

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Robustné PID-regulátory s obmedzeniami

Robust Constrained PID Control

Robust Design of Model Based PI_1 and PID_1 Controllers

Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.

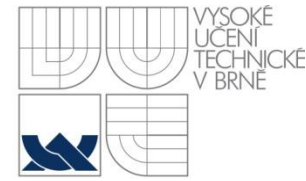


MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY

Contents



OP Vzdělávání
pro konkurenceschopnost



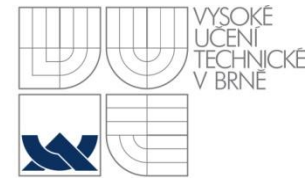
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

- Reconstruction & Compensation of Input Disturbances
- Reconstruction & Compensation of Output Disturbances
- Robust Tuning by the Performance Portrait Method
- Comparing with the Parallel PI controller
- Comparing with Smith Predictor

Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.



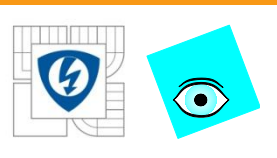
Contents



INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

- Stabilization of FOPDT plants with PD controller
- Model based PID_1 controllers for the FOPDT plants
- PID_1 controllers for 2nd order plants
- Conclusions

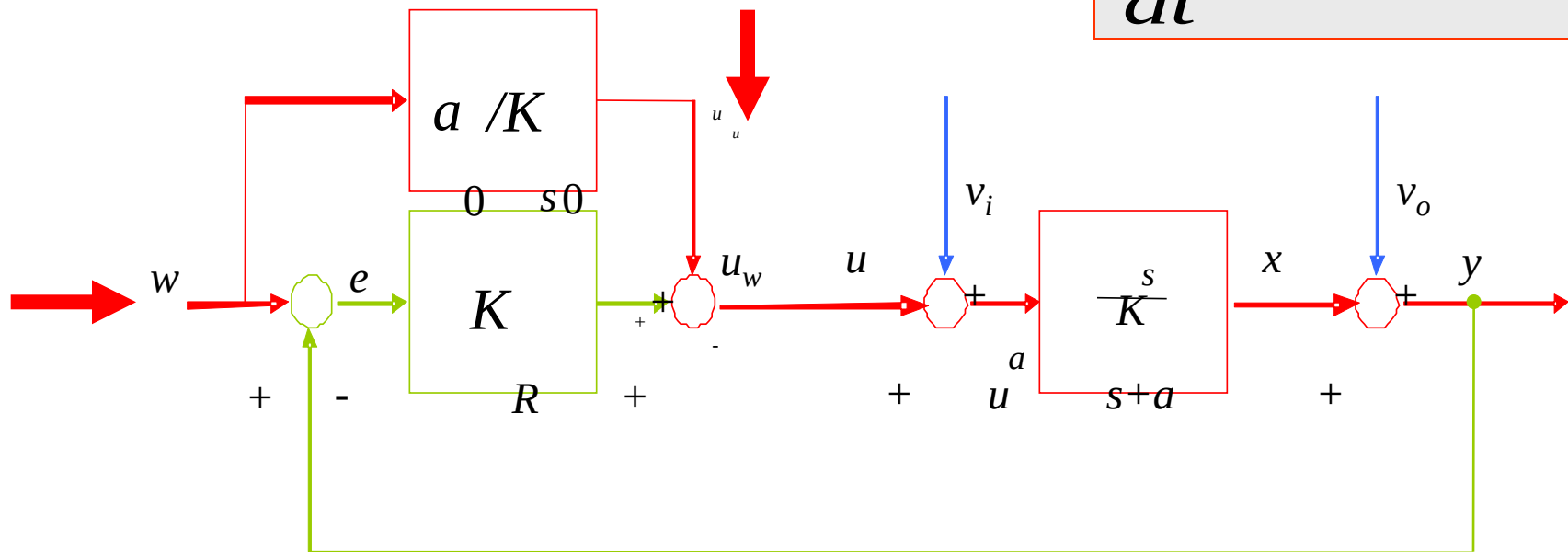
Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.



Dynamical Class “1”

Linear pole assignment P-control

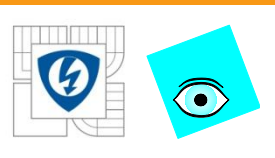
$$\frac{de}{dt} = \alpha e ; \quad \alpha < 0$$



$$e = w - y$$

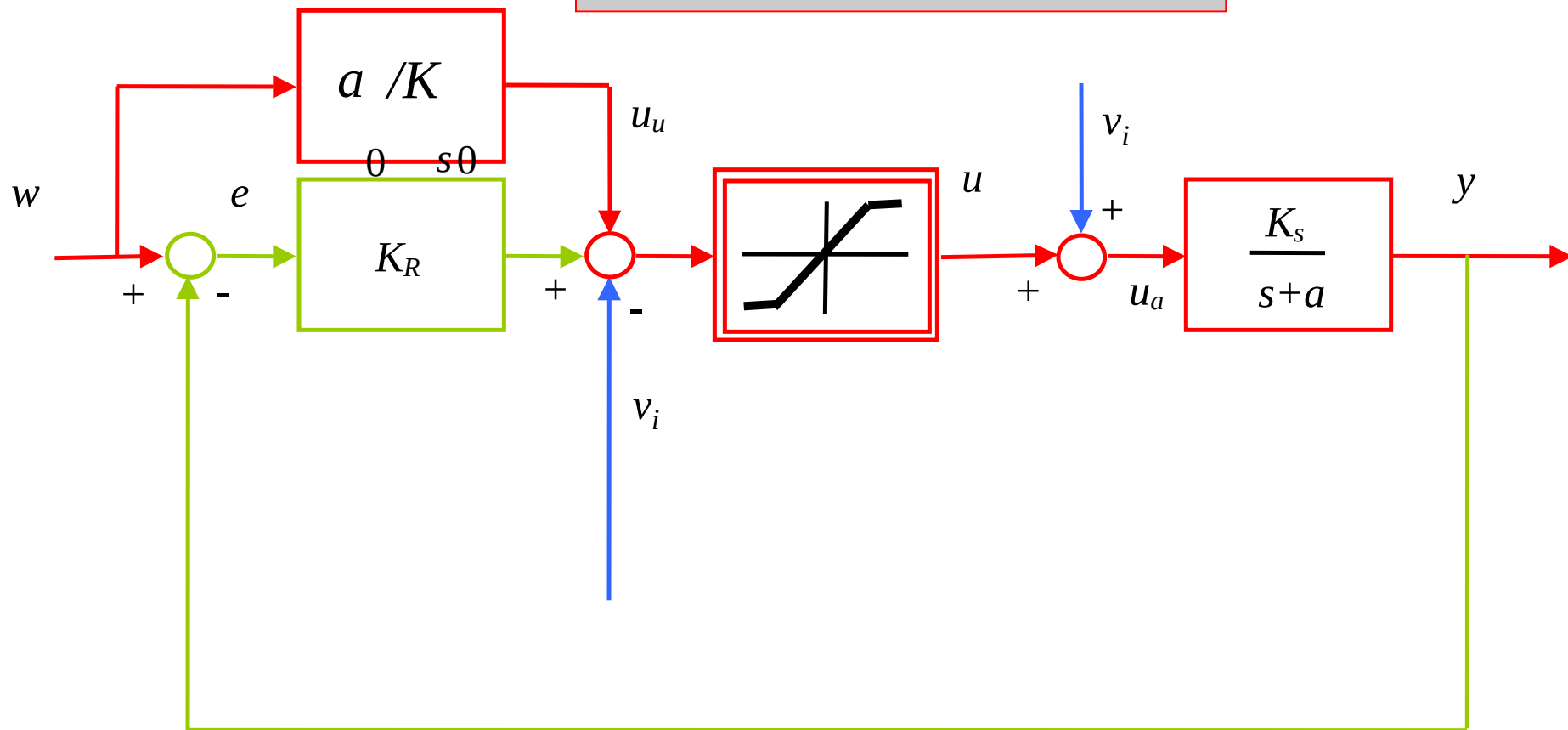
$$u = K_R e + u_w ; \quad K_R = -\frac{\alpha + a_0}{K_{s0}} = \frac{\Omega - a_0}{K_{s0}} ; \quad u_w = \frac{a_0}{K_{s0}} w$$

11.3.2011



Input Disturbance Compensation

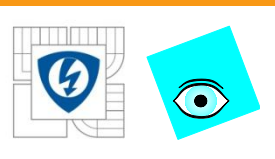
$$K_R = (\Omega - a_0) / K_{s0}$$



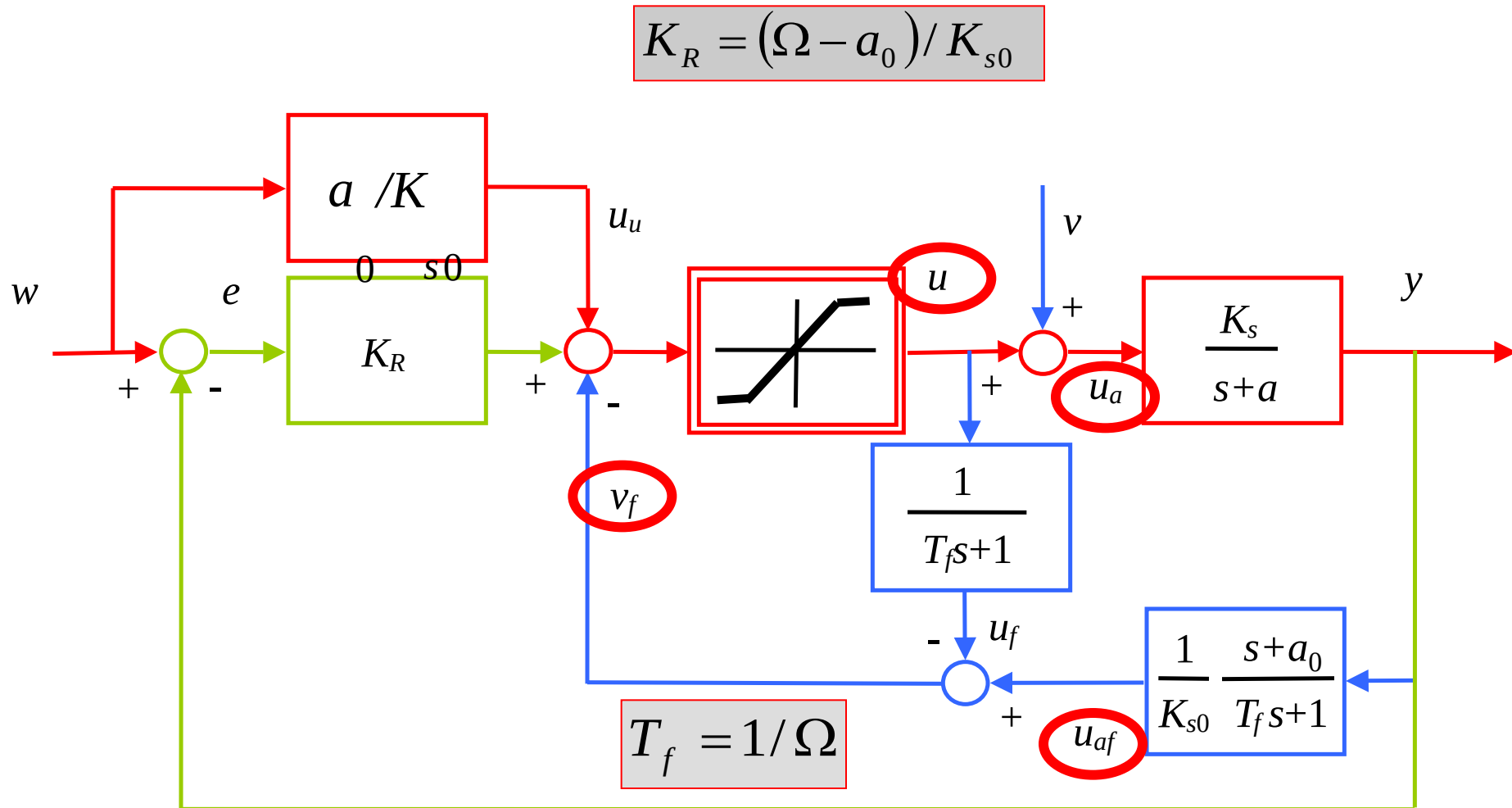
11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ



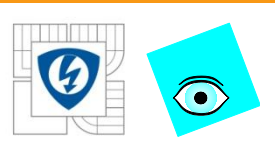


Input Disturbance Compensation



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ



Input Disturbance Compensation

- PI₁-IM Transfer functions

$$a_0 = a ; K_{s0} = K_s$$

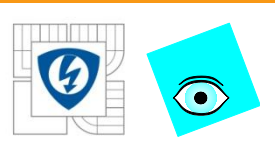
$$F_w(s) = \frac{K_s K_R + a}{s + K_s K_R + a} = \frac{1}{1 + T_w s} ; T_w = \frac{1}{K_s K_R + a} = \frac{1}{\Omega} ;$$

$$F_{vo}(s) = \frac{s(s+a)T_f T_w}{(1+T_w s)(1+T_f s)} ; F_{vi}(s) = \frac{K_s}{s+a} F_{vo} = \frac{s T_f T_w K_s}{(1+T_w s)(1+T_f s)}$$

$$F_{vo}(0) = 0 ; F_{vi}(0) = 0$$

- Nominal stability condition $T_f > 0, T_w > 0, K_R K_s + a > 0$
- Disturbance response has pole of the setpoint response + pole of the Disturbance Observer
- IM = Inverse Model

11.3.2011



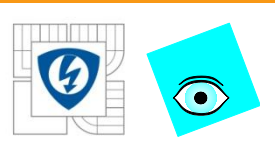
Output Disturbance Compensation

- PI₁-IM Transfer functions (uncertain case)

$$F_w(s) = \frac{(T_f s + 1)\Omega K_s}{K_{s0} T_f s^2 + [K_s T_f (\Omega - a_0) + K_{s0} T_f a + K_s] + K_s \Omega} ;$$
$$F_{vo}(s) = \frac{s(s + a) T_f K_{s0}}{K_{s0} T_f s^2 + [K_s T_f (\Omega - a_0) + K_{s0} T_f a + K_s] + K_s \Omega} ;$$
$$F_{vi}(s) = \frac{s T_f K_s K_{s0}}{K_{s0} T_f s^2 + [K_s T_f (\Omega - a_0) + K_{s0} T_f a + K_s] + K_s \Omega} ;$$

$$F_w(0) = 1 ; F_{vo}(0) = 0 ; F_{vi}(0) = 0$$

- Increased robustness of the setpoint responses for prefilter with $T_p = T_f$



Output Disturbance Compensation

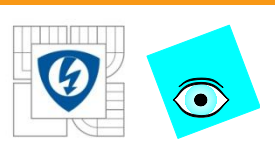
- PI₁-IM Transfer functions (uncertain case)

$$F_w(s) = \frac{(T_f s + 1)\Omega K_s}{K_{s0} T_f s^2 + [K_s T_f (\Omega - a_0) + K_{s0} T_f a + K_s] + K_s \Omega};$$

$$F_{vo}(s) = \frac{s(s + a)T_f K_{s0}}{K_{s0} T_f s^2 + [K_s T_f (\Omega - a_0) + K_{s0} T_f a + K_s] + K_s \Omega};$$

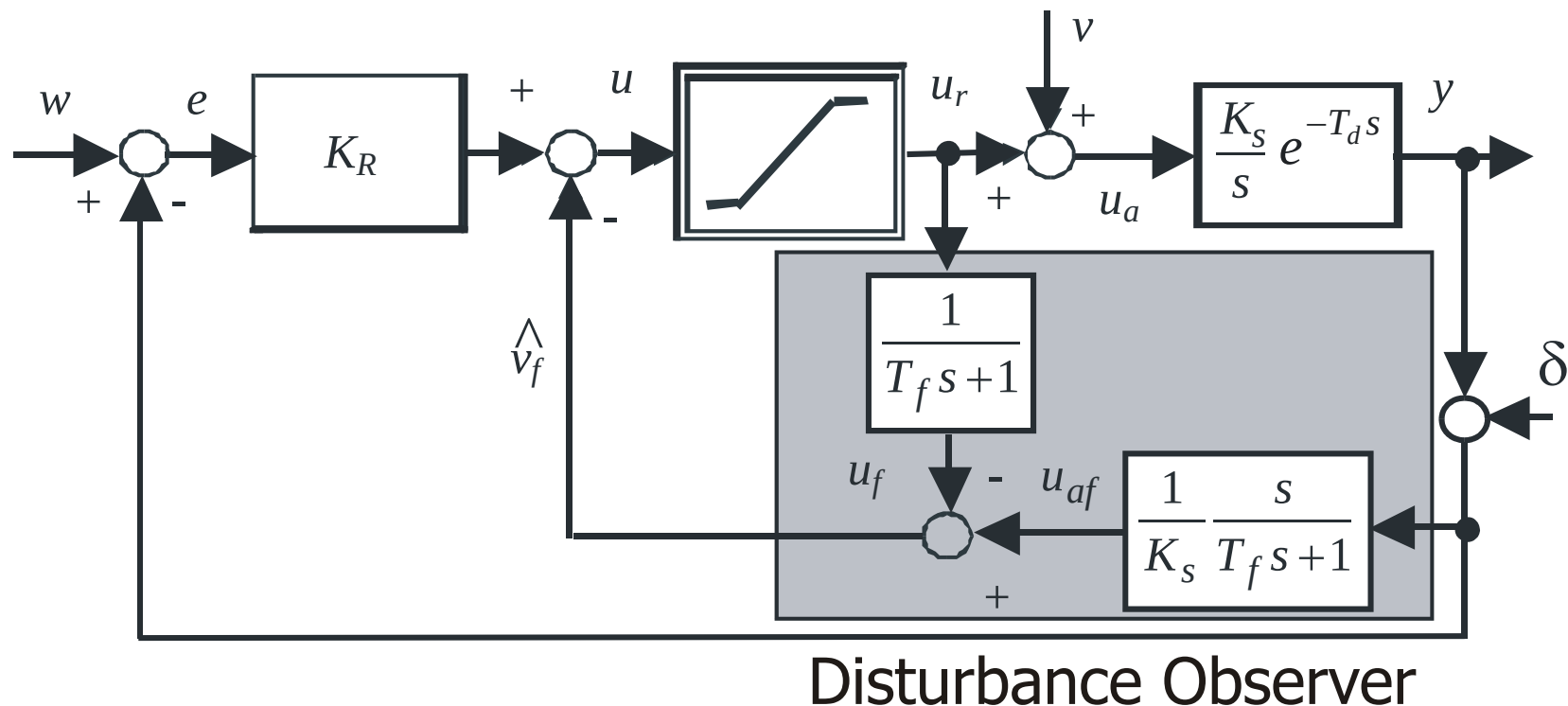
$$F_{vi}(s) = \frac{s T_f K_s K_{s0}}{K_{s0} T_f s^2 + [K_s T_f (\Omega - a_0) + K_{s0} T_f a + K_s] + K_s \Omega};$$

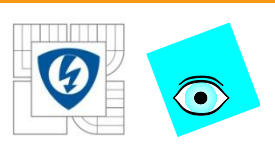
- Robust design parameters $K_{s0}/K_s, a, a_0, T_f \Omega$
- Positioning 2D UB corresponding to $(K_{s0}/K_s, a)$ in 5D (remaining coordinates are $a_0, T_f \Omega$)



PI₁-IM controller

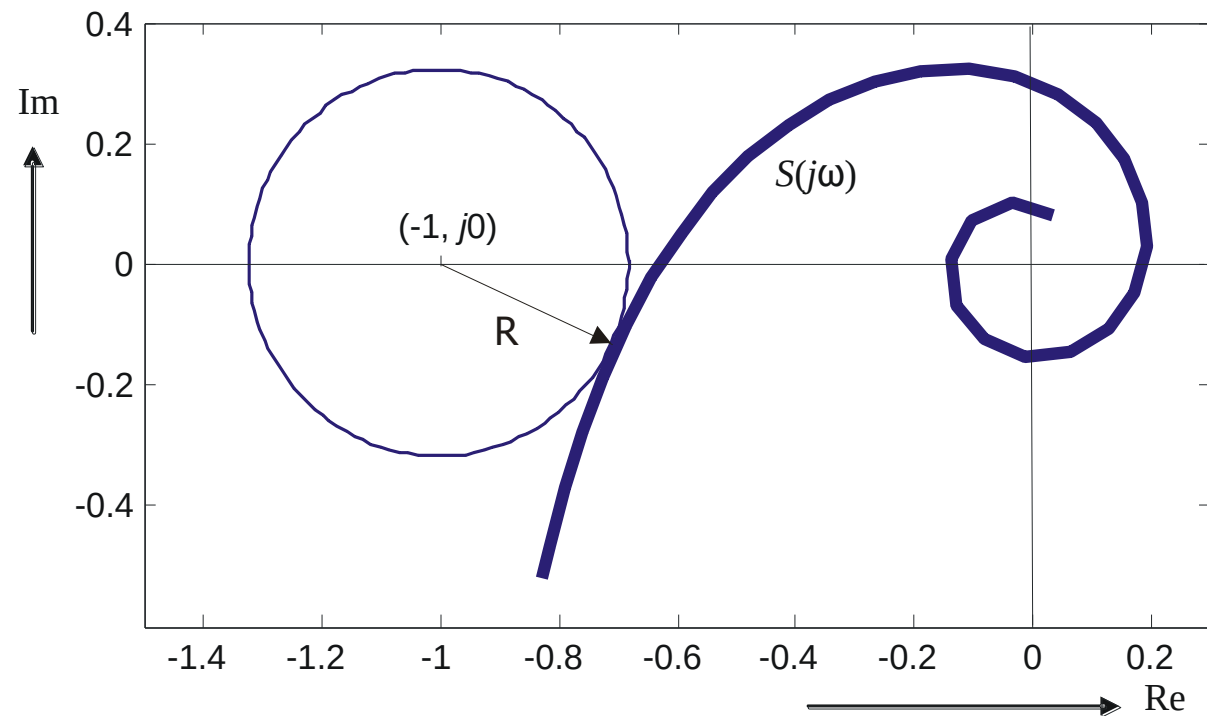
P controller + observer for input disturbance with inverse model ($a=a_0=0$)





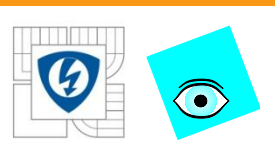
Maximal Sensitivity M_s

Maximal sensitivity M_s = equal to radius R of a circle with centre in critical point $(-1, 0j)$ touching the Nyquist curve of the closed loop system $S(j\omega)$

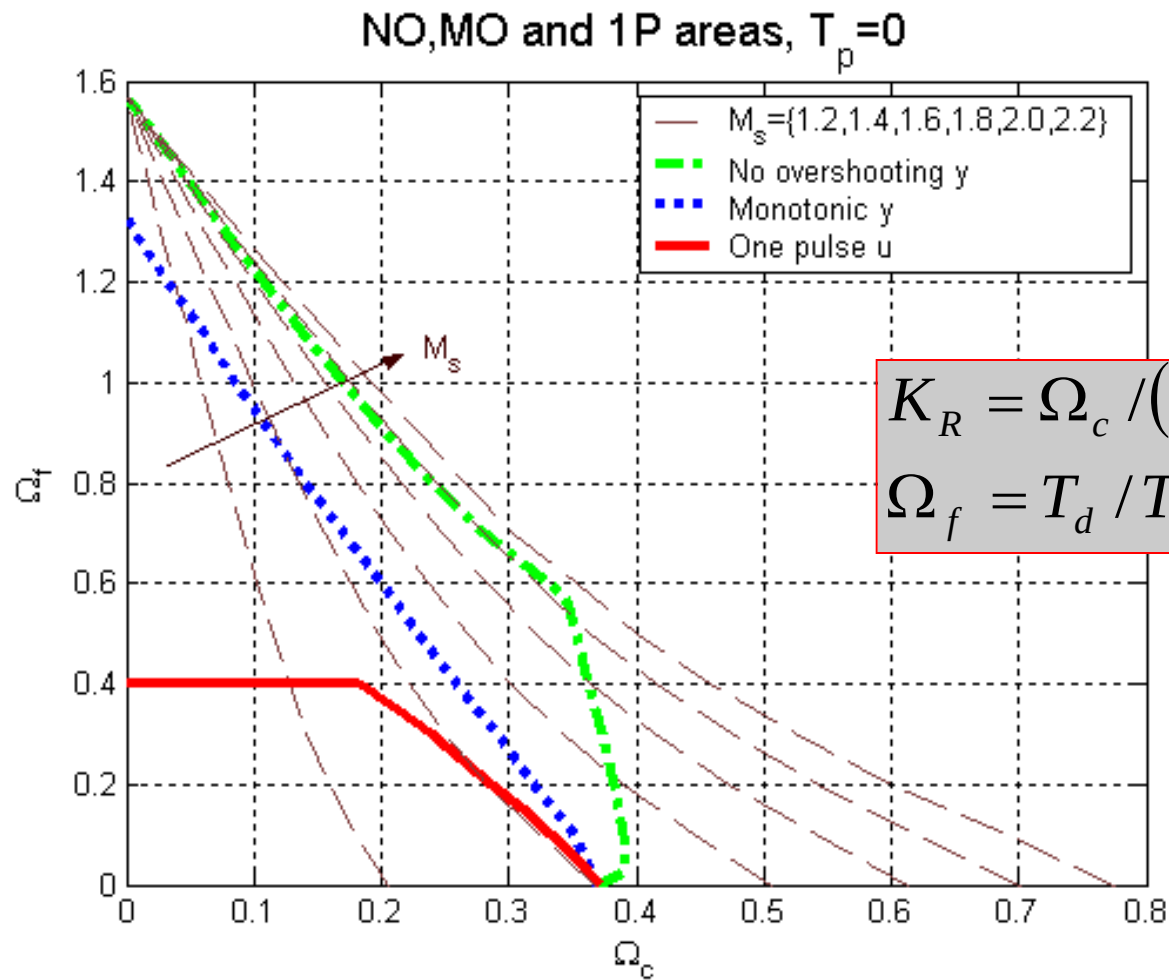


$$S(s) = 1/[1 + L(s)]$$

$$L(s) = R(s)F(s)$$



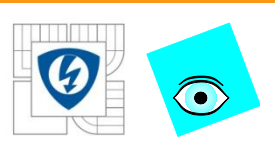
PI₁ Performance Portrait & M_s values



11.3.2011

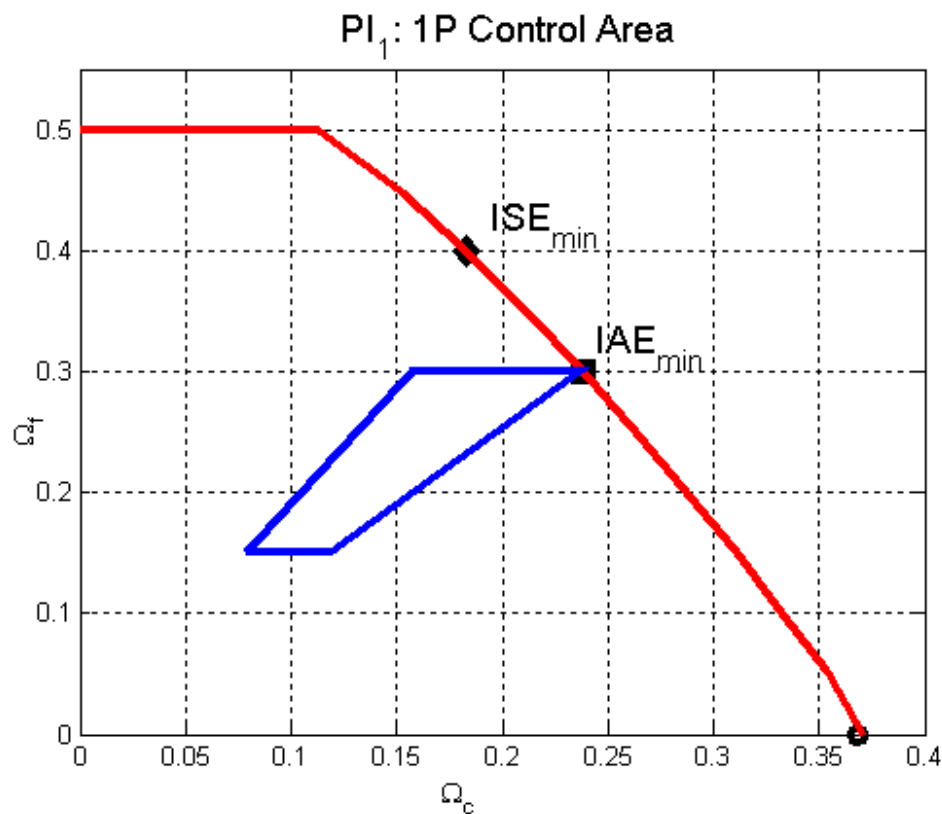
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ



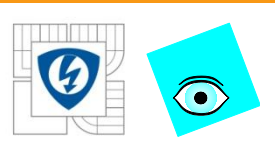


Uncertainty Set (US)

PI_1 controller – region 1P is convex, optimal point lying at its border!

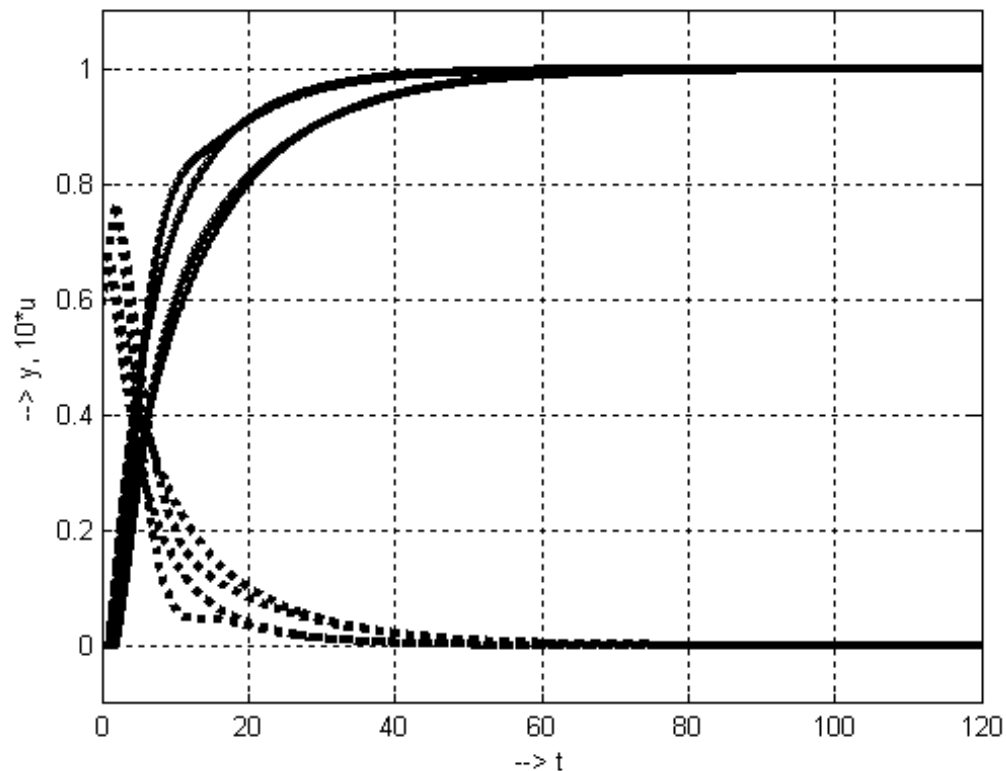


$$IAE_{min} = 4.12$$



Step responses PI_1 controller

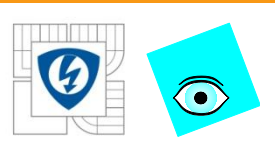
Checking results for limit values of K_s and T_d



11.3.2011

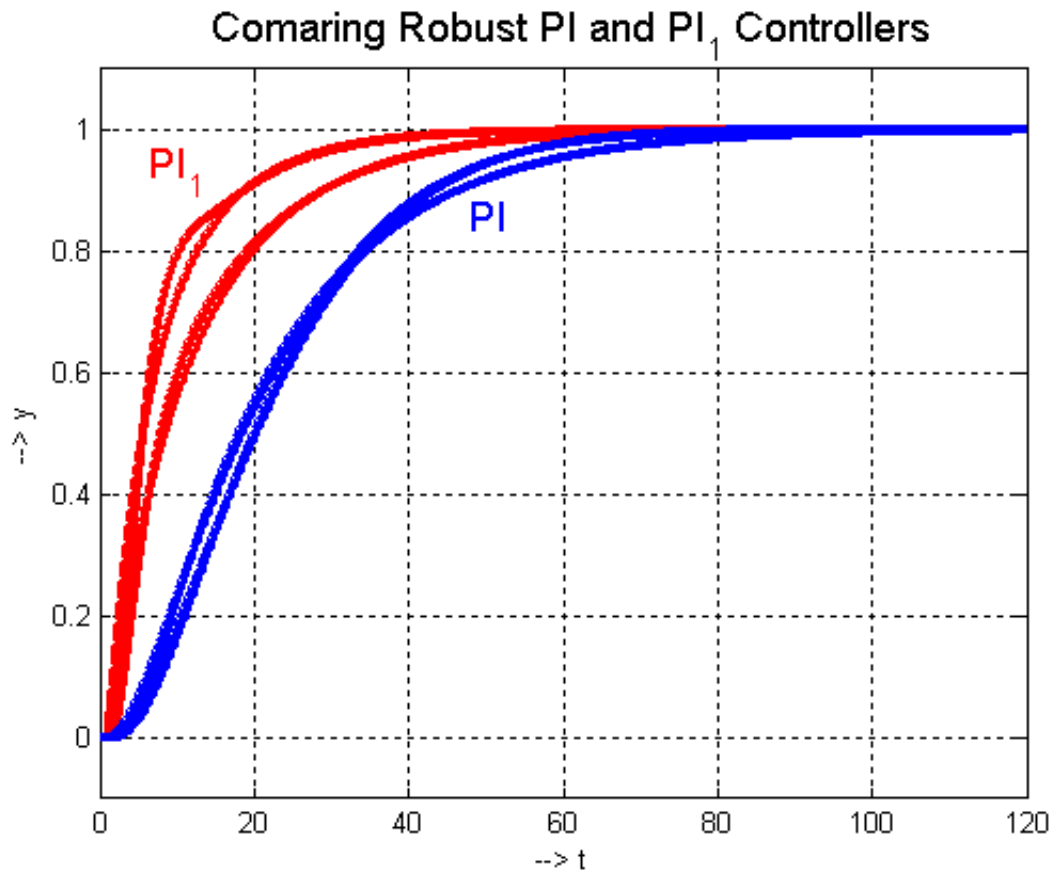
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





Step responses PI_1 -IM and PI control

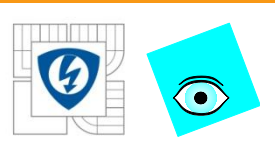
Comparing with the parallel PI controller/2D



11.3.2011

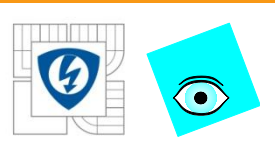
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





PI₁-DTIM controller

- P controller + Disturbance Observer with compensation of dead time T_d
- DTIM = Dead-Time + Inverse Model in Disturbance Observer



Input Disturbance Compensation

- PI₁-IM Transfer functions

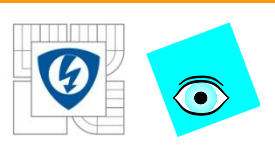
$$a_0 = a ; K_{s0} = K_s , T_{d0} = T_d$$

$$F_w(s) = \frac{(K_s K_R + a)e^{-T_d s}}{(s + K_s K_R e^{-T_d s} + a)} ;$$

$$F_{vo}(s) = \frac{(s + a)(1 - e^{-T_d s} + sT_f)}{(s + K_s K_R e^{-T_d s} + a)(1 + T_f s)} ; F_{vi}(s) = \frac{K_s}{s + a} F_{vo}$$

$$F_w(0) = 1 ; F_{vo}(0) = 0 ; F_{vi}(0) = 0$$

- Setpoint response has poles of simple P-control
- Nominal stability condition for $T_f > 0$ are the same as for the P controller
- Full robust design – 7D parameter space



Analytical Construction of PP

- Critical and optimal gains (aperiodicity border)

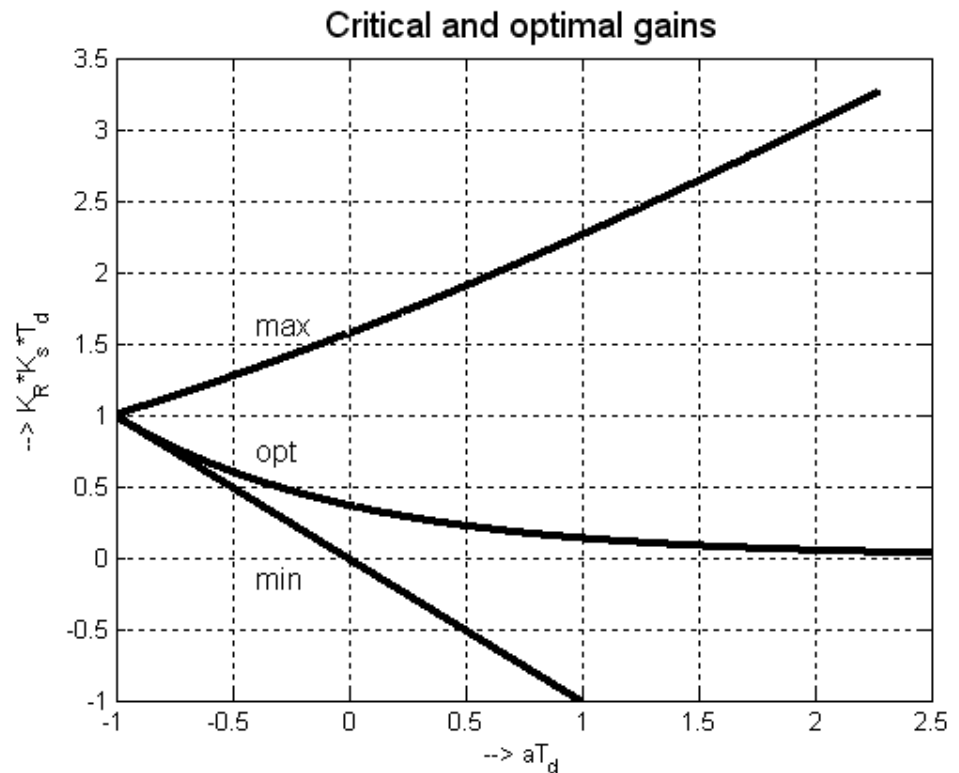
$$K_{min} < K < K_{max} ; K = K_R K_s T_d$$

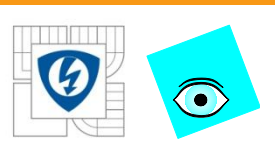
$$K_{max} = \frac{\tau_d}{\sin \tau_d} ; A = -\frac{\tau_d \cos \tau_d}{\sin \tau_d} ;$$

$$\tau_d \in (0, \pi/2) \cup (\pi/2, \pi)$$

$$K_{min} = -A = -aT_d$$

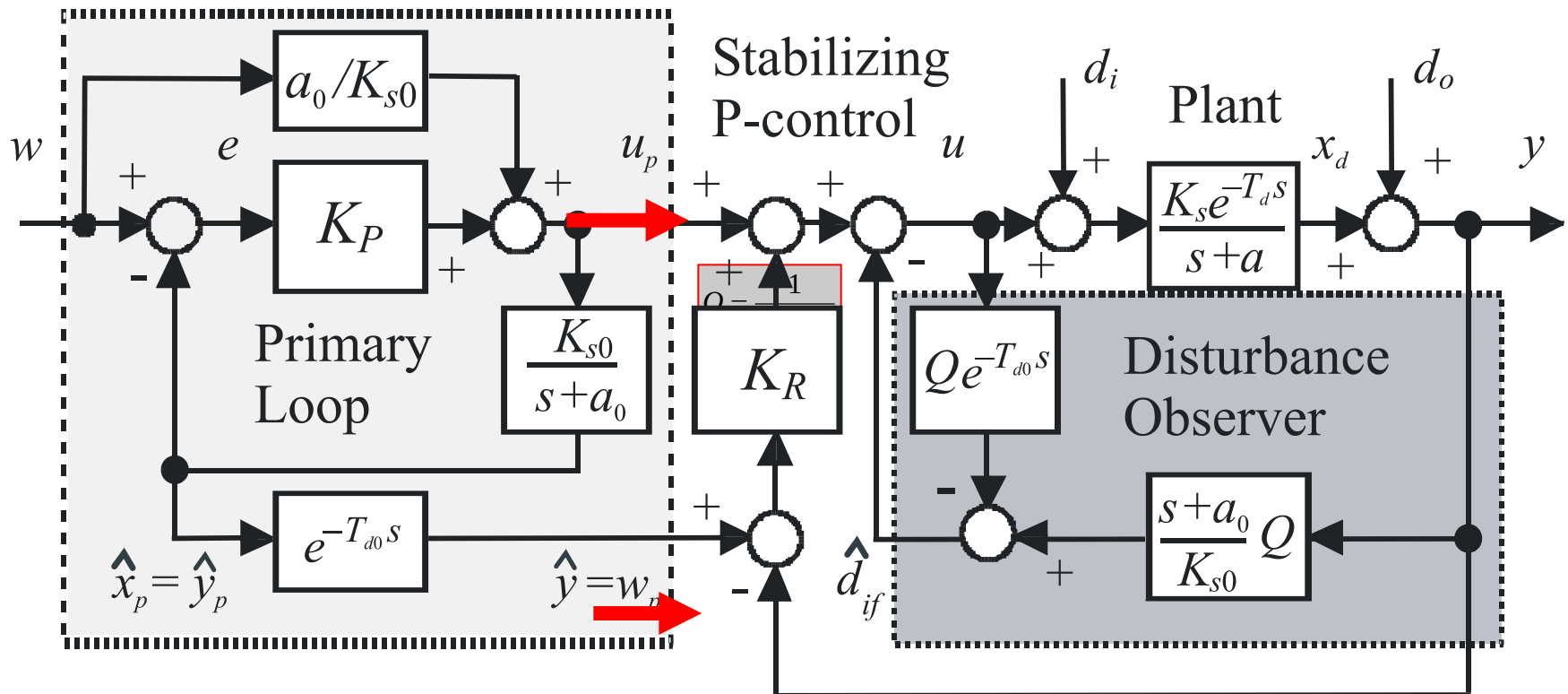
$$K_{opt} = e^{-(1+aT_d)}$$

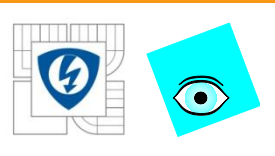




PI₁-DFF-DTIM controller

P control + dynamical feedforward + disturbance observer with inverse model and compensation of dead time T_d





Input Disturbance Compensation

- PI₁-IM Transfer functions

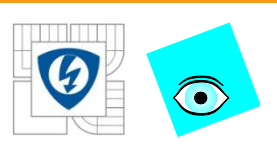
$$a_0 = a ; K_{s0} = K_s , T_{d0} = T_d$$

$$F_w(s) = \frac{(K_s K_P + a)e^{-T_d s}}{(s + K_s K_P + a)} = \frac{e^{-T_d s}}{T_w s + 1} ; T_w = \frac{1}{K_s K_P + a}$$

$$F_{vo}(s) = \frac{(s + a)(1 - e^{-T_d s} + sT_f)}{(s + K_s K_R e^{-T_d s} + a)(1 + T_f s)} ; F_{vi}(s) = \frac{K_s}{s + a} F_{vo}$$

$$F_w(0) = 1 ; F_{vo}(0) = 0 ; F_{vi}(0) = 0$$

- Nominal setpoint response without T_d in denominator
- Nominal stability condition for $T_f > 0$ are the same as for the P controller
- Full robust design – 8D parameter space



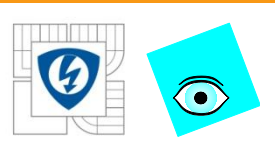
Conclusions 1

- Modular controller pack enabling extension from the simplest controllers not considering nonmodelled dynamics up to generalization of Smith Predictor
- Controller Tuning based on the Stability/Quality analysis of simple P controller
- Separate tuning of the setpoint and disturbance response
- Enabling unified treatment of both stable, integral and unstable FOPDT systems
- Setpoint response reasonably improved against parallel PI control

11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

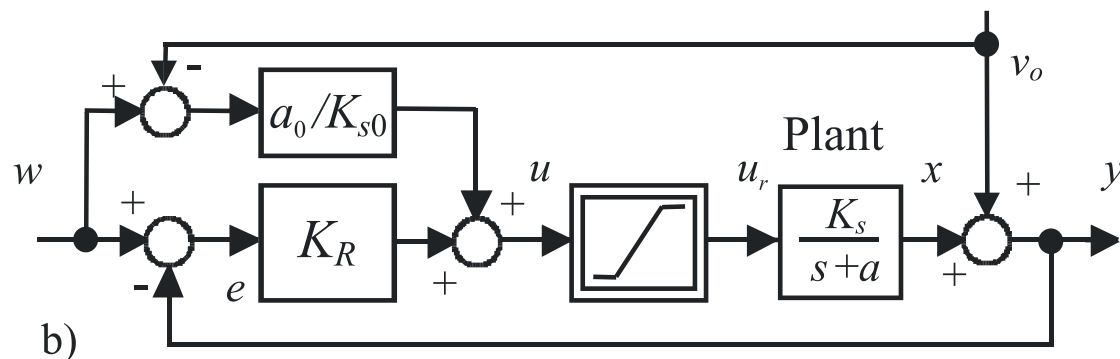
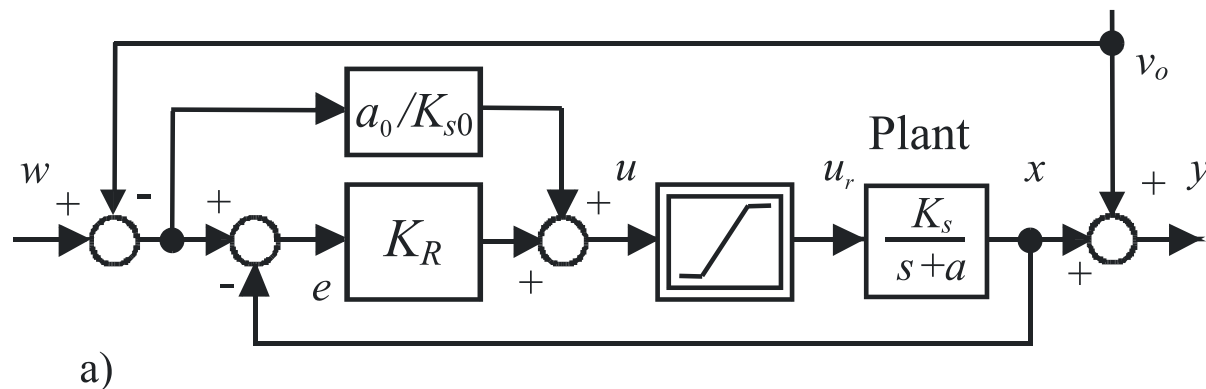


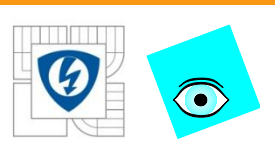


Output Disturbance Compensation

- Measurable output disturbance compensation

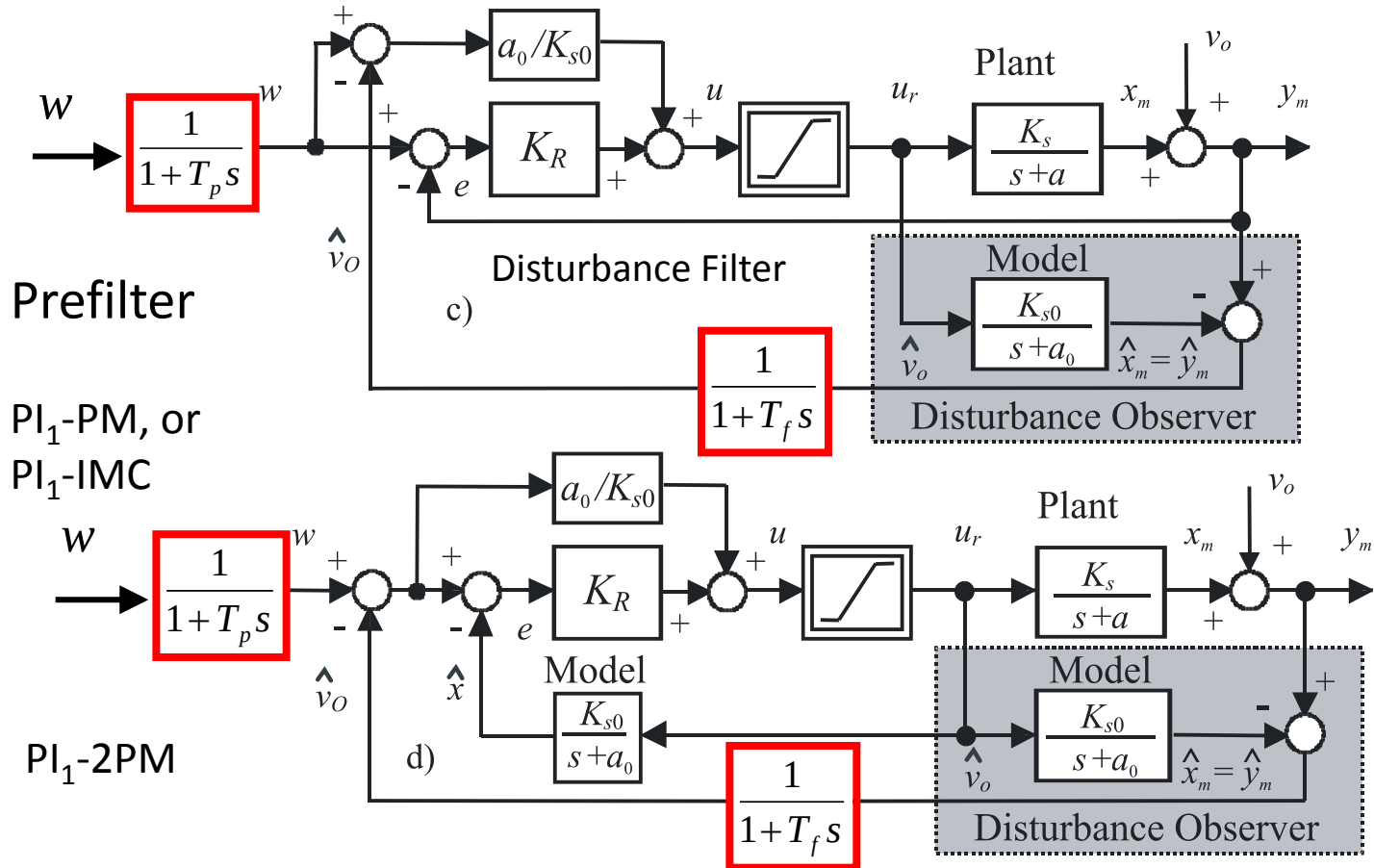
$$K_R = (\Omega - a_0) / K_{s0}$$

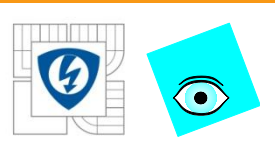




Output Disturbance Compensation

- Disturbance reconstruction & compensation





Output Disturbance Compensation

- PI₁-PM Transfer functions

$$F_w(s) = \frac{1}{1 + T_w s} ; T_w = \frac{1}{K_s K_R + a} = \frac{1}{\Omega} ;$$

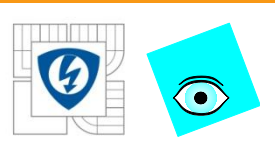
$$F_{vo}(s) = \frac{s T_w}{1 + T_w s} ; F_{vi}(s) = \frac{K_s}{s + a} F_{vo}$$

$$F_{vo}(0) = 0 ; F_{vi}(0) = 0 \text{ (for } a \neq 0 \text{)}$$

$$a_0 = a ; K_{s0} = K_s$$

- nominal stability condition $K_R K_s + a > 0$
- but, already in the nominal case with step input disturbance, there is:
- a permanent steady state error for $a=0$ and
- unstable input disturbance response for $a < 0$

11.3.2011



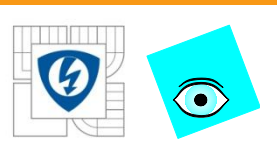
Output Disturbance Compensation

- PI₁-PM Transfer functions (uncertain case)

$$F_w(s) = \frac{Y(s)}{W(s)} = \frac{K_s(s+a_0)\Omega}{K_s(s+a_0)\Omega + K_{s0}s(s+a)};$$
$$F_{vo}(s) = \frac{Y(s)}{V_o(s)} = \frac{sK_{s0}(s+a)}{K_s(s+a_0)\Omega + K_{s0}s(s+a)};$$
$$F_{vi}(s) = \frac{Y(s)}{V_i(s)} = \frac{sK_sK_{s0}}{K_s(s+a_0)\Omega + K_{s0}s(s+a)};$$

$$F_w(0)=1; F_{vo}(0)=0; F_{vi}(0)=0$$

- Does not hold for F_{vi} a $a=0, a_0=0$
- Increased robustnes for prefilter with $T_p=1/a_0$



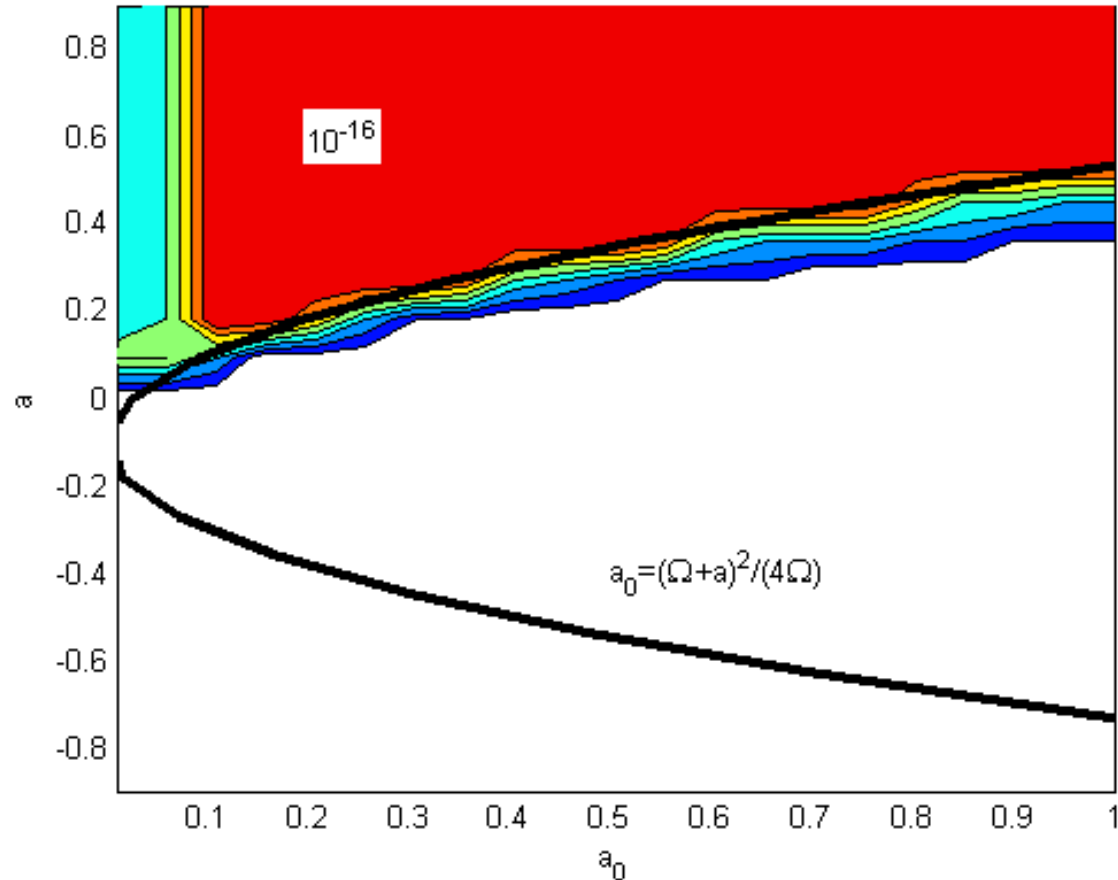
Analytical Tuning ver. PP for $K_{s0}=K_s$

$$A(s) = s^2 + (\Omega_r + a)s + \Omega_r a_0$$

PI₁-IMC, u-1P & y-MC, $\Omega=0.1$, $k=1$

Aperiodicity
Border

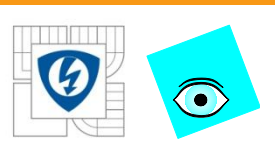
$$\frac{(\Omega_r + a)^2}{4\Omega_r} = a_0$$



11.3.2011

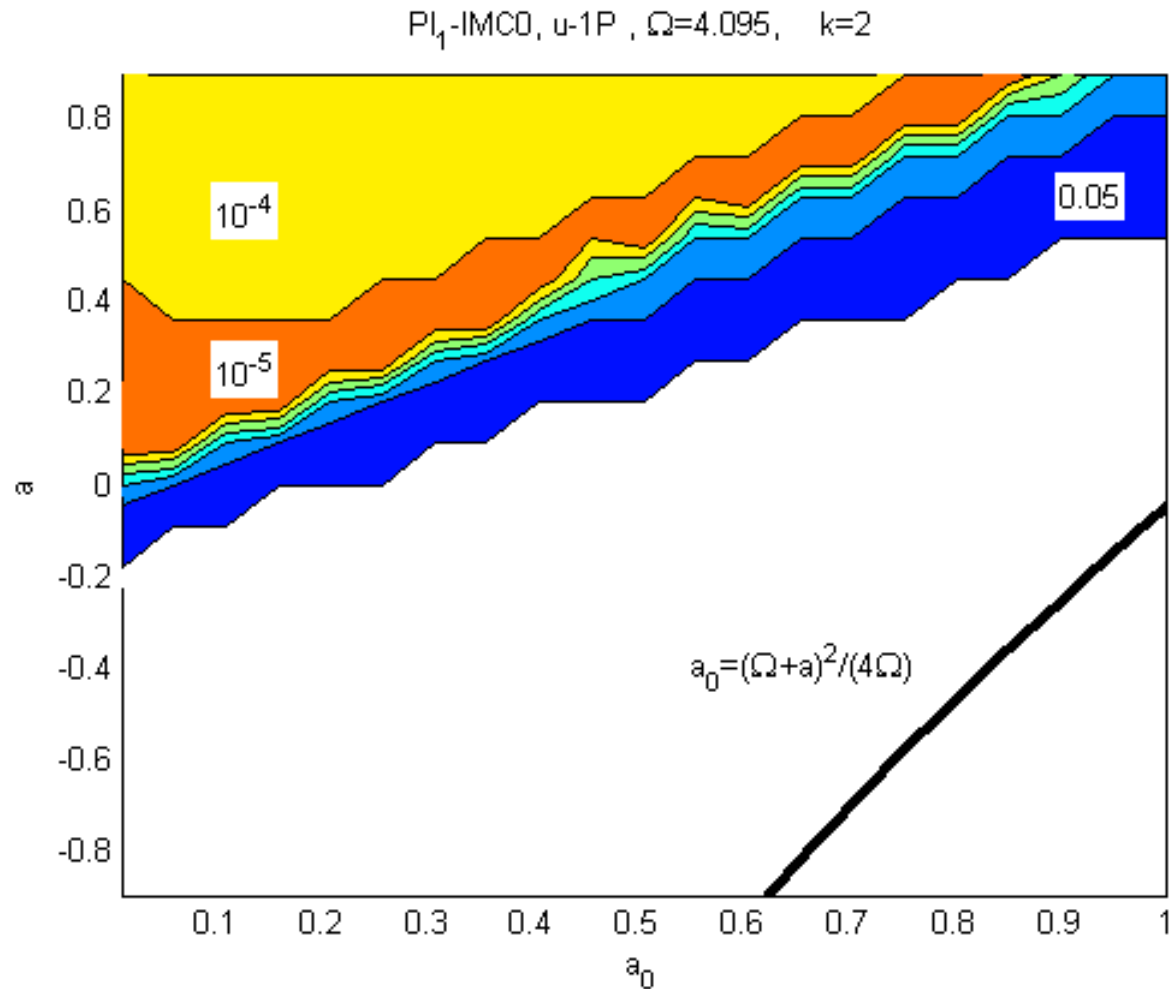
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





Analytical Tuning ver. PP for $K_{s0}=K_s$

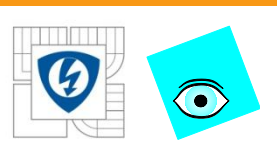
For increasing values the Aperiodicity Border gives no significant information



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

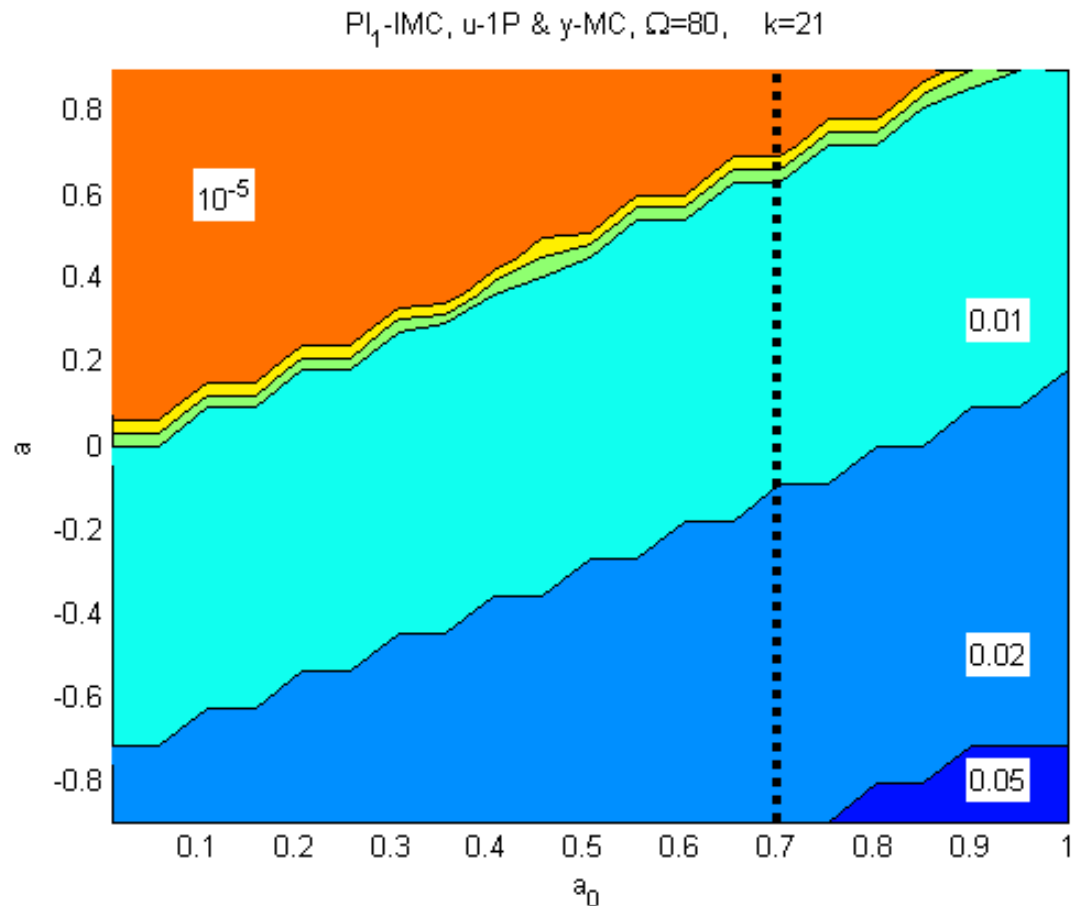




Robust Tuning based on PP for $K_{s0}=K_s$

Localizing
Uncertainty Line
Segment for
uncertain a

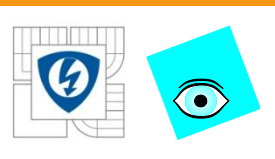
$$a \in \langle -0.9, 0.9 \rangle$$



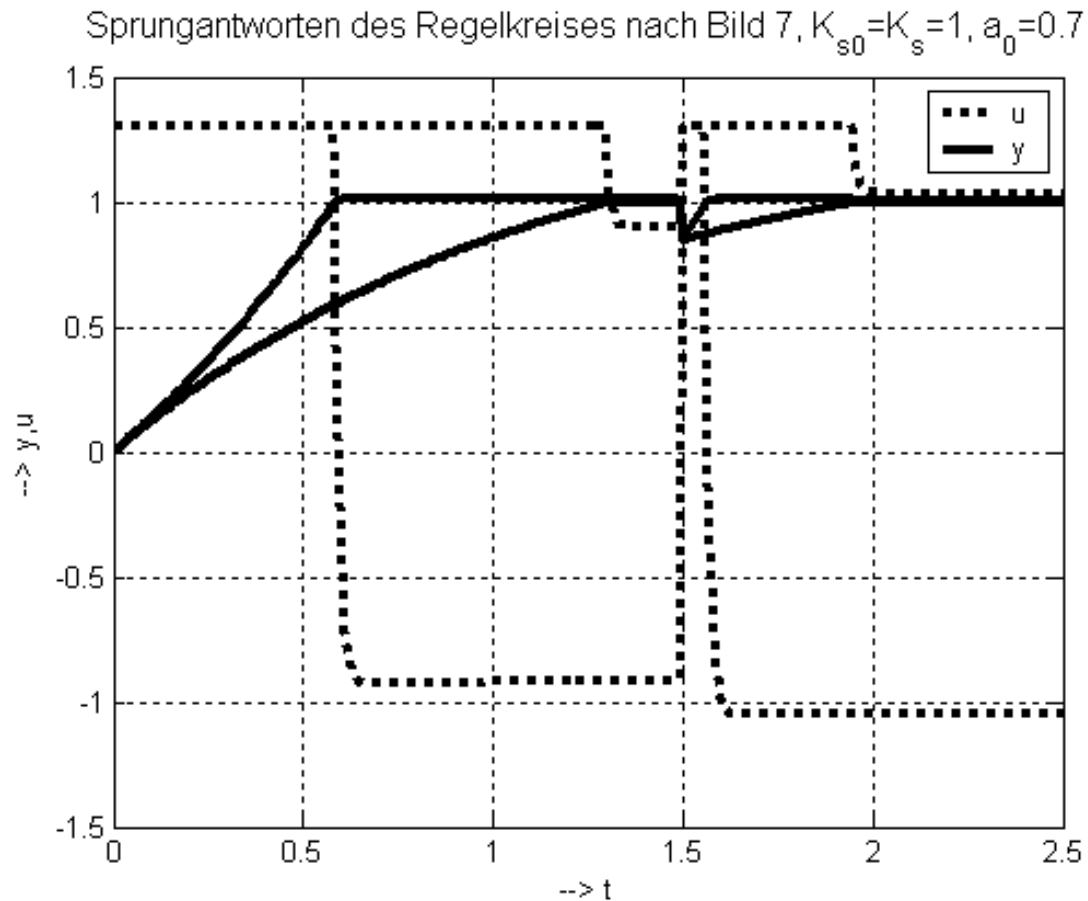
11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





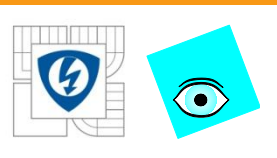
Corresponding transient responses



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





PP for uncertain a and K_s

$$\kappa = \frac{K_{s0}}{K_s} \in \langle 0.05, 2 \rangle$$

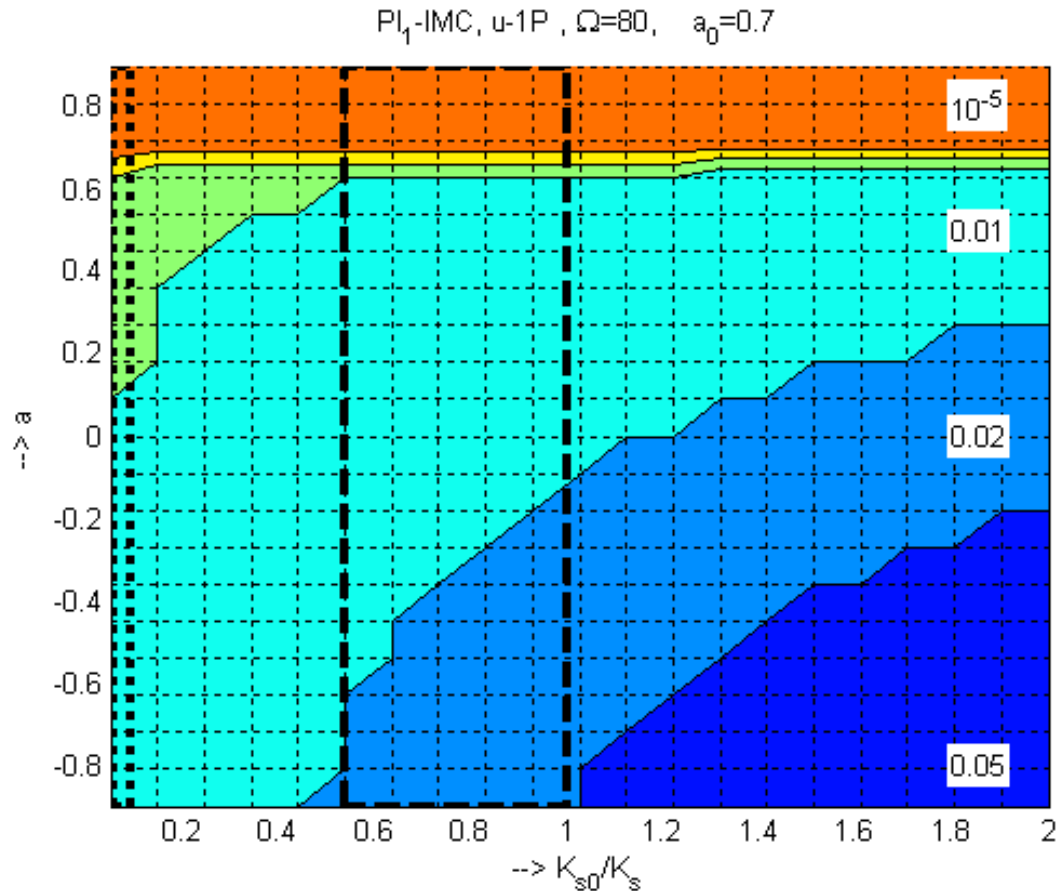
$$a \in \langle -0.9, 0.9 \rangle$$

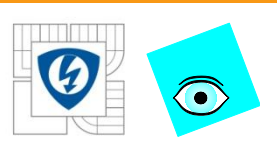
$$a_0 = 0.7; \Omega_r = 80$$

Localizing UB
for variable a
and K_s

$$\varepsilon = 2\%$$

$$\kappa = K_{s0} / K_s < 1$$

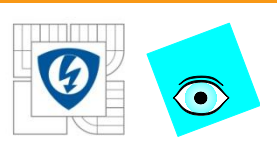




Output Disturbance Compensation

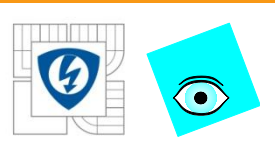
- It is enough to use the measured output for disturbance reconstruction
- The same transfer functions may be derived for the PI_1 -2PM controller – both solutions are equivalent
- Basic Problem: *Integral and Unstable Plants*
- It is not possible to stabilize two parallel unstable plants by a single input (divergence of the output disturbance reconstruction error)
- Divergence of the output disturbance equivalent to a constant input (load) disturbance – computer overflow problem
- Well known within the Internal Model Control (IMC)

11.3.2011



Output Disturbance Compensation

- Basic Problem: *Integral and Unstable Plants*
- **First solution:**
- Choosing $a_0 > 0$ even if $a \leq 0$
- Effect of the imperfect approximation may be reduced by robust tuning procedure
- Basic question: how far does the imperfect approximation influence the achievable performance?
- Response possible by the Performance Portrait method



Output Disturbance Compensation

- Basic Problem: *Integral and Unstable Plants*
- **Second solution:**
- Modifying the output disturbance reconstruction for unstable plants

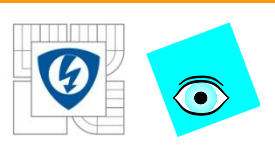
$$y = x + v_o ; v_o = const \Rightarrow \frac{dy}{dt} = \frac{dx}{dt}$$

- From the plant model

$$\hat{x} = \frac{1}{a} \left[K_s u - \frac{dx}{dt} \right] = \frac{1}{a} \left[K_s u - \frac{dy}{dt} \right]$$

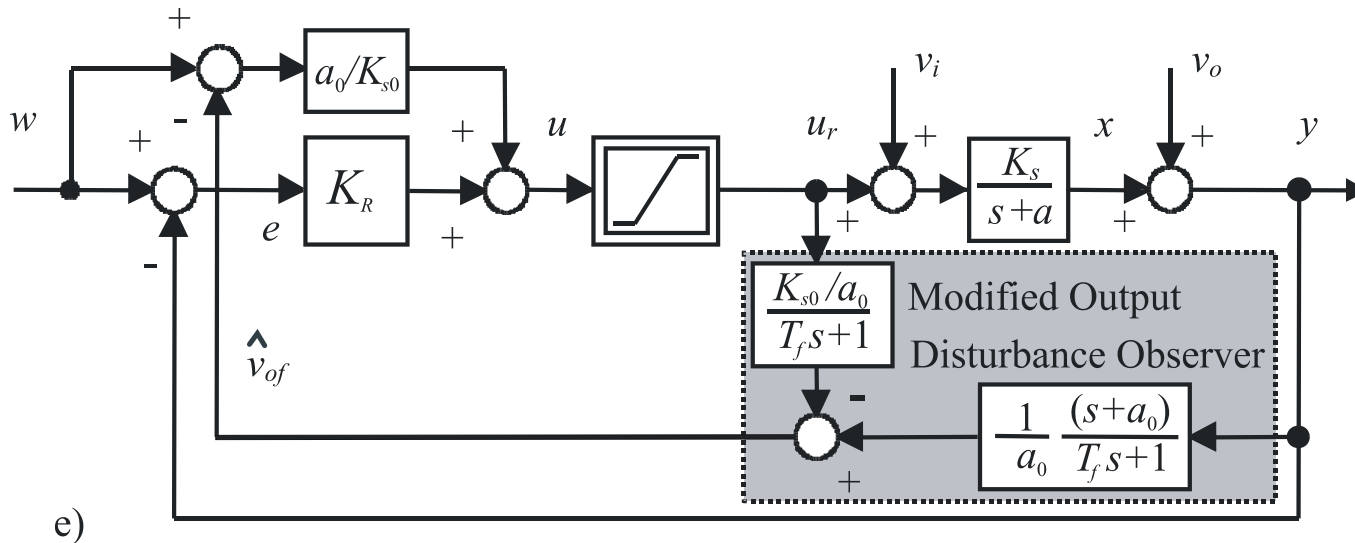
- Modified reconstruction algorithm

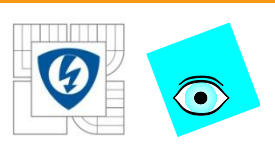
$$\hat{v}_o = y - \hat{x} = y - \frac{1}{a} \left[K_s u - \frac{dy}{dt} \right]$$



Output Disturbance Compensation

- Basic Problem: *Integral and Unstable Plants*
- **Second solution:** Control Scheme
- Modified output disturbance reconstruction for unstable plants – *disturbance observer filter*
- Close to the input disturbance reconstruction



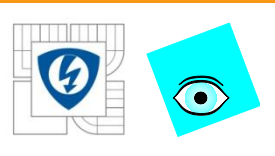


Output Disturbance Compensation

- In general, output disturbance equivalent to a constant load disturbance is not constant
- Usability Check: Loop transfer functions

$$F_w(s) = \frac{1}{1 + T_w s} ; T_w = \frac{1}{K_s K_R + a} ; a_0 = a ; K_{s0} = K_s$$
$$F_{vo}(s) = \frac{s T_f T_w (s + a)}{(1 + T_w s)(1 + T_f s)} ; F_{vi}(s) = \frac{s T_f T_w K_s}{(1 + T_w s)(1 + T_f s)}$$
$$F_{vo}(0) = 0 ; F_{vi}(0) = 0 \text{ (for } a \neq 0 \text{)}$$

- Integral case should again be approximated by $a_0 \neq 0$ – but now it is possible to choose also $a_0 < 0$



Output Disturbance Compensation

- PI₁-2PM Transfer functions (general case)

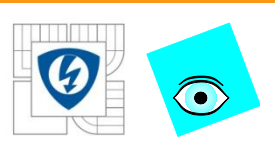
$$F_w(s) = \frac{Y(s)}{W(s)} = \frac{K_s (T_f s + 1) \Omega}{K_{s0} T_f s(s+a) + K_s (1 + T_f (\Omega - a_0)) s + K_s \Omega} ;$$

$$F_{vo}(s) = \frac{Y(s)}{V_o(s)} = \frac{s(s+a) K_{s0} T_f}{K_{s0} T_f s(s+a) + K_s (1 + T_f (\Omega - a_0)) s + K_s \Omega} ;$$

$$F_{vi}(s) = \frac{Y(s)}{V_i(s)} = \frac{s K_s K_{s0} T_f}{K_{s0} T_f s(s+a) + K_s (1 + T_f (\Omega - a_0)) s + K_s \Omega} ;$$

$$F_w(0) = 1 ; F_{vo}(0) = 0 ; F_{vi}(0) = 0$$

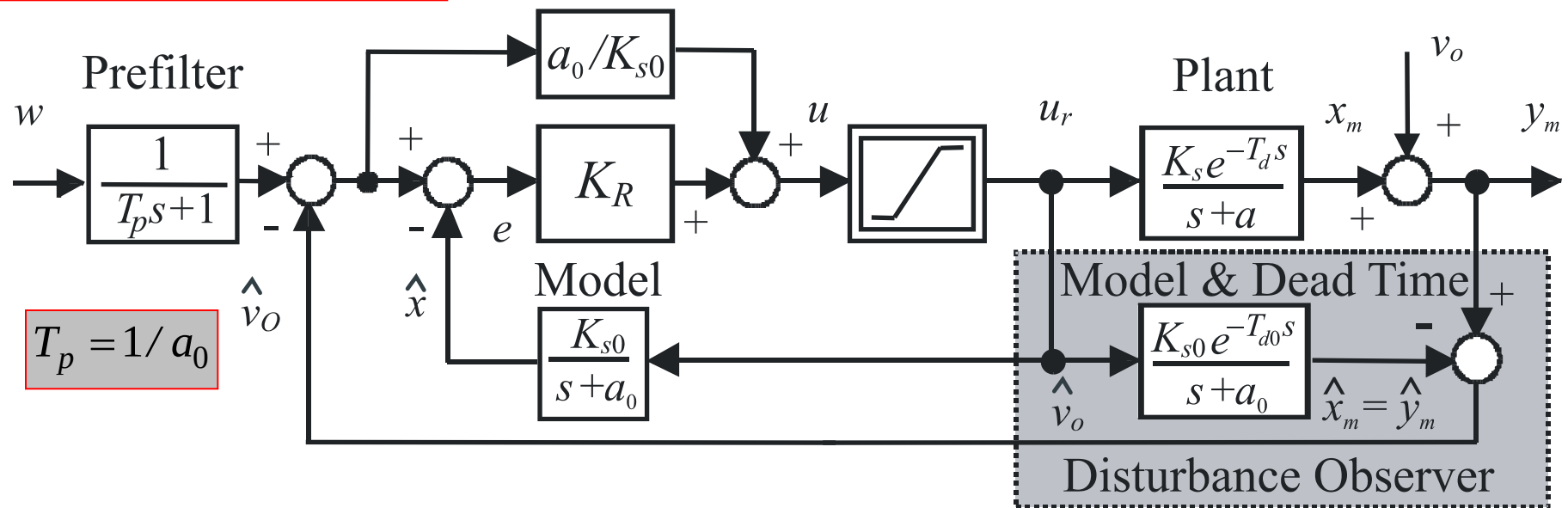
- This holds also for F_{vi} and $a=0$, $a_0=0$, but the reconstruction formulas are not defined
- Increased robustness for prefilter with $T_p = T_f$



Output Disturbance Compensation

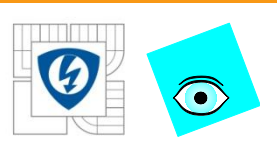
- $T_{d0} \neq 0$
- Output disturbance reconstruction & compensation (parallel model – IMC like structure – Smith predictor)

$$K_R = (\Omega_r - a_0) / K_{s0}$$



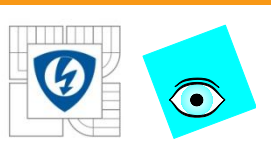
11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ



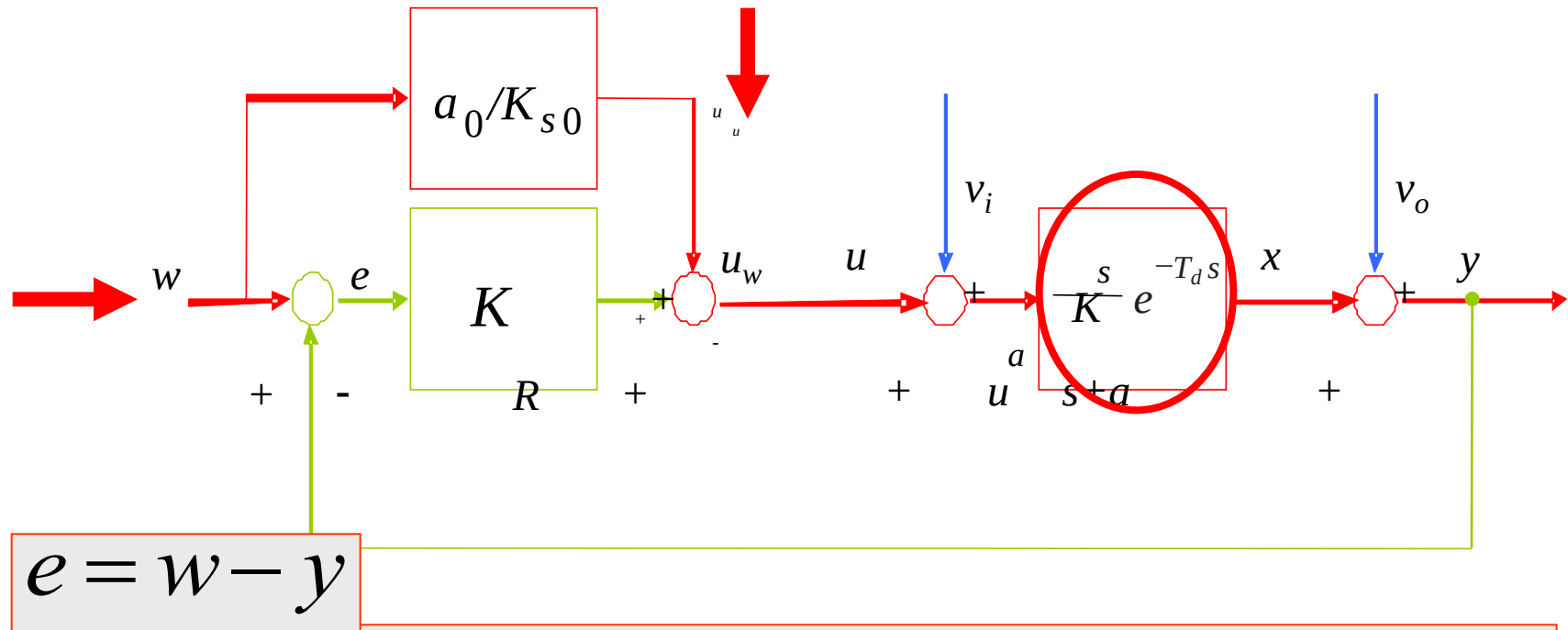
Conclusions 2

- Two classes of model based PI controller from the dynamical class 1 were introduced and compared
- The IMC like structures with parallel plant model seem to be simpler (no disturbance filter is strictly required), but they are limited just to controlling stable plants and the disturbance filter is also here welcomed due to improved robustness
- Solutions with Inverse Model seem to be more universal (no problems with control of unstable plants).



Dynamic class 1

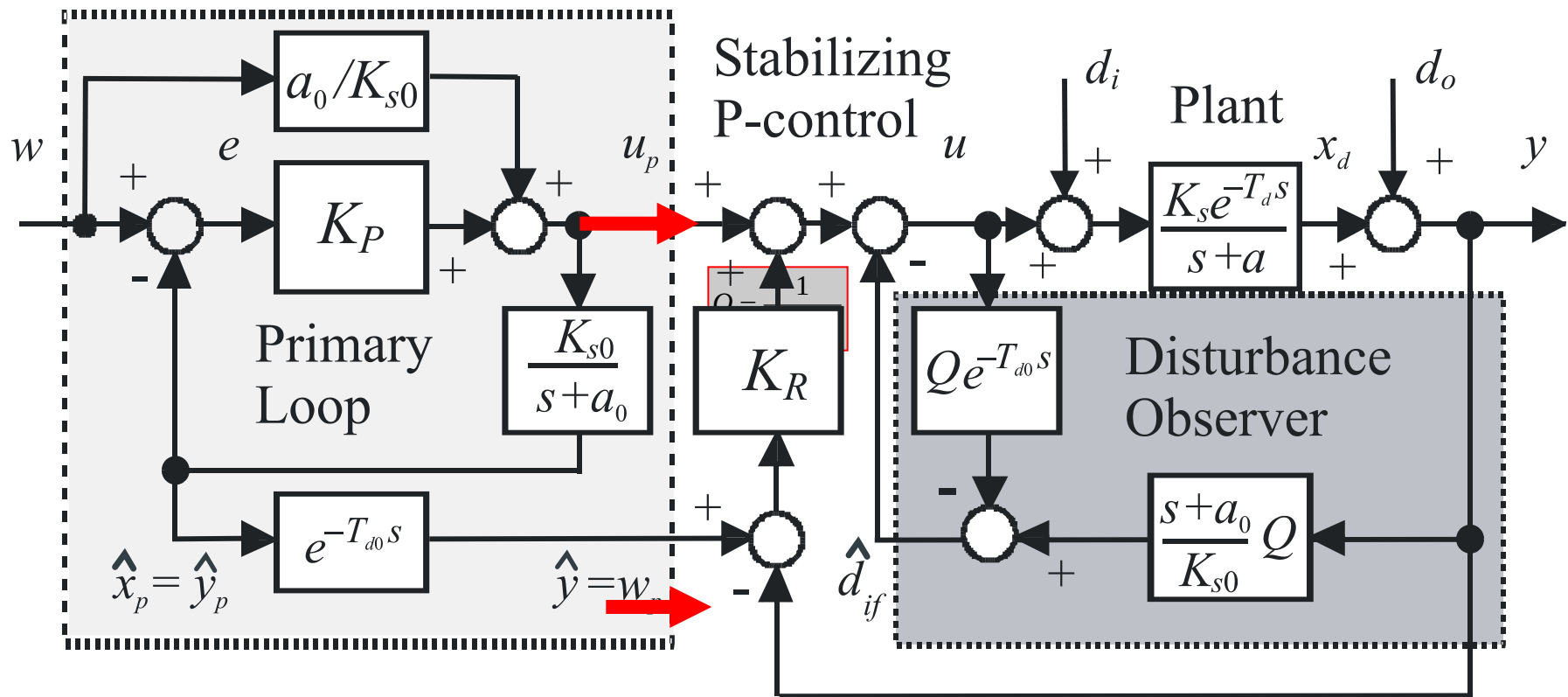
- Linear pole assignment P-controller



$$u = K_R e + u_w ; K_R = \frac{e^{-(1+a_0 T_{d0})}}{K_{s0} T_{d0}} ; u_w = \frac{a_0}{K_{s0}} w$$

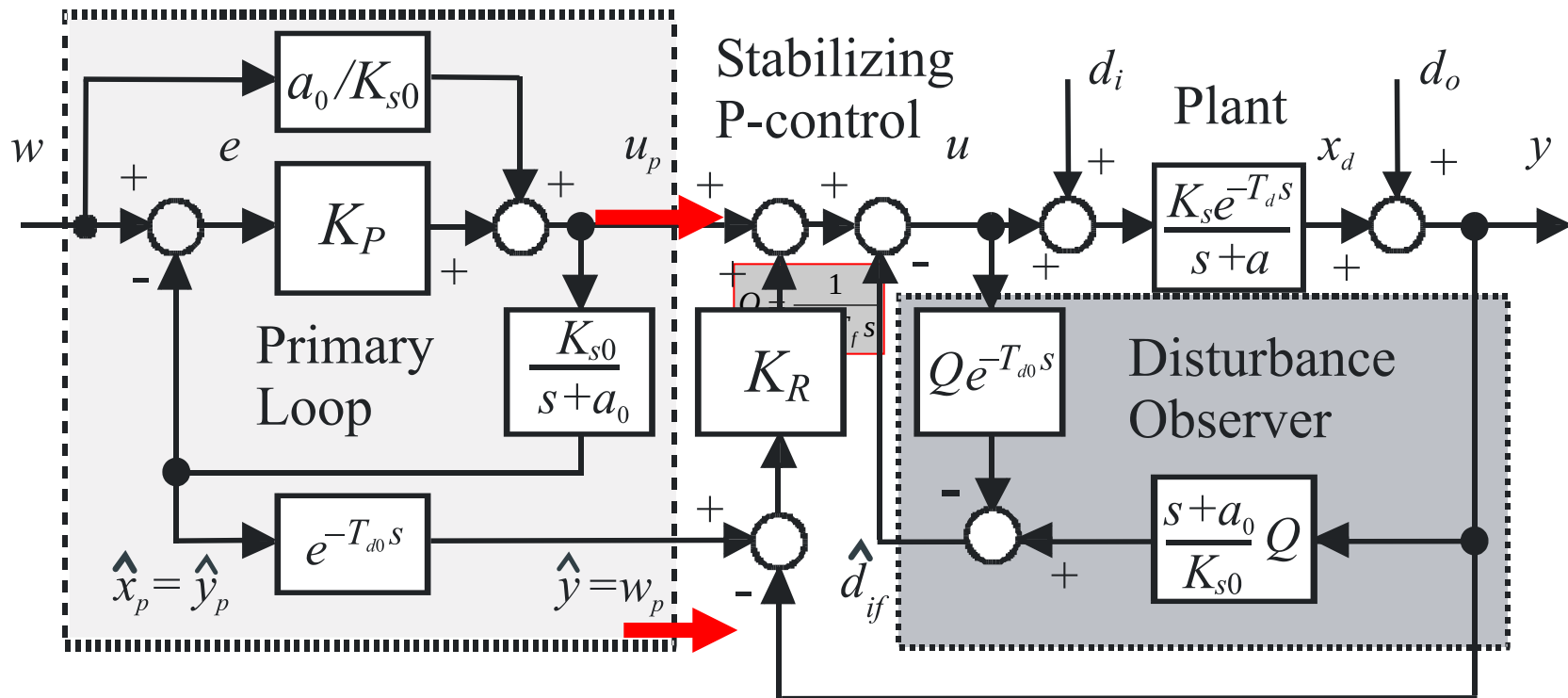
PI₁-DFF-DTIM controller

P controller + dynamical feedforward + observer for input disturbance with IM and compensation of dead time T_d



PI₁-DFF-DTIM controller

P controller + dynamical feedforward + observer for input disturbance with IM and compensation of dead time T_d





The diagram illustrates a control system architecture with the following components and signal flow:

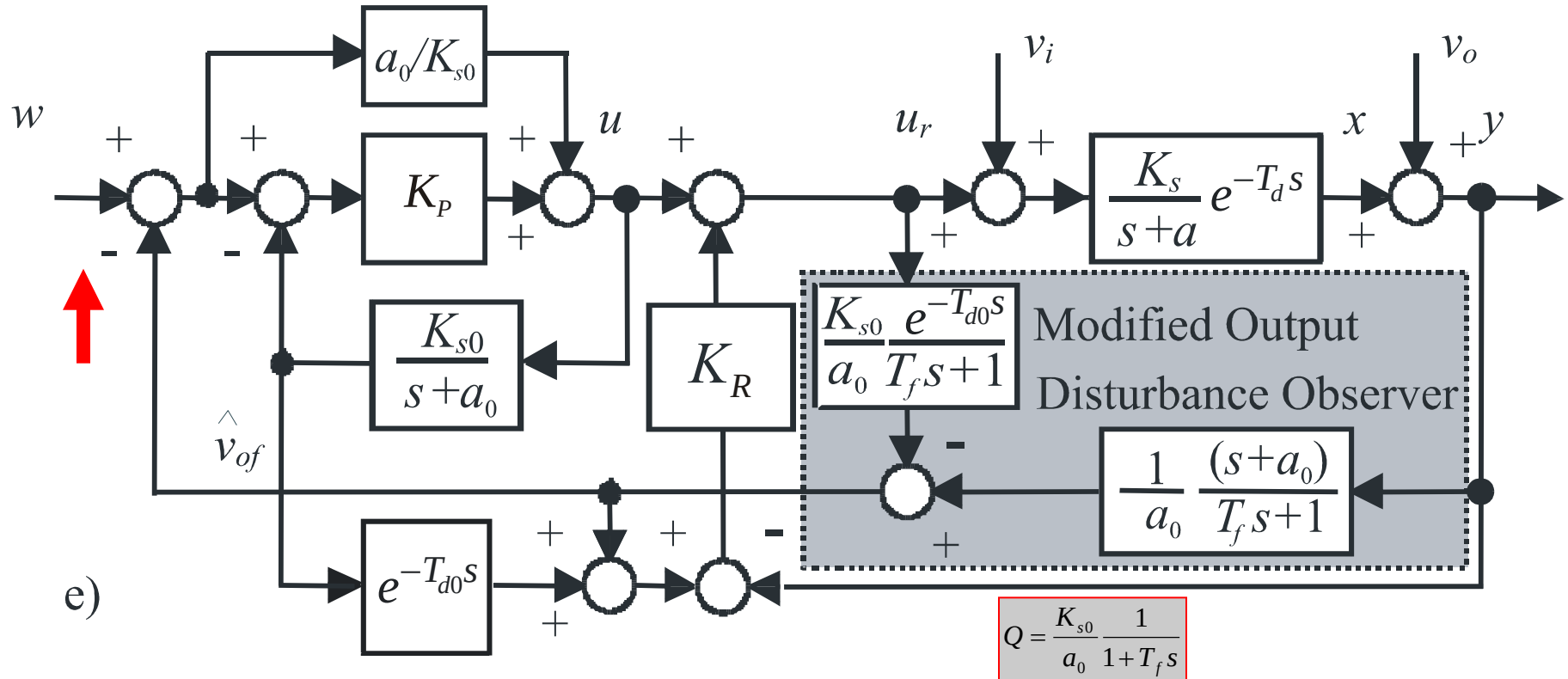
- Primary Loop:** Processes the reference w and feedback $\hat{y}_p = \hat{x}_p$ to generate the error e . The error e is fed into a proportional gain K_P . The output of K_P is summed with a feedforward path (gain a_0/K_{s0}) and a feedback path (transfer function $K_{s0}/(s+a_0)$) to produce the control signal u_p .
- Stabilizing P-control:** Receives u_p and feedback from the output y through a gain K_R (highlighted with a red circle). Its output is summed with disturbance d_i to produce the input u to the plant.
- Plant:** A system with transfer function $\frac{K_s e^{-T_d s}}{s+a}$ that takes input u and produces output y .
- Disturbance Observer:** Estimates the disturbance $\hat{d}_{if}$ based on the output y . It consists of a forward path with transfer function $Q e^{-T_{d0} s}$ and a feedback path with transfer function $\frac{s+a_0}{K_{s0}} Q$.

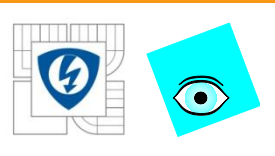
Red arrows indicate key signal paths: the feedforward path from e , the K_R feedback path, and the disturbance estimate $\hat{d}_{if}$ being summed into the control signal.





-MPM = Modified Parallel Model

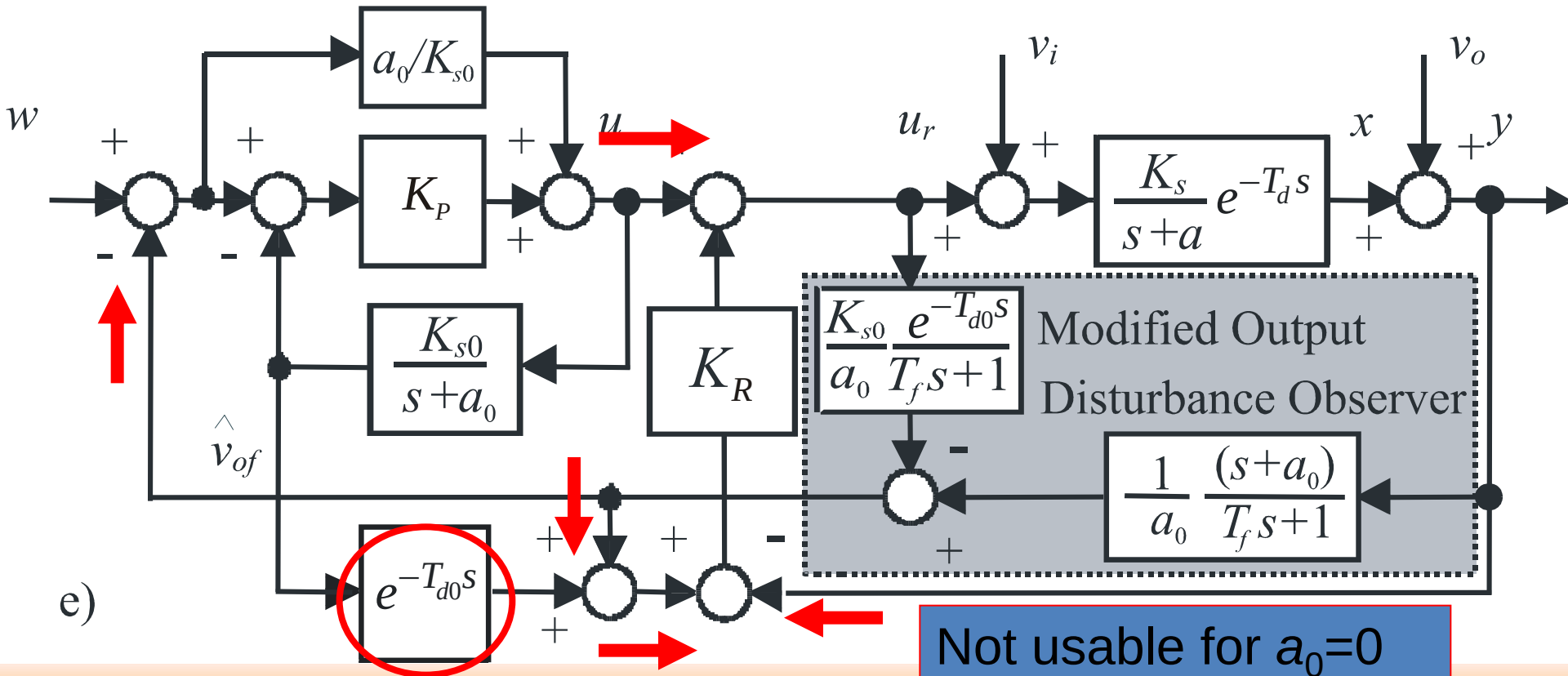


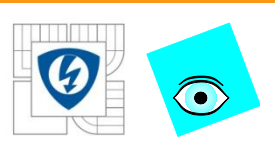


PI1-DFF-DTMPM controller

P controller + dynamical feedforward + observer for output disturbance with MPM and compensation of dead time T_d

- MPM = Modified Parallel Model





Analytical Construction of PP

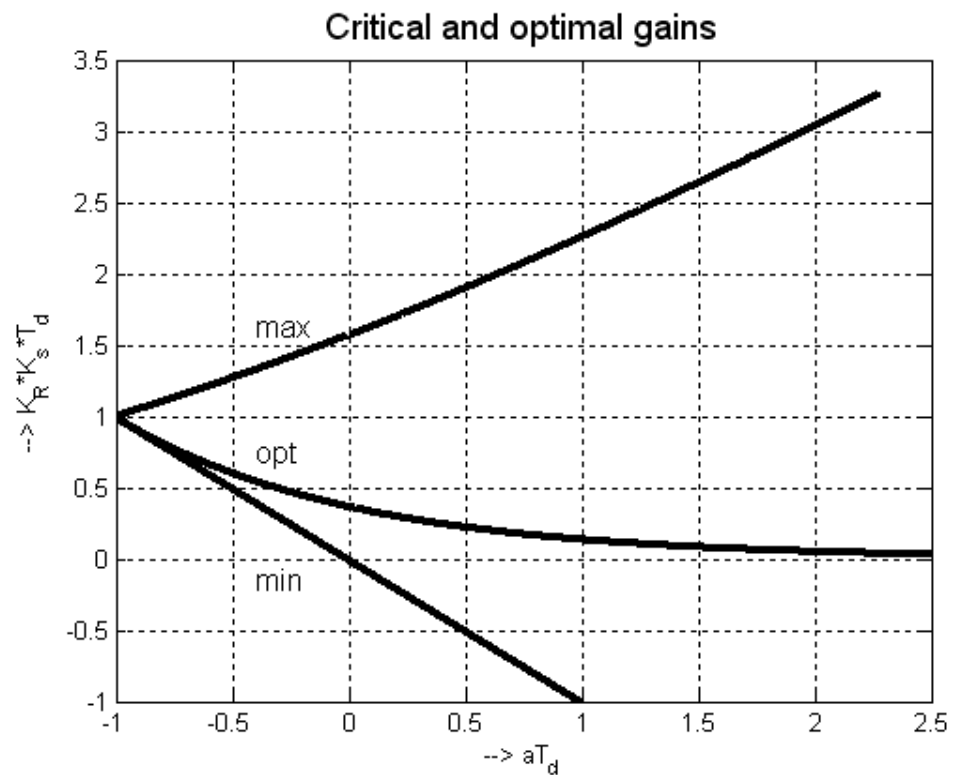
- Critical and optimal gains (aperiodicity border)

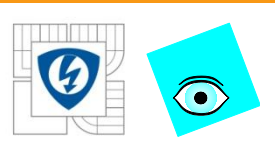
$$K_{min} < K < K_{max} ; K = K_R K_s T_d$$

$$K_{max} = \frac{\tau_d}{\sin \tau_d} ; A = -\frac{\tau_d \cos \tau_d}{\sin \tau_d} ;$$
$$\tau_d \in (0, \pi/2) \cup (\pi/2, \pi)$$

$$K_{min} = -A = -aT_d$$

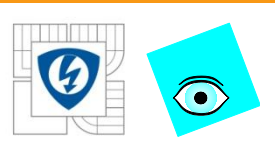
$$K_{opt} = e^{-(1+aT_d)}$$





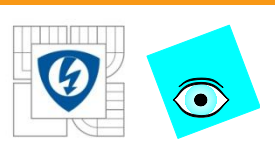
Basic Questions

- Is it not possible to design nominal dead-time compensator with parallel model for reconstruction and compensation of an output disturbance of pure integrator?
- Is it not possible to control unstable plants with $aT_d < -1$
- Is it not possible to improve the disturbance response?



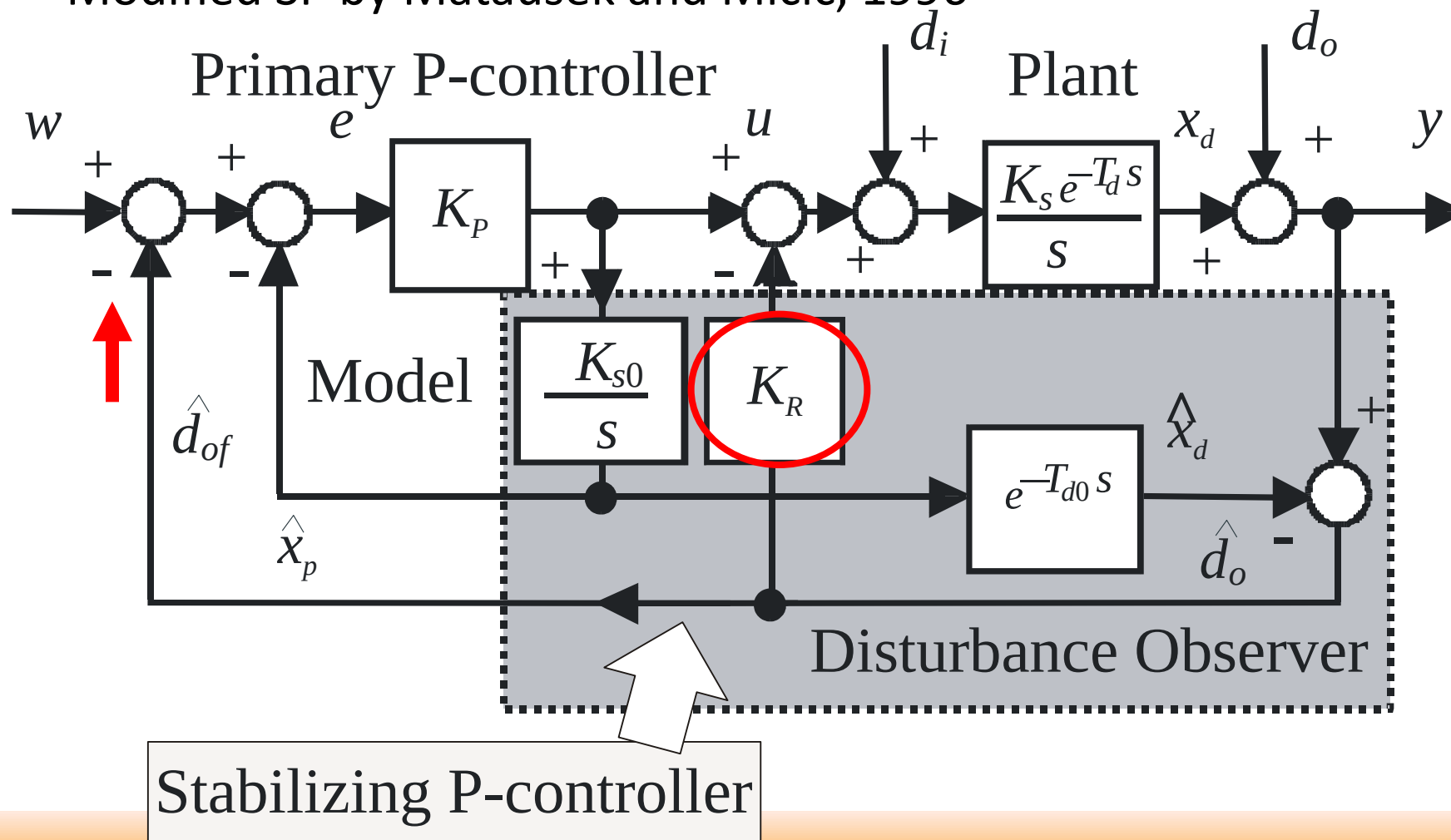
Modifications of SP for IPDT Systems

- C. C. Hang and F. S. Wong, "Modified Smith Predictors for the control of processes with dead time," in Proc. ISA Annual Conf., 1979, 33-44.
- K. Watanabe and M. Ito, "A process-model control for linear systems with delay," IEEE Trans. Automat. Contr., vol. AC-26, no. 6, Dec. 1981, 1261-1266.
- Åström, K. J., Hang, C. C., & Lim, B. C. (1994). A new Smith predictor for controlling a process with an integrator and long dead-time. IEEE Transactions on Automatic Control. 39(2). 343-345.
- Matausek, M.R. and Micic, A.D.: 'A modified Smith predictor for controlling a process with an integrator and long dead-time'. IEEE Trans. Autom. Control, 1996, 41, pp. 1199-1203
- Matausek, M.R. and Micic, A.D: On the modified Smith predictor for controlling a process with an integrator and long dead-time, IEEE Trans. Autom. Control, 1999, 44, 1603-1606.
- Zhong, Q.C. and J.E. Normey-Rico: Control of integral processes with dead-time. Part 1: Disturbance observer-based ZDOF control scheme. IEE Proc.-Control Theory Appl., Vol. 149, No. 4, July 2002, 285-290.
- Normey-Rico J.E., Camacho E.F.: Unified approach for robust dead-time compensator design. (2009) Journal of Process Control, 19 (1), pp. 38-47.



Stabilized SP (SSP)

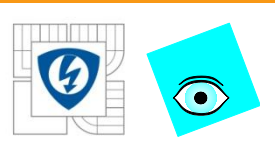
Modified SP by Mataušek and Micič, 1996



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





Stabilized SP (SSP)

Modified SP by Mataušek and Micič, 1996

- Transfer functions

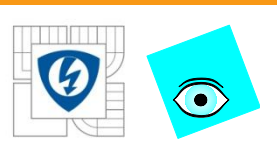
$$a_0 = a ; K_{s0} = K_s , T_{d0} = T_d$$

$$F_w(s) = \frac{K_s K_p e^{-T_d s}}{(s + K_s K_p)} = \frac{e^{-T_d s}}{T_w s + 1} ; T_w = \frac{1}{K_s K_p}$$

$$F_{vo}(s) = \frac{s(s + K_s K_p - K_s K_p e^{-T_d s})}{(s + K_s K_R e^{-T_d s})(s + K_p K_s)} ; F_{vi}(s) = \frac{K_s}{s} F_{vo}$$

$$F_w(0) = 1 ; F_{vo}(0) = 0 ; F_{vi}(0) = 0 \text{ for } K_p K_s > 0$$

$$K_{Ropt} = 1/(K_s T_d e) \approx 1/(2K_s T_d)$$



Comparing PI_1 -DTIM and SSP

- IM has dist. response independent from T_w

$K_s=1$

$a=0$

$T_d=1$

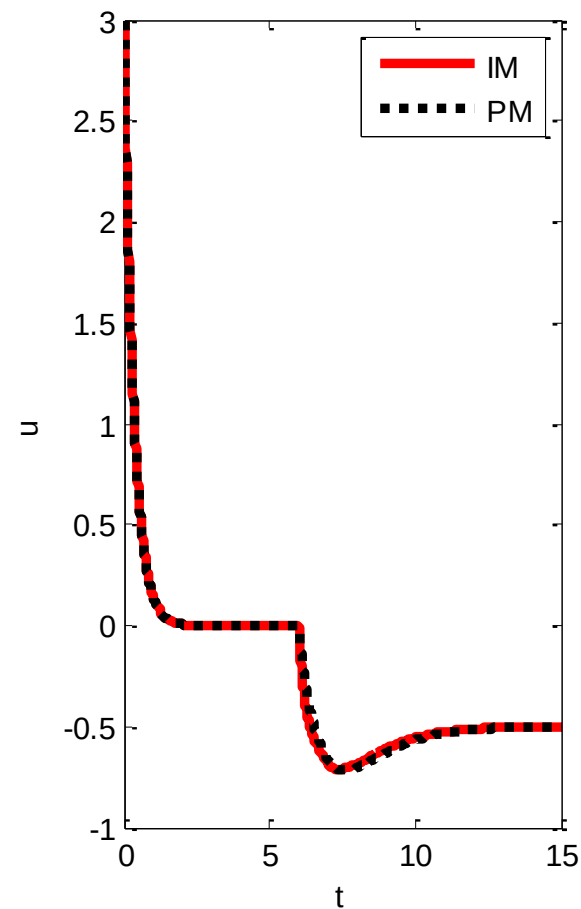
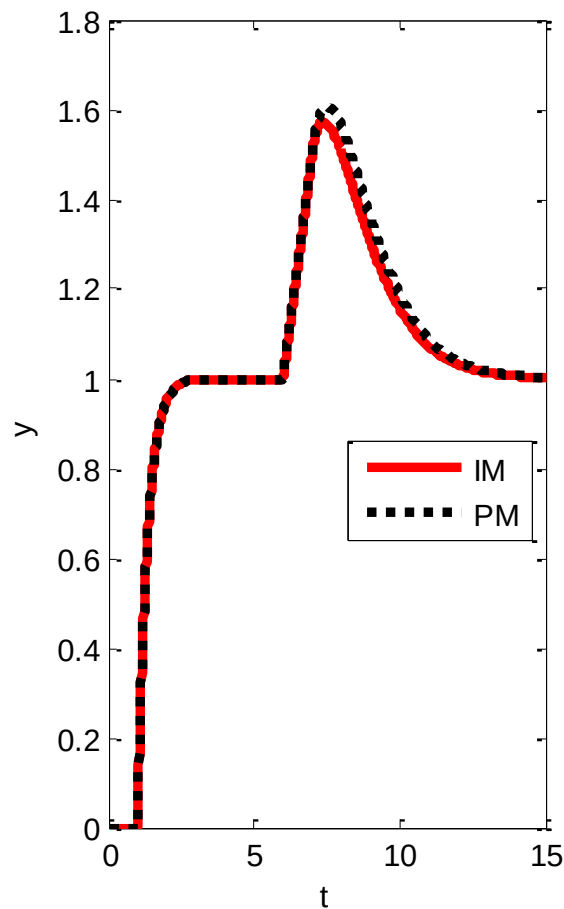
$K_p=3$

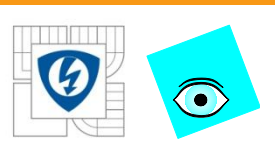
$T_w=0.33$

$T_f=0.2$

$v_i=0.5$

$K_R=K_{Ropt}$





Comparing PI_1 -DTIM and SSP

- Stabilizing P-controller of SSP compensates input disturbances (black)
- Stabilizing P-controller of PI_1 -DTIM is active just during transients

$K_s=1$

$a=-0$

$T_d=1$

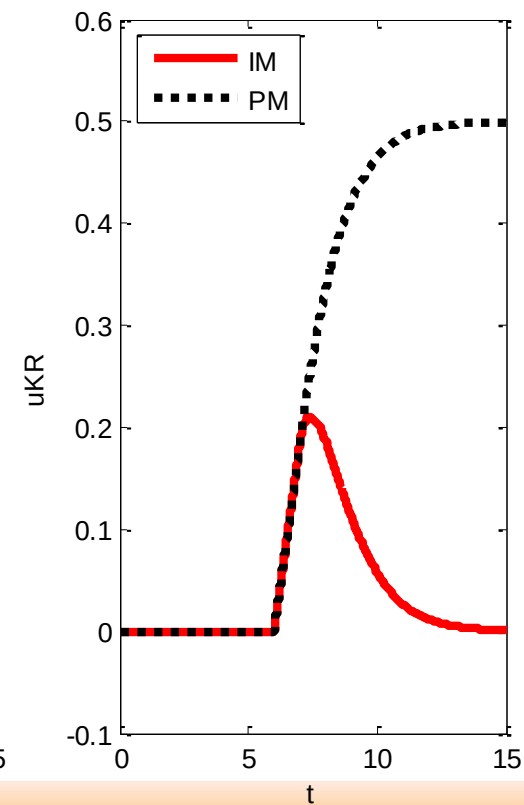
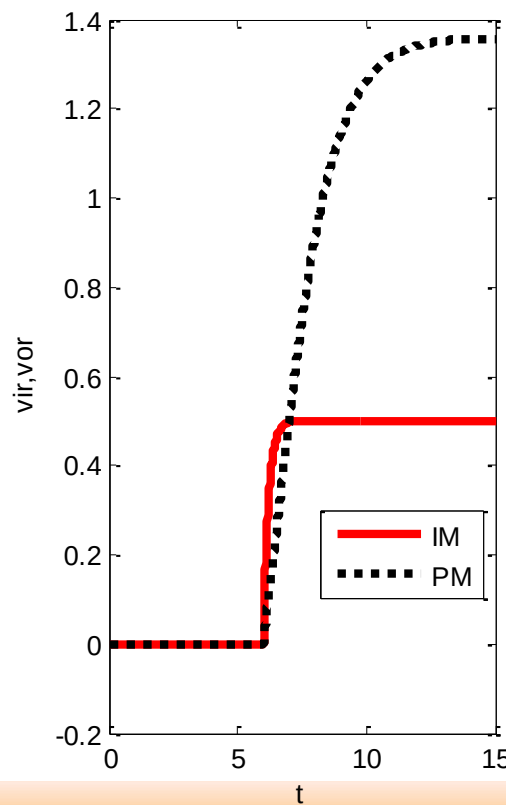
$K_p=3$

$T_w=0.33$

$T_f=0.2$

$v_i=0.5$

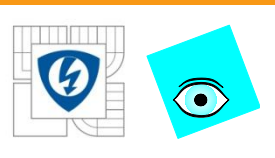
$K_R=K_{Ropt}$



11.3.2011

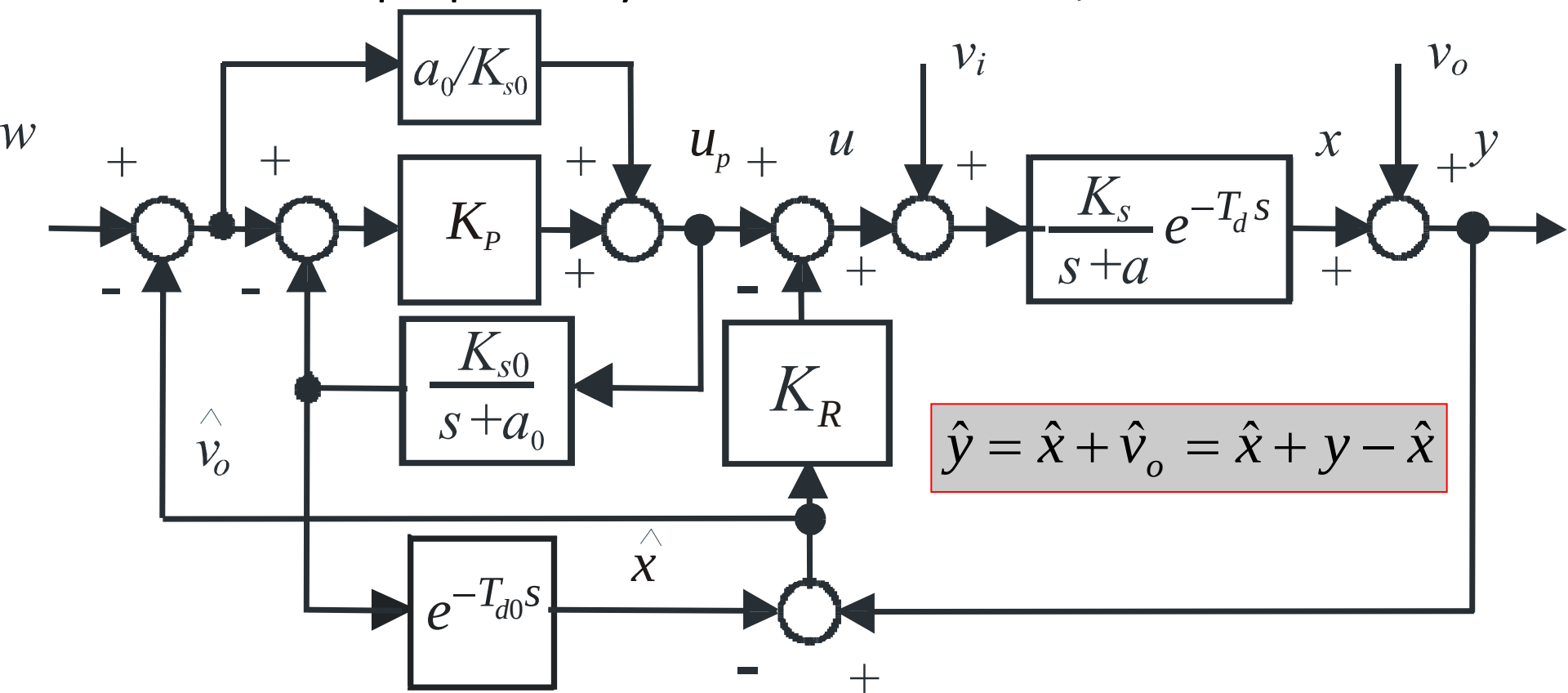
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





Generalized Stabilized SP (GSSP)

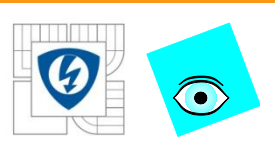
Generalization of
solution proposed by Mataušek and Micič, 1996



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





Generalized Stabilized SP (GSSP)

Generalization of solution by Mataušek and Micič, 1996

- Transfer functions

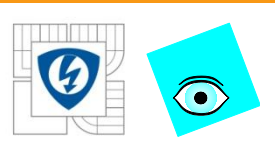
$$a_0 = a ; K_{s0} = K_s , T_{d0} = T_d$$

$$F_w(s) = \frac{(K_s K_P + a)e^{-T_d s}}{(s + K_s K_P)} = \frac{e^{-T_d s}}{T_w s + 1} ; T_w = \frac{1}{K_s K_P + a}$$

$$F_{vo}(s) = \frac{(s + a)(s + a + K_s K_P - (K_s K_P + a)e^{-T_d s})}{(s + K_s K_R e^{-T_d s} + a)(s + K_P K_s + a)} ; F_{vi}(s) = \frac{K_s}{s + a} F_{vo}$$

$$F_w(0) = 1 ; F_{vo}(0) = 0 ; F_{vi}(0) = 0 \text{ for } K_P K_s + a > 0$$

$$K_{Ropt} = \exp(-(1 + aT_d)) / (K_s T_d)$$



PI_1 -DTIM, MPM and GSSP

- IM has dist. response independent from T_w

$K_s=1$

$a=-0.5$

$T_d=1$

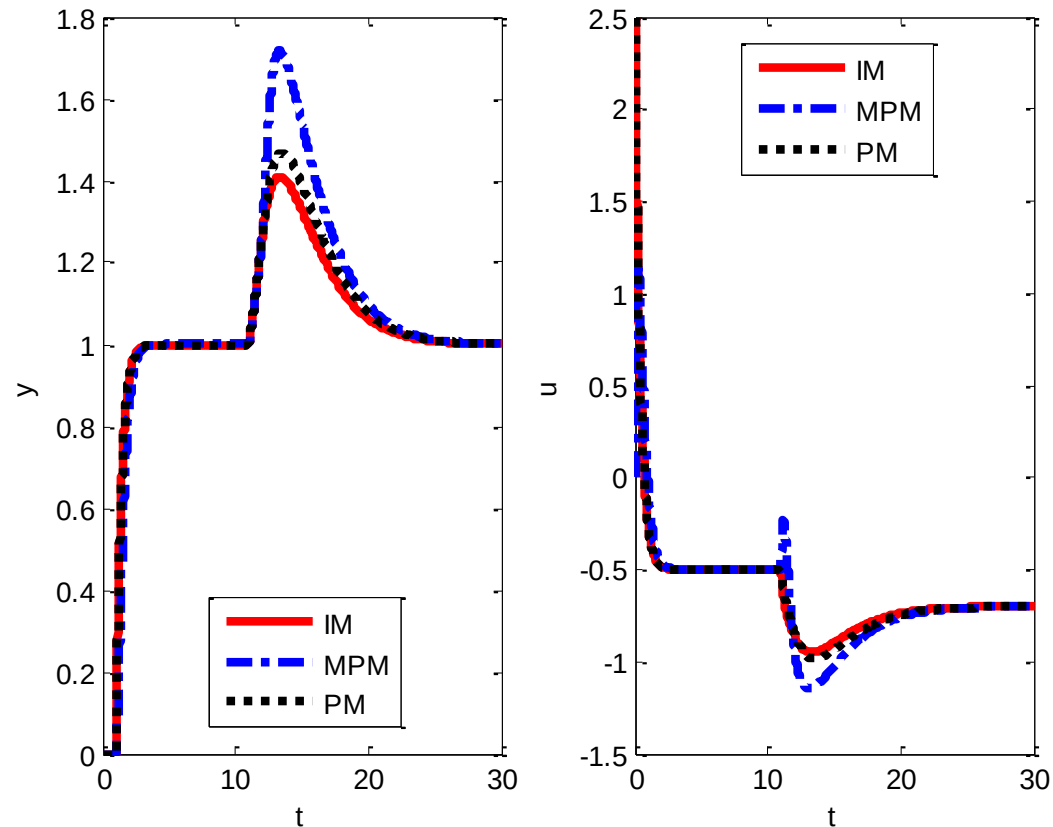
$K_p=3$

$T_w=0.4$

$T_f=0.2$

$v_i=0.2$

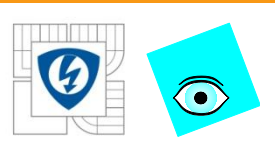
$K_R=K_{Ropt}$



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





PI₁-DTIM, MPM and GSSP

- Stabilizing P-controller compensates inp.dist.

$K_s=1$

$a=-0.5$

$T_d=1$

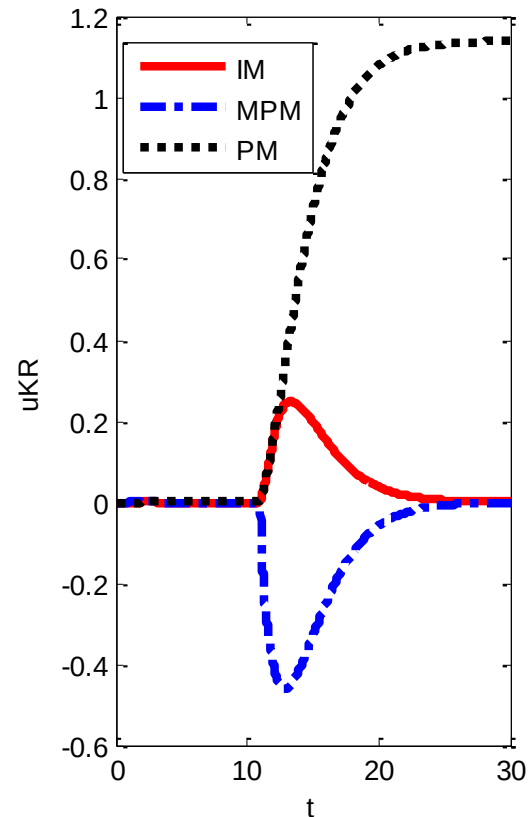
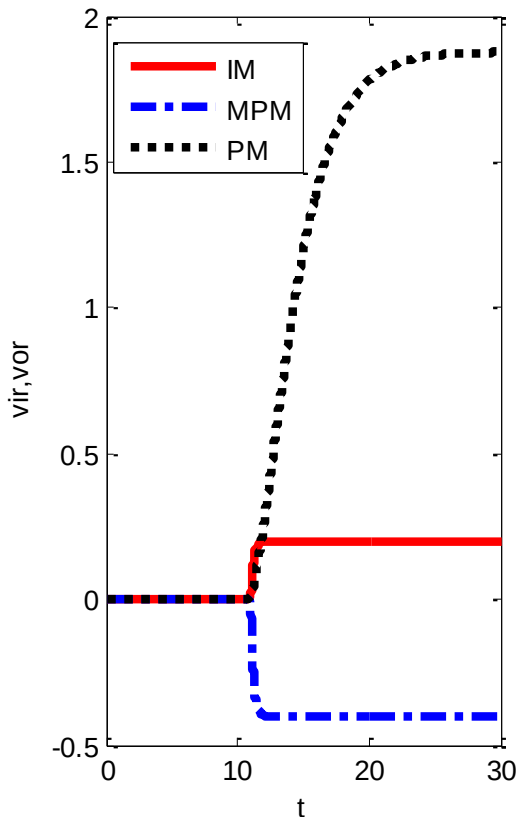
$K_p=3$

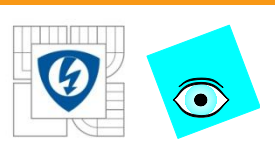
$T_w=0.4$

$T_f=0.2$

$v_i=0.2$

$K_R=K_{Ropt}$





PI₁-DTIM, MPM and GSSP

- IM has slightly better disturbance response

$K_s=1$

$a=0.5$

$T_d=1$

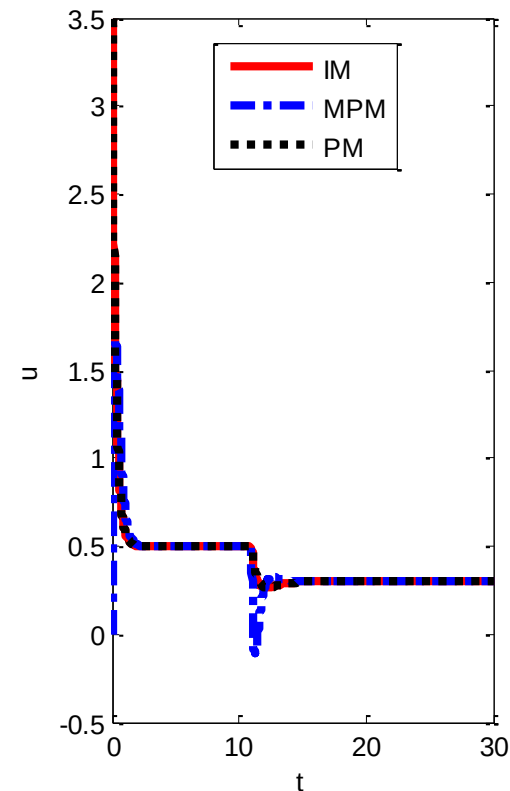
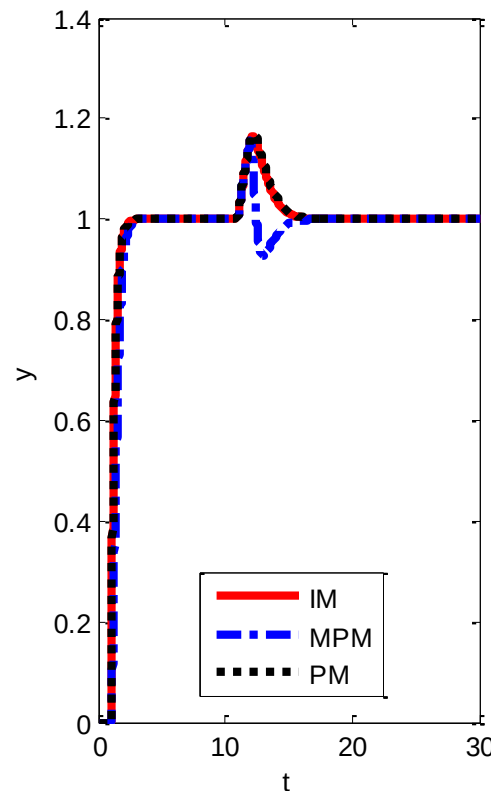
$K_p=3$

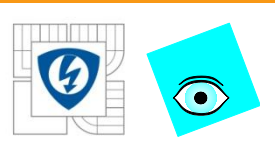
$T_w=0.286$

$T_f=0.2$

$v_i=0.2$

$K_R=K_{Ropt}$





PI_1 -DTIM, MPM and GSSP

- Stabilizing P-controller compensates inp.dist.

$K_s=1$

$a=0.5$

$T_d=1$

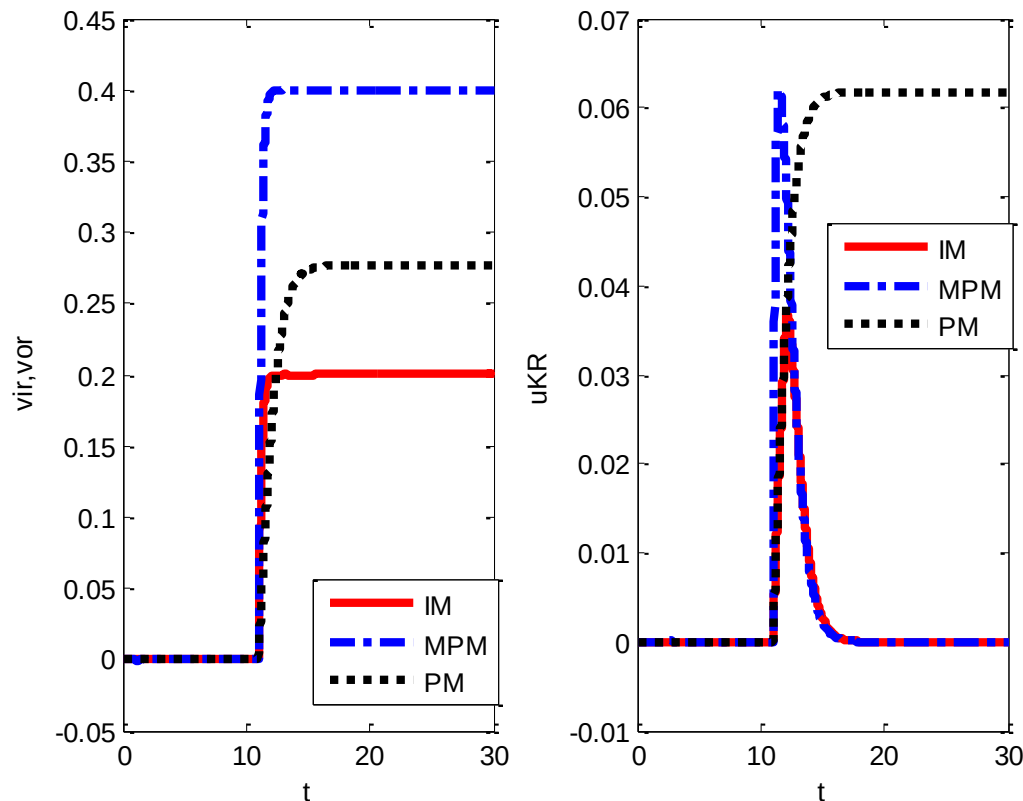
$K_p=3$

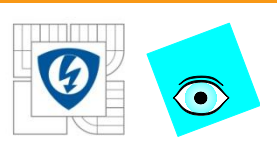
$T_w=0.286$

$T_f=0.2$

$v_i=0.2$

$K_R=K_{Ropt}$





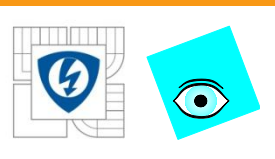
Conclusions 3

- Several alternatives for constructing P-controllers with dynamical feedforward for integral and unstable FOPDT plants
- All of them keep the disturbance observer+stabilizing controller structure – important for constrained control – no windup
- All of them guarantee no influence of disturbances on the output in steady states, but internally they are different
- All of them keep poles of simple P-controller with FOPDT plant – easy tuning of stabilizing controller
- However, transients after disturbance steps are different – the same will hold for robustness

11.3.2011

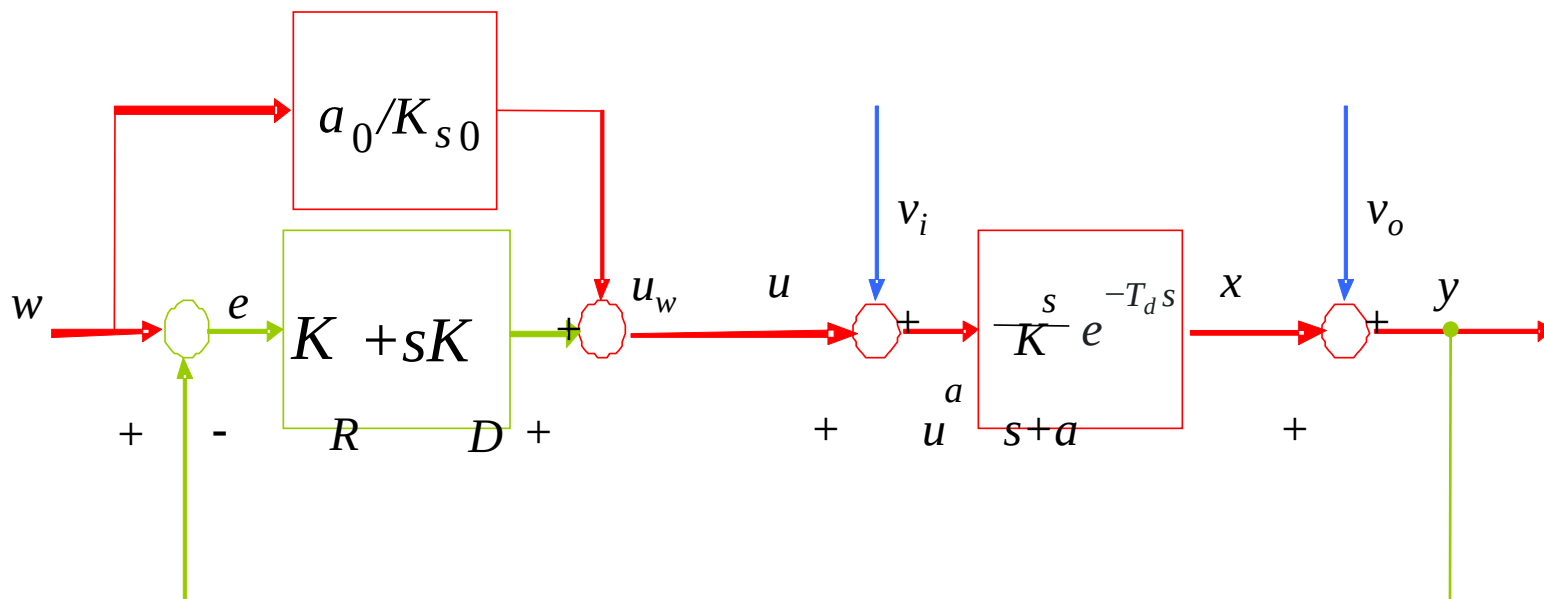
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





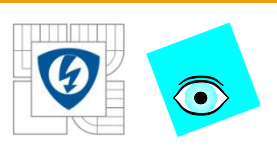
Dynamical class „1“

Linear pole assignment PD-controller



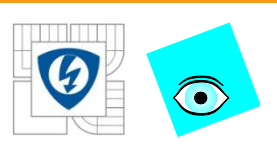
$$u = K_R e + K_D \frac{de}{dt} + u_w ; K_R = \frac{(4 + a_0 T_d) e^{-(2 + a_0 T_d)}}{K_{s0} T_{d0}} ;$$

$$u_w = \frac{a_0}{K_{s0}} w ; K_D = \frac{e^{-(2 + a_0 T_d)}}{K_{s0}} ; T_D = \frac{K_D}{K_R} = \frac{T_{d0}}{4 + a_0 T_d}$$



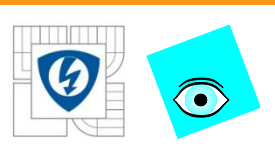
Conclusions 4

- Introduction of D-action can ideally double the admissible negative product aT_d from -1 up to -2
- This, however, holds just for ideal PD controller without filter in denominator, so the real extension will be less and depends on the signal quality (measurement noise)
- Due to this it is also not realistic to think about more advanced controllers with higher order derivatives of control error

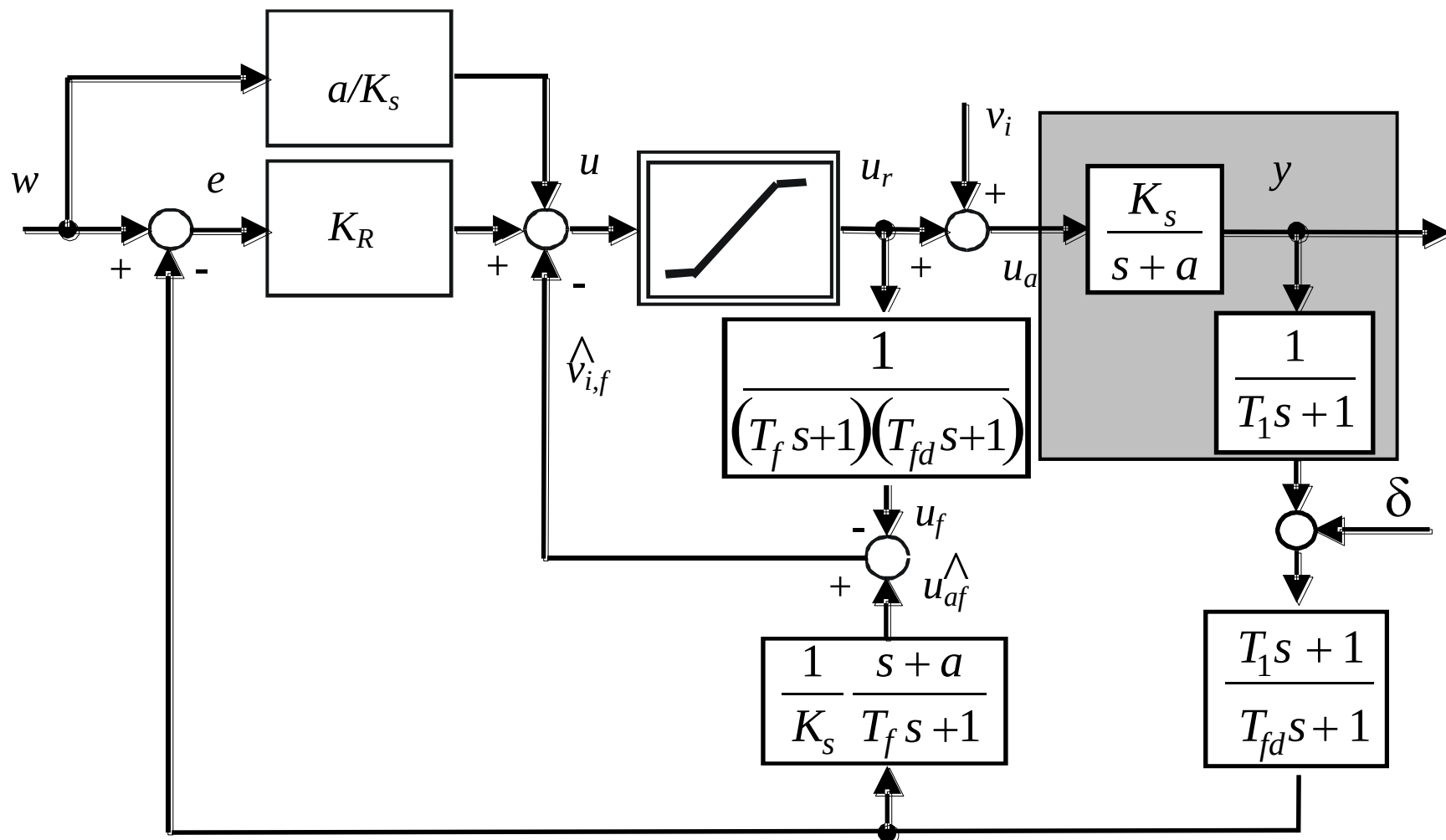


PID₁-controller

- Loop approximation with the second order dominant plant
- Stable time constant is located in the feedback (e.g. the sensor dynamics)
- Gives reliable results also in the case with this time constant in the feedforward path (e.g. the actuator time constant) – however, just for the DC1
- Optimal solutions for such configuration are already from DC2



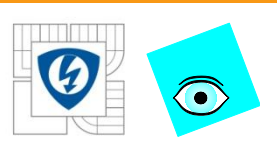
Windupless PID_1 Controller



11.3.2011

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ





Advantages of PI_1 and PID_1 controllers

- Do not need Anti-Windup
- Tuning of lower number of parameters,
- Convex regions of MO and 1P control
- Optimal nominal point may be included in US – less steep quality decrease
- Simple performance portrait generation
- Modular structure
- Simple compensation of dead time T_d
- Possibility to use more complex filters (increased filter properties, increased robustness)