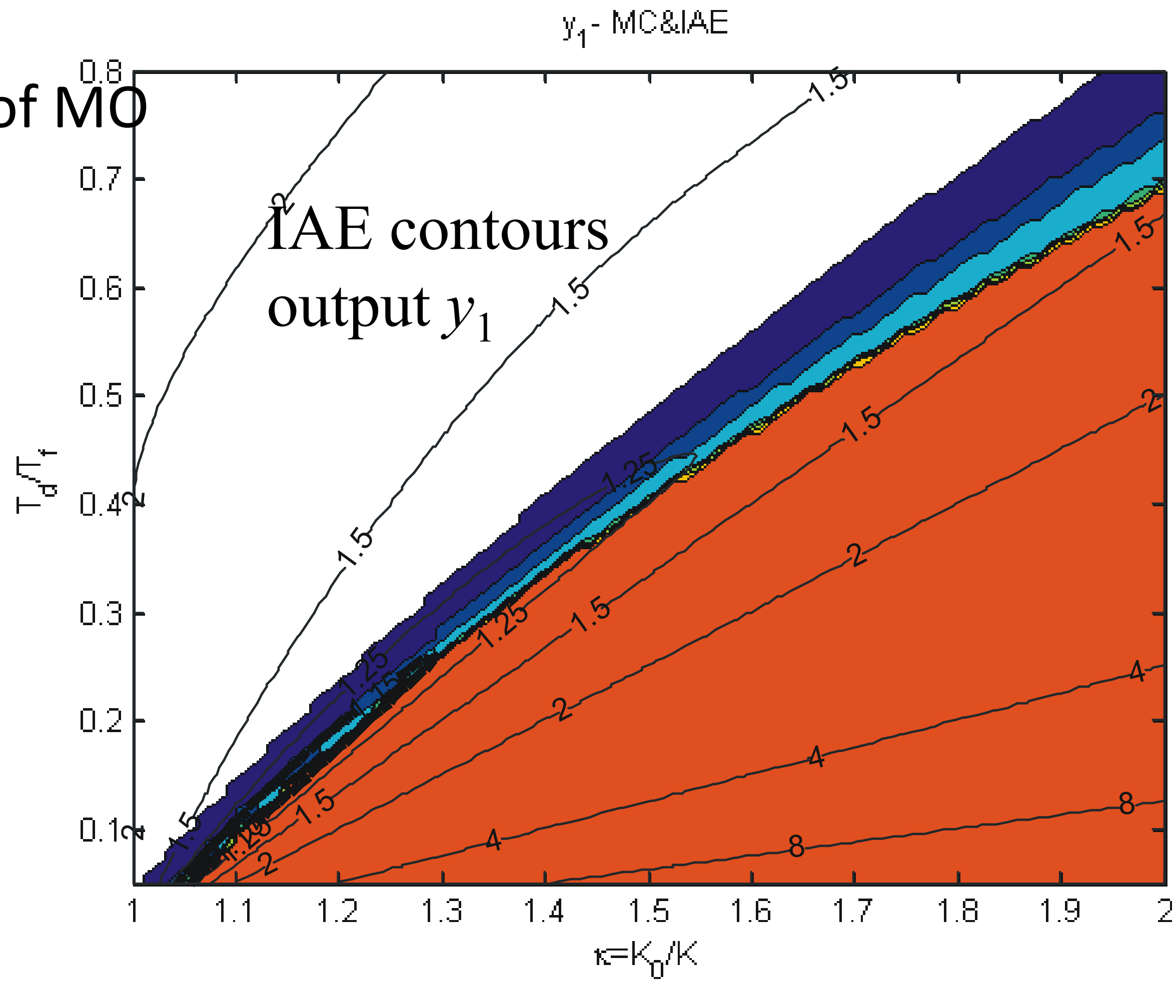
$$F_{w1}(p) = \frac{Y_1(p)}{W(p)} = (1 + p/\Omega)\frac{\Omega/\kappa}{pe^p + \Omega/\kappa}$$

I_0 controller+dead time T_d

Performance Portrait
tolerated deviations of MO & NO ε:

- 0 (red)
- 0.00001
- 0.0001
- 0.001
- 0.01
- 0.02
- 0.05
- 0.1 (darkblue)

with respect to
κ and T_d/T_f

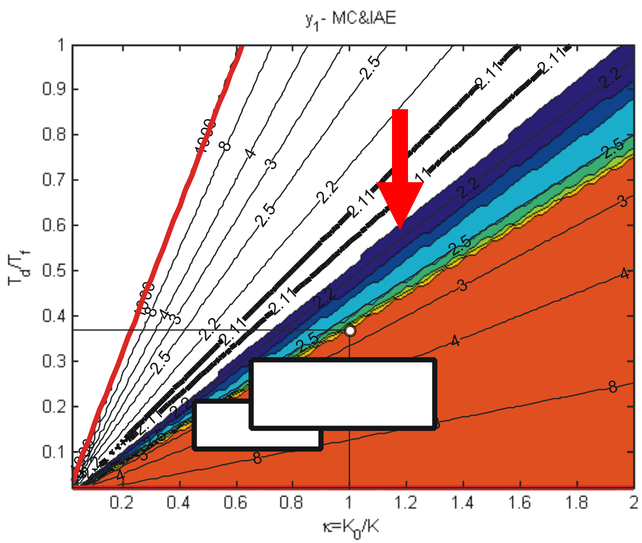


FI_0 versus I_0 controller+dead time T_d

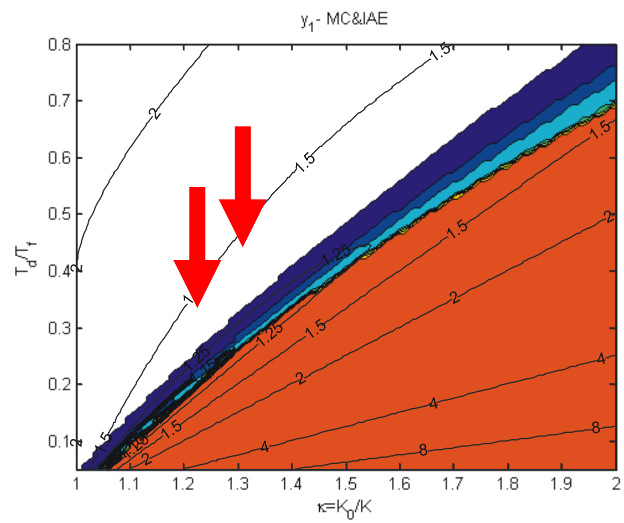
For I_0 minimal IAE values separate the area of strictly NO&MO transients from those with a tolerable overshooting = much more convenient localization of the operating point

K_0 and T_f may now be tuned separately (for FI_0 in a product)

FI_0



I_0

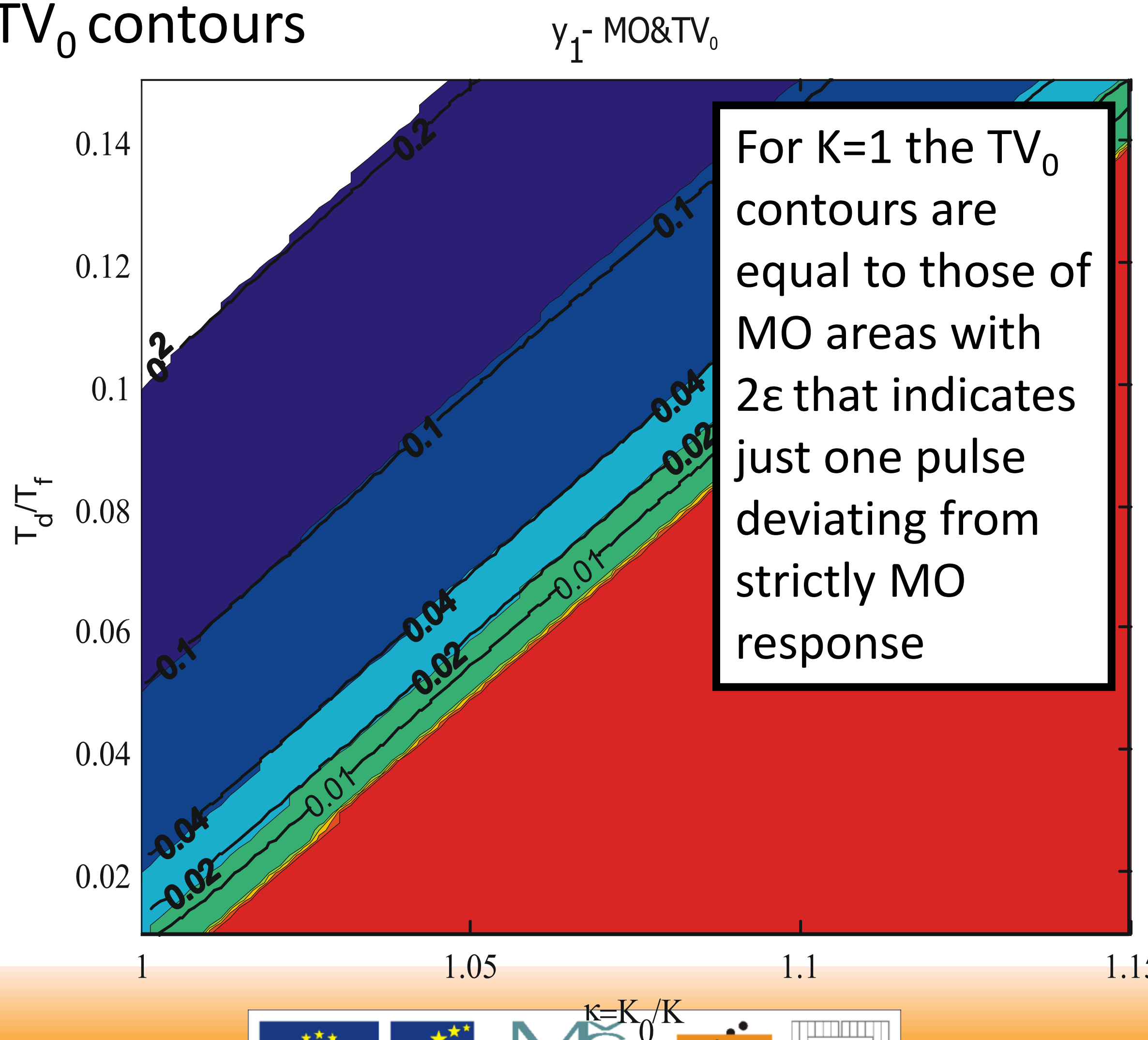


I_0 controller+dead time T_d

ε-MO vs. TV_0 contours

Performance Portrait tolerated deviations of MO & NO ε:

0.00001 (red)
0.0001
0.001
0.01
0.02
0.05
0.1 (darkblue)



FI_0: Conclusions

- Structure of FI_0 (I controller) = static feedforward control + DOB with the first order filter + the first order prefilter with the time constant equal to that of the DOB
- A reliable controller tuning requires approximation of the nonmodelled dynamics by time constant, or more frequently by dead time

FI_0: Conclusions

- To guarantee a tolerated overshooting, robust controller tuning must respect the maximal plant gain K_{max} and the maximal value of the nonmodelled dynamics (T_{amax}, or T_{dmax})
- Sensitivity to fluctuations of the time parameter is the same as sensitivity to plant gain changes

FI_0: Conclusions

- Higher requirements on control quality (lower tolerated overshooting) lead to increased sensitivity to model uncertainty
- For the time constant and lower tolerated overshooting this sensitivity is higher than for the dead time and conversely, for higher tolerated overshooting the sensitivity is higher in the case of dead time than for a time constant

I_0: Conclusions

- Structure of I_0 controller = static feedforward control + DOB with the first order filter
- In comparing with FI_0, the setpoint responses may be reasonably improved
- A reliable controller tuning requires approximation of the nonmodelled dynamics (by a time constant, or more frequently by a dead time).

I_0: Conclusions

- To guarantee a tolerated overshooting, robust controller tuning must respect the maximal plant gain K_{max} and the maximal value of the nonmodeled dynamics (T_{amax}, or T_{dmax})
- Sensitivity to fluctuations of the time parameter is different from the sensitivity to plant gain changes.

I_0: Conclusions

- Requirement to keep the disturbance response (i.e. $T_f K_0$=const) leads finally to dynamics comparable with those achieved by reasonably simpler FI_0
- Therefore, we will deal mostly just with the simpler FI_0 = I-controller