EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Robustné PID-regulátory s obmedzeniami
Robust Constrained PID Control
Robust Design of Parallel PI Controller

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Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.

Objectives

To explain:

- basic possibilities for disturbance compensation in controlling 1st order systems
- To explain some basic rules for robust tuning of the traditional parallel PI controller
- to compare these methods with tuning based on the Performance Portrait method
- To explain basic possibilities for eliminating windup effect
- To explain basic possibilities for avoiding windup effect by different Disturbance Observers

Fundamental Controllers of DC1

- DC1 represents the 2nd row of the Table of fundamental PID controllers
- It includes controllers that are already non-linear, typically depending on effect of constraints, frequently equipped with anti-windup circuitry
- Control signal corresponding to monotonic setpoint step response shows typically one pulse of control

FF = static feedforward control

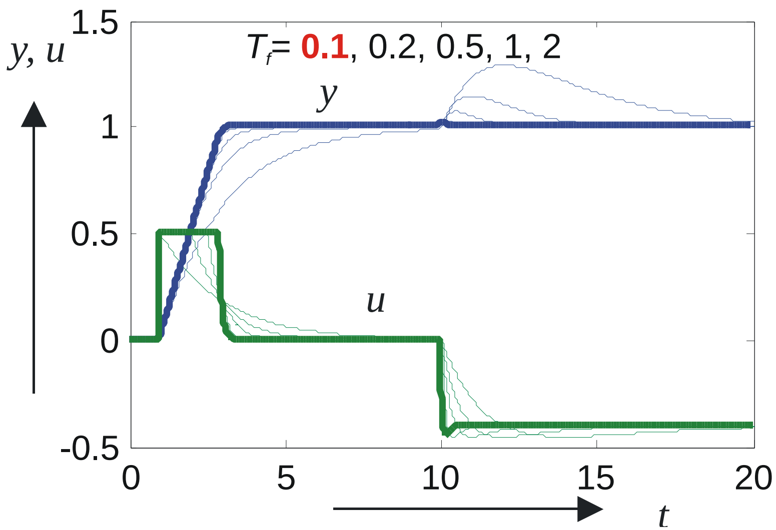
Pr = predictive (dead time) controllers

		Dominant dynamics							
Dynamic class	I-action	K	Ke^{-T_d}	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}$	$\left[\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}\right]e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	$\boxed{\text{PI}}$	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	PD	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

Fundamental Solutions DC1

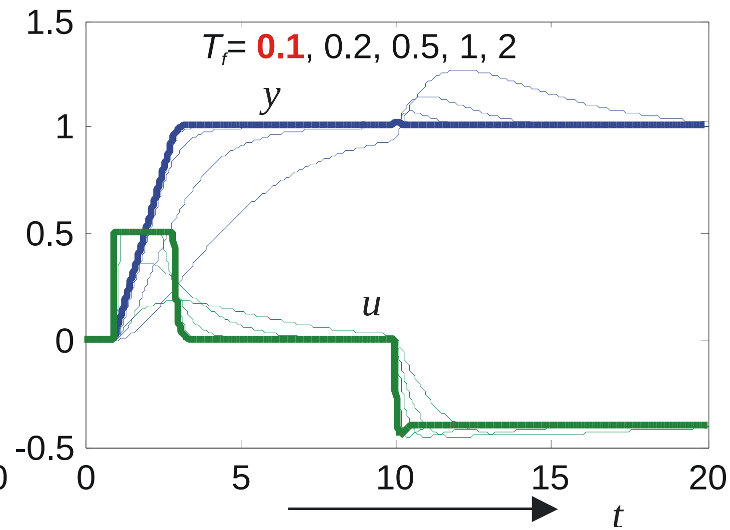
- DC1 includes controllers that are already non-linear, typically depending on effect of constraints, frequently equipped with anti-windup circuitry
- Control signal corresponding to monotonic setpoint step response shows typically one pulse of control

Continuous for $t > 0$



Basic Controllers

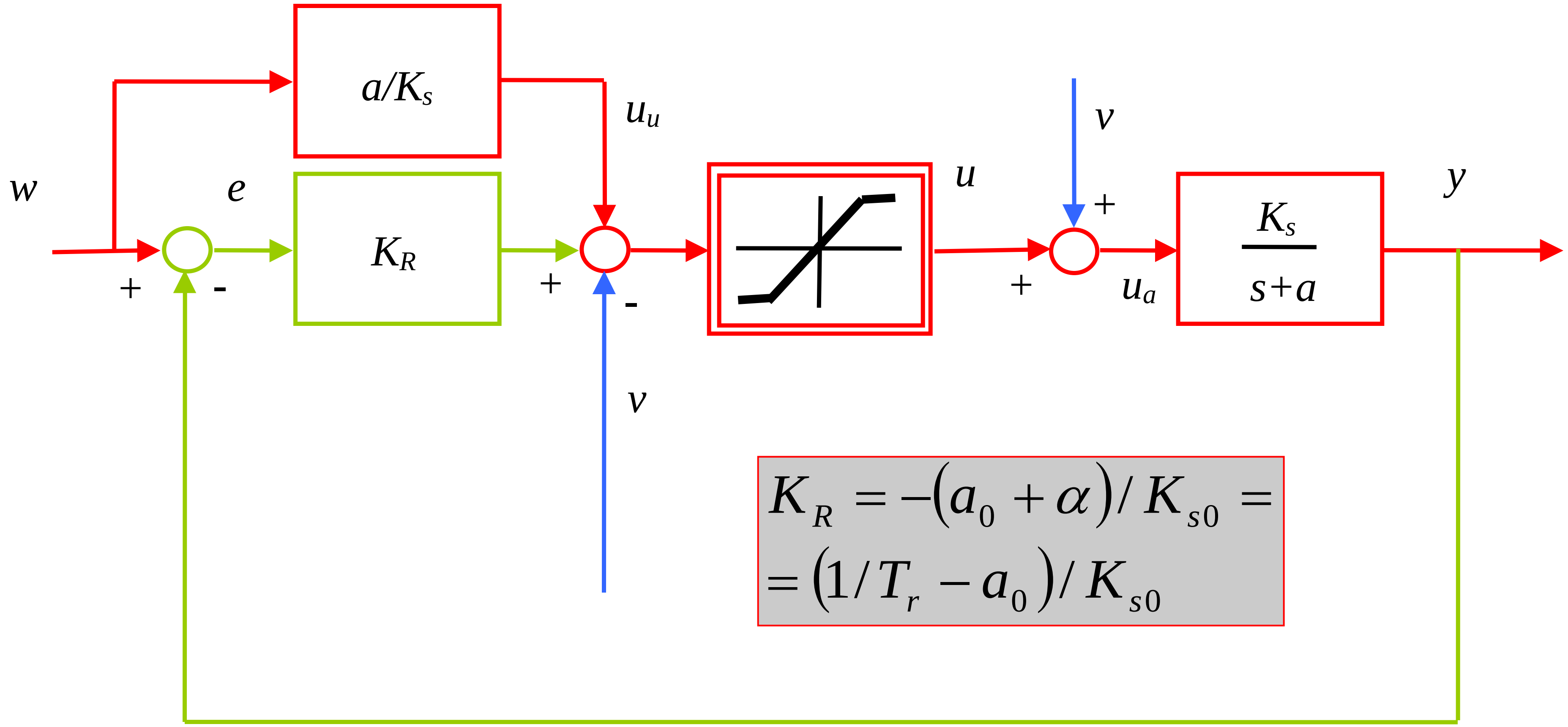
Continuous for $\forall\ t$



Filtred Controllers

Input Disturbance of 1st Order Plant

Static feedforward + P-controller + disturbance compensation



$$K_R = -(a_0 + \alpha)/K_{s0} = \\ = (1/T_r - a_0)/K_{s0}$$

Measurable disturbance may be compensated

Non-measured disturbances may be reconstructed

Parallel PI controller

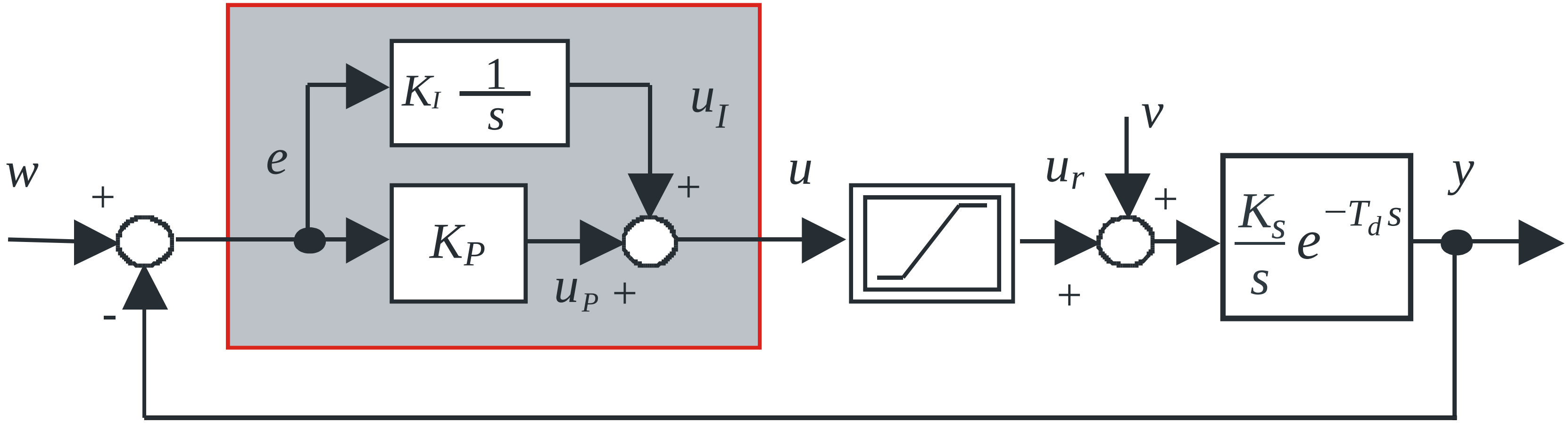
P controller – loop stabilization

I action – disturbance reconstruction (observer)

– disturbance compensation/rejection

In steady state u_I=-v

$$\frac{du_I}{dt} = K_I e \; ; \; u_I = \int_{-\infty}^{t} e(t)dt$$



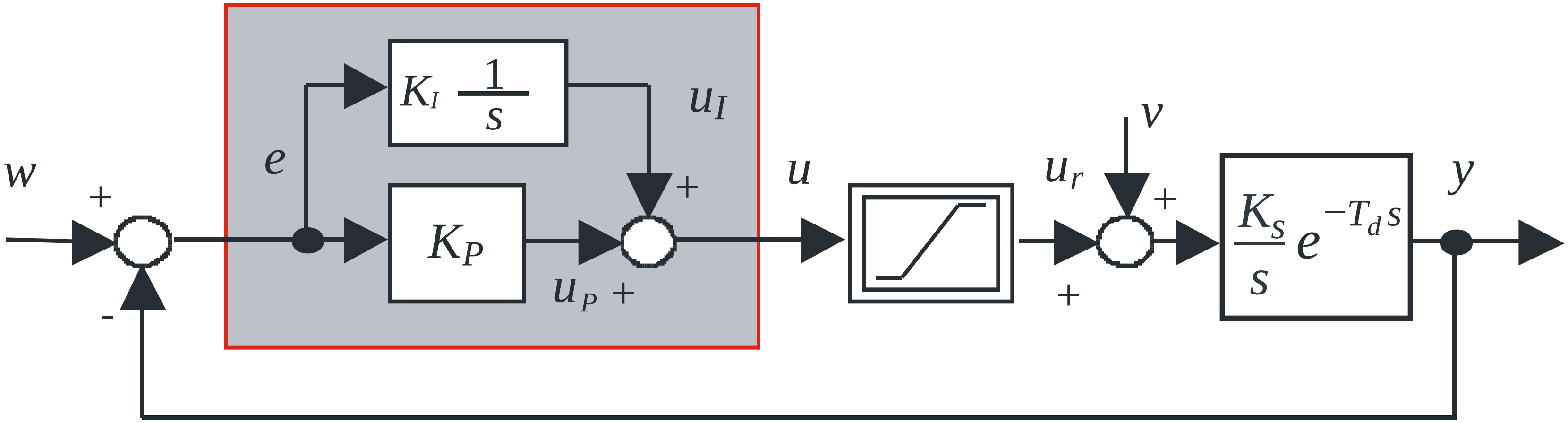
Problem 1

Nominal system

$$K_s = K_{s\,max} = K_{s\,min} > 0$$
$$T_d = T_{d\,max} = T_{d\,min} > 0$$

What is the optimal tuning (for T_d=0)?

$$R(s) = \frac{U(s)}{E(s)} = K_P + K_I \frac{1}{s} = K_P\left(1 + \frac{1}{T_i s}\right) = \frac{K_P T_i s + 1}{T_i s}$$



Problem 1

Output transients have always overshooting

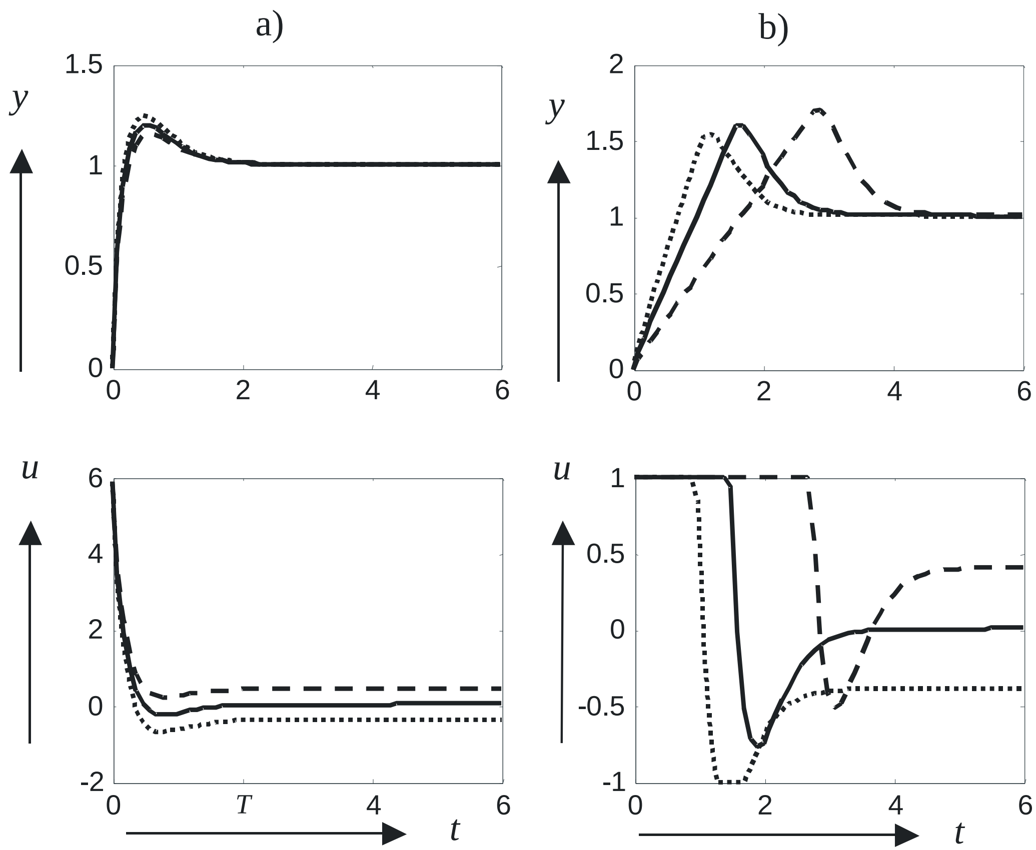
One explanation – it is caused by zero in the closed loop transfer function

Other explanation – I action integrates during the initial phase of transient responses even in situations with no disturbance – the accumulated signal can be cleared just by changing sign of the control error

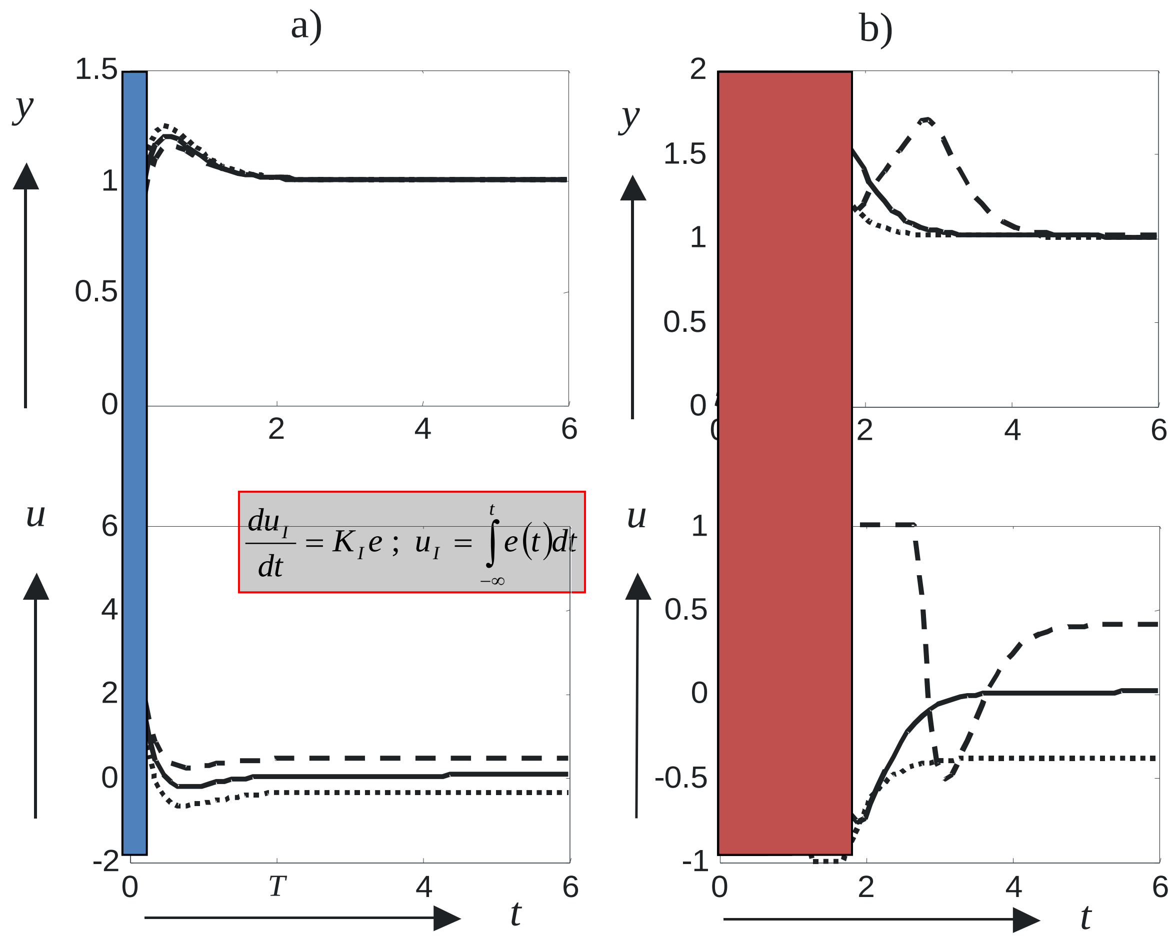
Control constraints prolong phase of wrong integration and enlarge its product – windup effect

Problem 1

Examples of transient responses for different disturbance *v* and unconstrained/constrained case



Problem 1



$$\frac{du_I}{dt} = K_I e \; ; \; u_I = \int_{-\infty}^{t} e(t)\,dt$$

Problem 1

Setpoint-to-output and disturbance-to-output transfer functions (T_d=0)

$$F_w(s)=\frac{Y(s)}{W(s)}=\frac{K_sK_P(T_Is+1)}{s^2T_I+K_PK_s(T_Is+1)}$$

$$F_v(s)=\frac{V(s)}{W(s)}=\frac{sK_sT_I}{s^2T_I+K_PK_s(T_Is+1)}$$

$$A(s)=s^2T_I+K_PK_s(T_is+1)=T_I(s-s_1)(s-s_2)$$

In steady-states the disturbance is compensated

Problem 1

Roots of $A(s)$ (for $T_d=0$)

$$A(s) = s^2 T_I + K_P K_s (T_i s + 1) = T_I (s - s_1)(s - s_2)$$

$$s_{1,2} = \frac{-K_P K_s \pm K_P K_s \sqrt{1 - \frac{4}{K_P K_s T_I}}}{2}$$

Aperiodic transients (see e.g. Skogestad, 2003)

Skogestad, S. Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control* 13, 2003, 291–309

Problem 1

Overshooting and windup are casued by zero of the setpoint-to-output TF. This can be cancelled by prefilter numerator with time constant T_I

Transients may be speeded up by cancelling one pole of $A(s)$ by prefilter numerator $1+bT_Is$

Vítečková, M., Víteček, A.: Two-degree of Freedom Controller Tuning for Integral Plus Time Delay Plants. ICIC Express Letters. An International Journal of Research and

Surveys. Volume 2, Number 3, September 2008, Japan , pp. 225-229

$$F_w(s)=\frac{Y(s)}{W(s)}=\frac{K_sK_P(T_Is+1)}{s^2T_I+K_PK_s(T_Is+1)}\;;\;-\frac{1}{bT_I}=s_1=-K_PK_s\left(1+\sqrt{1-4[(K_PK_sT_I)}\right)$$

Problem 1

Prefilter

- Denominator canceling zero of $F_w(s)$
- Numerator canceling slower real pole

$$F_p(s) = \frac{bT_i s + 1}{T_i s + 1}$$

Equivalent solution

Two-Degree-of-Freedom

2DOF PI Controller

$$U(s) = K_P\left[bW(s) - Y(s)\right] + \frac{K_P}{sT_i}\left[W(s) - Y(s)\right]$$

Problem 1

Optimal PI with prefilter (2DOF PI)

Pole-zero cancellation

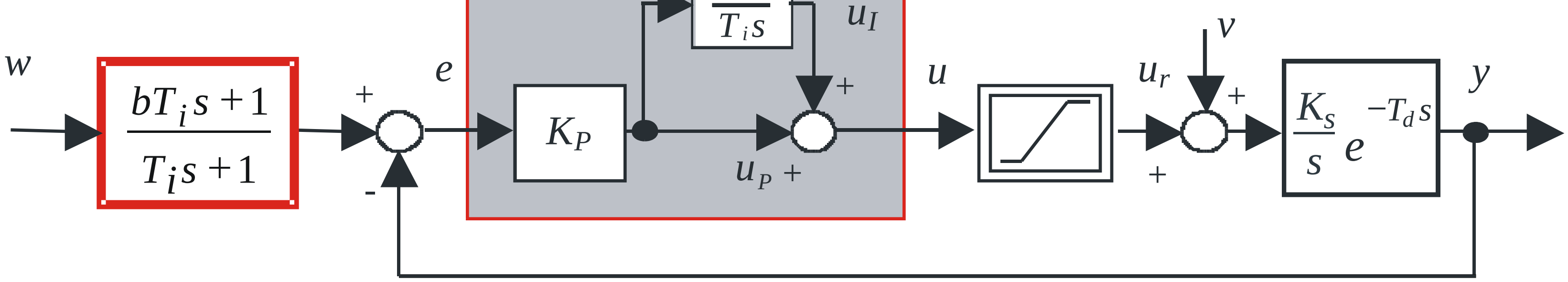
Monotonic transients

Low IAE values

What to do for $T_d > 0$?

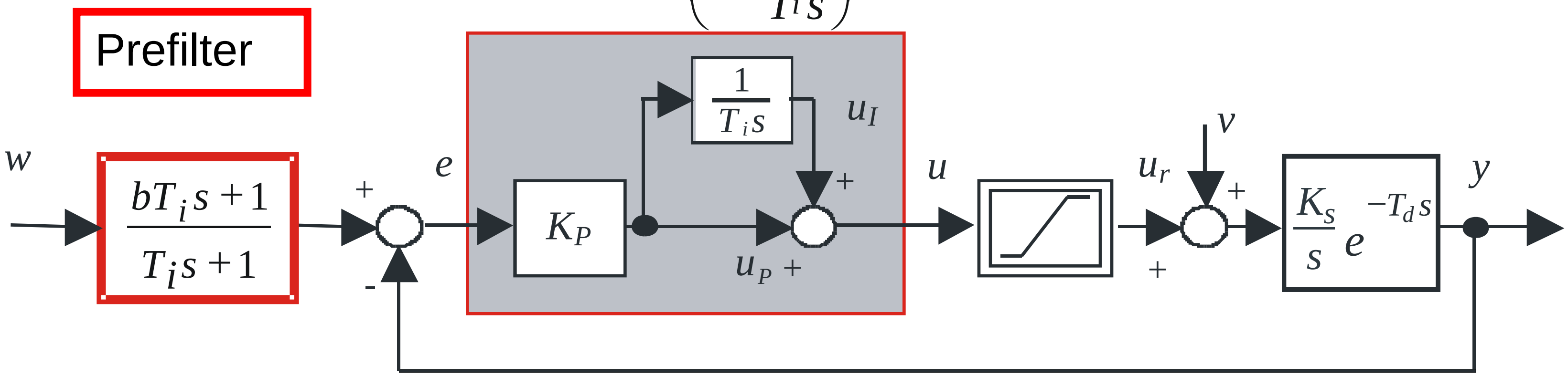
$$R(s) = K_P\left(1 + \frac{1}{T_i s}\right)$$

Prefilter



Optimal PI – Tripple Real Dominant Pole TRDP (Vítečková & Víteček, 2008)

$$R(s) = K_P\left(1+\frac{1}{T_i s}\right)$$



Problem 2

Setpoint-to-Output Transfer Function for T_d>0

$$F_w(s) = \frac{Y(s)}{W(s)} = \frac{K_s K_P (T_i s + 1)}{s^2 T_i e^{T_d s} + K_P K_s (T_i s + 1)}$$

$$A(s) = s^2 T_i e^{T_d s} + K_P K_s (T_i s + 1)$$

Tripple Real Dominant Pole of $A(s)$ corresponds to

$$A(s) = (s - s_0)^3$$

Problem 2

TRDP fulfills conditions

$$A(s_0)=0\ ;\ \ \dot{A}(s_0)=0\ ;\ \ \ddot{A}(s_0)=0$$

$$A(s)=s^2T_ie^{T_ds}+K_PK_s(T_is+1)$$

$$\dot{A}(s)=2sT_ie^{T_ds}+s^2T_dT_ie^{T_ds}+K_PK_sT_i$$

$$\ddot{A}(s)=2T_ie^{T_ds}+4sT_dT_ie^{T_ds}+s^2T_d^2T_ie^{T_ds}$$

Solution gives

$$s_0=-\left(2-\sqrt{2}\right)/T_d$$

$$K_P=2\left(\sqrt{2}-1\right)e^{\sqrt{2}-2}/(K_sT_d)\approx 0.461/(K_sT_d)$$

$$T_i=\left(2\sqrt{2}+3\right)T_d\approx 5.828T_d$$

Problem 2

Prefilter

- Denominator canceling zero of $F_w(s)$
- Numerator canceling one pole s_0

$$F_p(s) = \frac{bT_i s + 1}{T_i s + 1}$$

$$bT_i = -1/s_0 \;\Rightarrow\; b = \frac{2-\sqrt{2}}{2} \approx 0.293$$

Equivalent solution – 2DOF PI Controller

$$U(s) = K_P\left[bW(s) - Y(s)\right] + \frac{K_P}{sT_i}\left[W(s) - Y(s)\right]$$

Problem 2

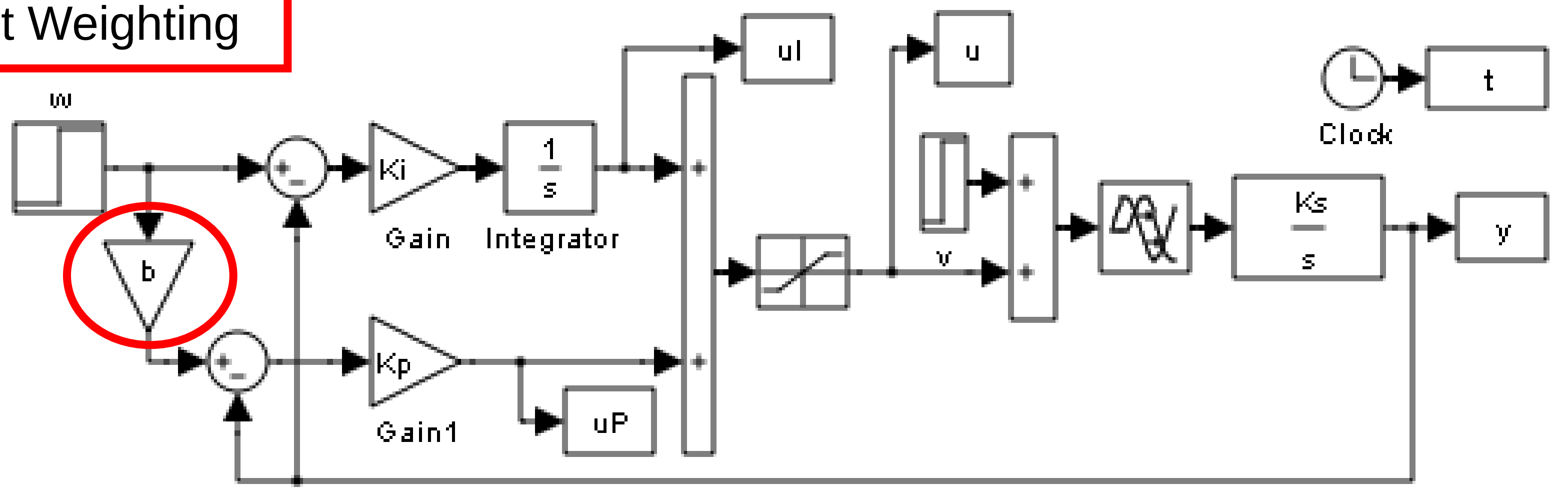
Equivalent solution

Monotonic transients

Low IAE values

I action does not equal to negative disturbance!

Setpoint Weighting

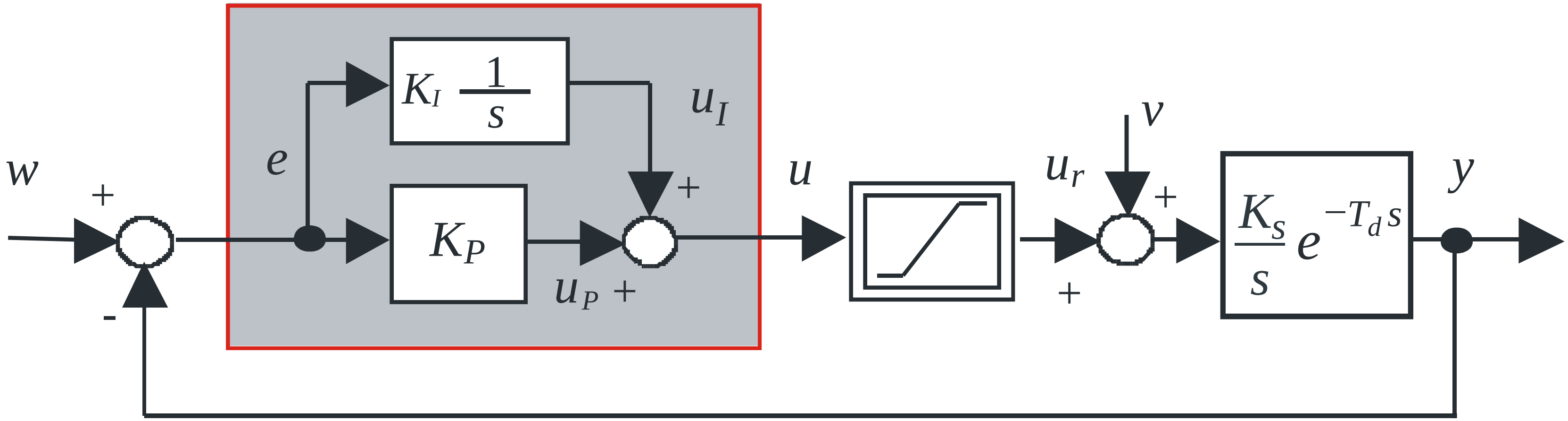


Problem 3

Interval Plant Parameters

$$K_s \in \langle K_{s\,min}, K_{s\,max} \rangle;\ K_{s\,max} \geq K_{s\,min} > 0$$
$$T_d \in \langle T_{d\,min}, T_{d\,max} \rangle;\ T_{d\,max} \geq T_{d\,min} > 0$$

Looking for PI controller tuning guaranteeing monotonic setpoint step responses with minimal mean IAE over uncertainty set



Problem 3

Closed loop transfer functions for normed variables

$$F_w(s)=\frac{Y(s)}{W(s)}=\frac{K_sK_P(1+bT_is)}{s^2T_ie^{T_ds}+K_PK_s(T_is+1)}$$

$$p=T_ds;\ \Omega_c=K_sK_PT_d\ ;\ \tau_i=T_i/T_d\ ;\ \Omega_f=1/\tau_i=T_d/T_i$$

$$\Downarrow$$

$$F_w(p)=\frac{\Omega_c(bp+\Omega_f)}{p^2e^p+\Omega_c(p+\Omega_f)}=\frac{B(p)}{A(p)}$$

Problem 3

Conditions for triple real dominant pole

$$A(p_0)=\dot{A}(p_0)=\ddot{A}(p_0)=0$$
$$A(p)=p^2e^p+\Omega_c p+\Omega_c\Omega_f$$
$$\dot{A}(p)=\left(p^2+2p\right)e^p+\Omega_c$$
$$\ddot{A}(p)=\left(p^2+4p+2\right)e^p=\left[(p+2)^2-2\right]e^p$$

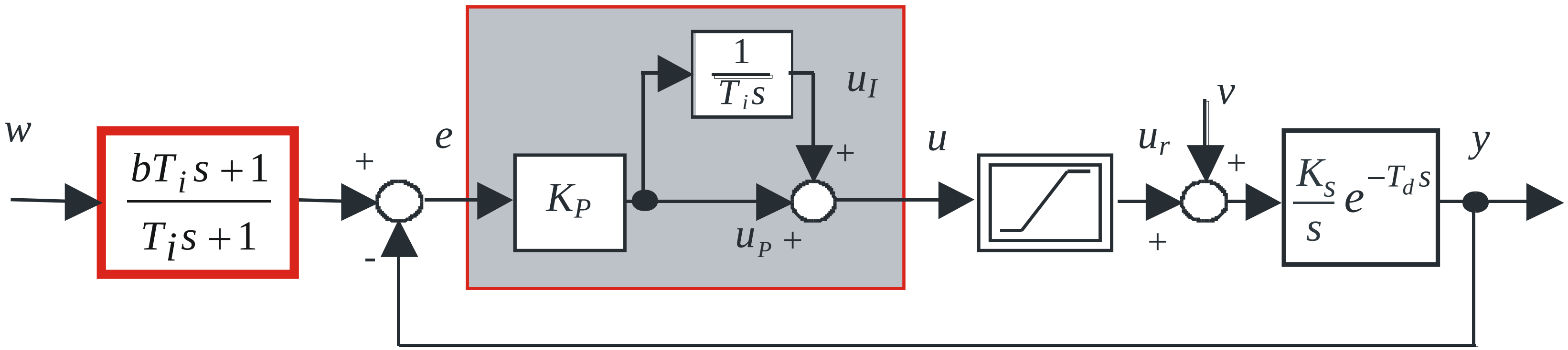
$$\ddot{A}(p_0)=0\Rightarrow p_0+2=\pm\sqrt{2}\Rightarrow p_0=\sqrt{2}-2$$
$$\dot{A}(p_0)=0\Rightarrow \Omega_{c0}=-\left(p_0^2+2p_0\right)e^{p_0}=2\left(\sqrt{2}-1\right)e^{\sqrt{2}-2}$$
$$A(p_0)=0\Rightarrow \Omega_{f0}=-\frac{p_0^2e^{p_0}}{\Omega_{c0}}-p_0=\frac{p_0}{(p_0+2)}-p_0=\frac{\sqrt{2}-2}{\sqrt{2}}-\sqrt{2}+2=3-2\sqrt{2}$$

Problem 3

How to tune prefilter (coefficient *b*)? – iterative solution - real dominant pole (it exists at least one) – canceling with prefilter numerator!

Mapping the Performance Portrait in 2D (Ω_c,Ω_f) with chosen steps and limits for K_p and T_i

$$R(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$

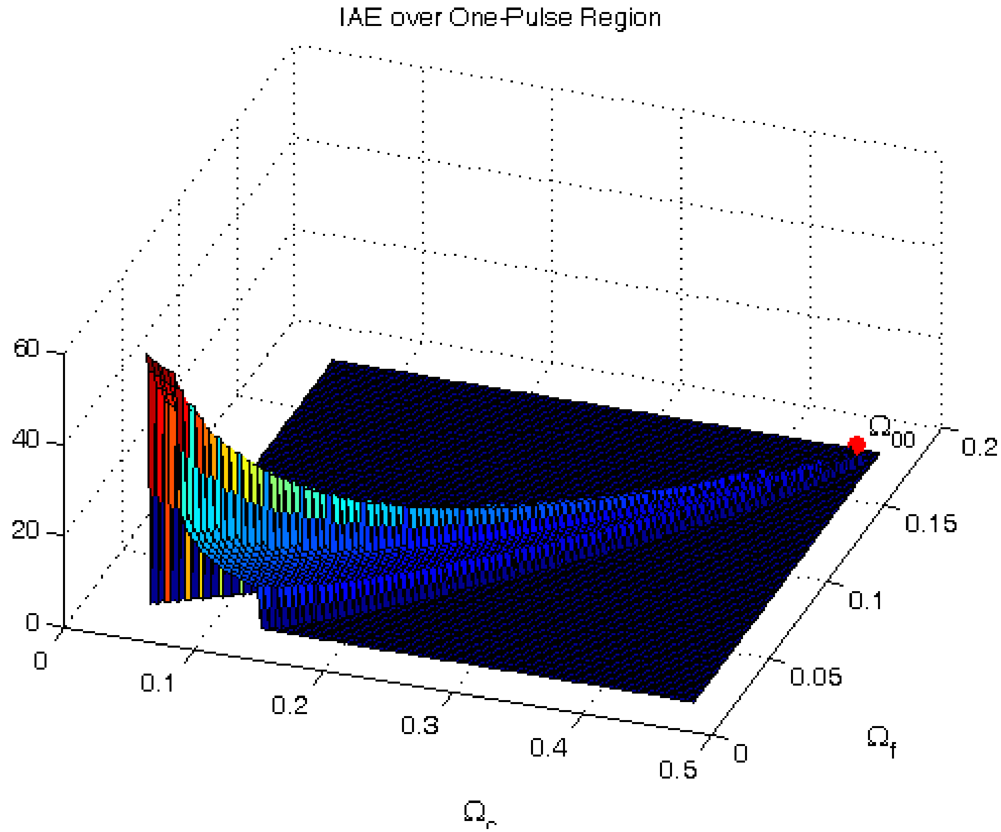


Performance Portrait

IAE values over region of y-MO & u-1P control

$$\Omega_c = K_P K_s T_d$$

$$\Omega_f = T_d / T_i$$



Uncertainty Set US

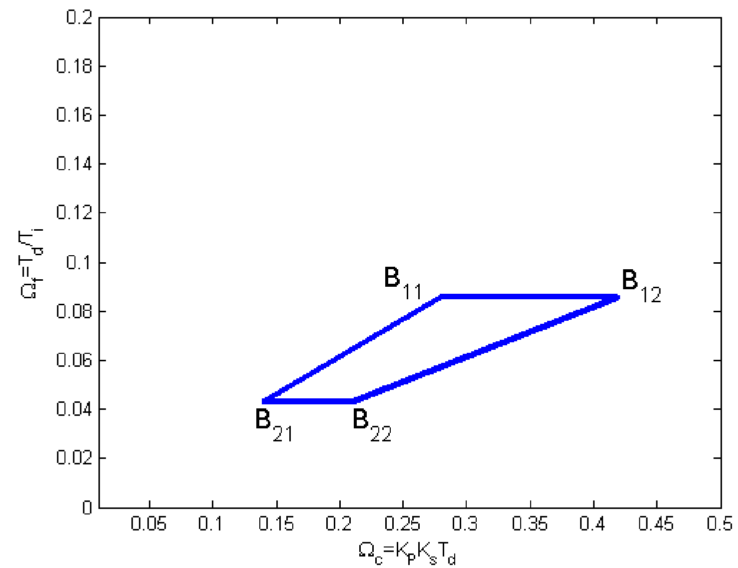
Uncertainty Set – Trapezoid

$$US = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}; \qquad \Omega_c = K_P K_s T_d \;;\; \Omega_f = T_d / T_i$$

$$B_{11} = \left(K_P K_{s\min} T_{d\max}, \Omega_{f\max}\right);\; B_{12} = \left(K_P K_{s\max} T_{d\max}, \Omega_{f\max}\right)$$

$$B_{21} = \left(K_P K_{s\min} T_{d\min}, \Omega_{f\min}\right);\; B_{22} = \left(K_P K_{s\max} T_{d\min}, \Omega_{f\min}\right)$$

Uncertainty in T_d is influencing all 4 vertices – irregularly!

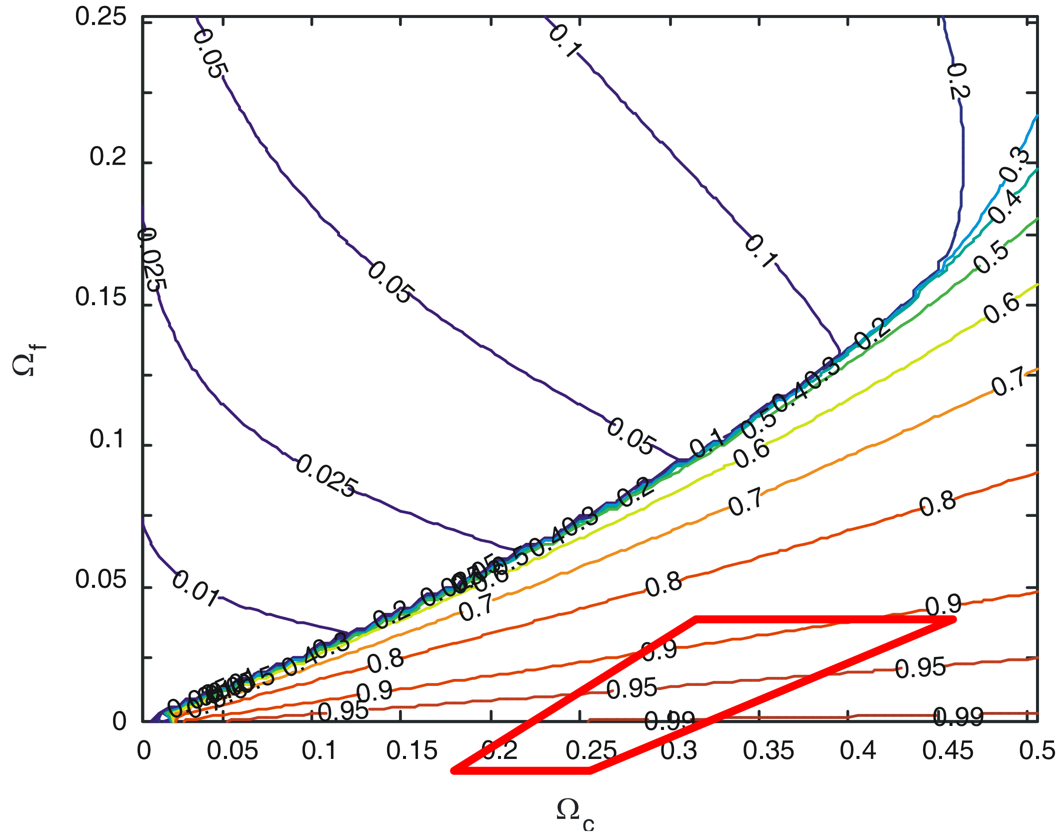


Performance Portrait

Setpoint Weighting *b*

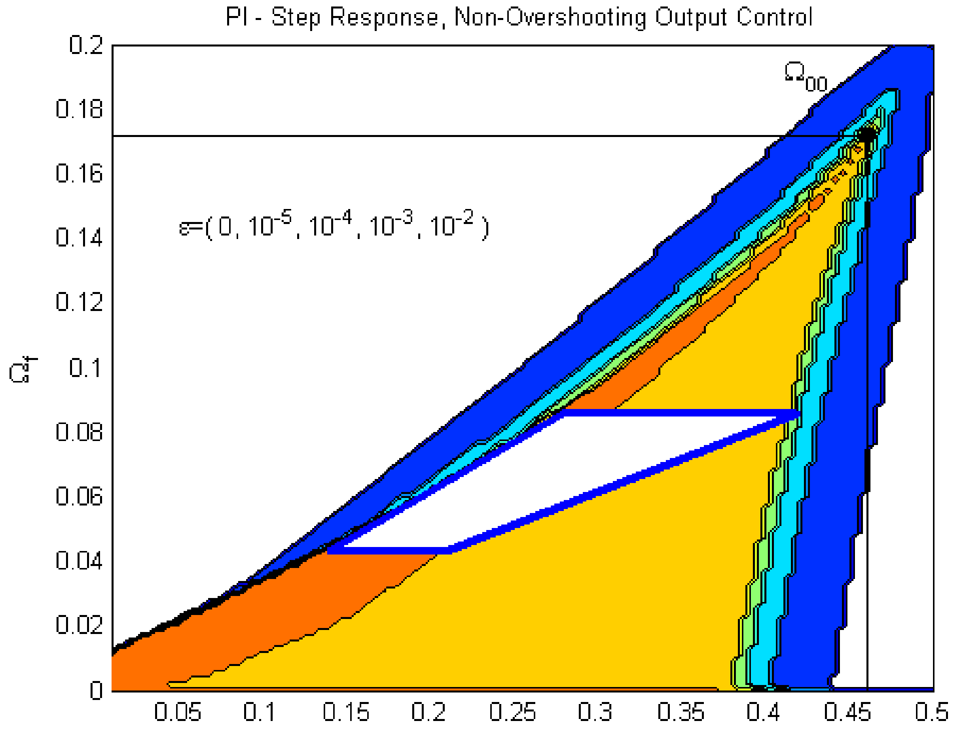
$$\Omega_c = K_P K_s T_d$$
$$\Omega_f = T_d / T_i$$

Robust monotonic responses => minimal *b* over US = slightly conservative



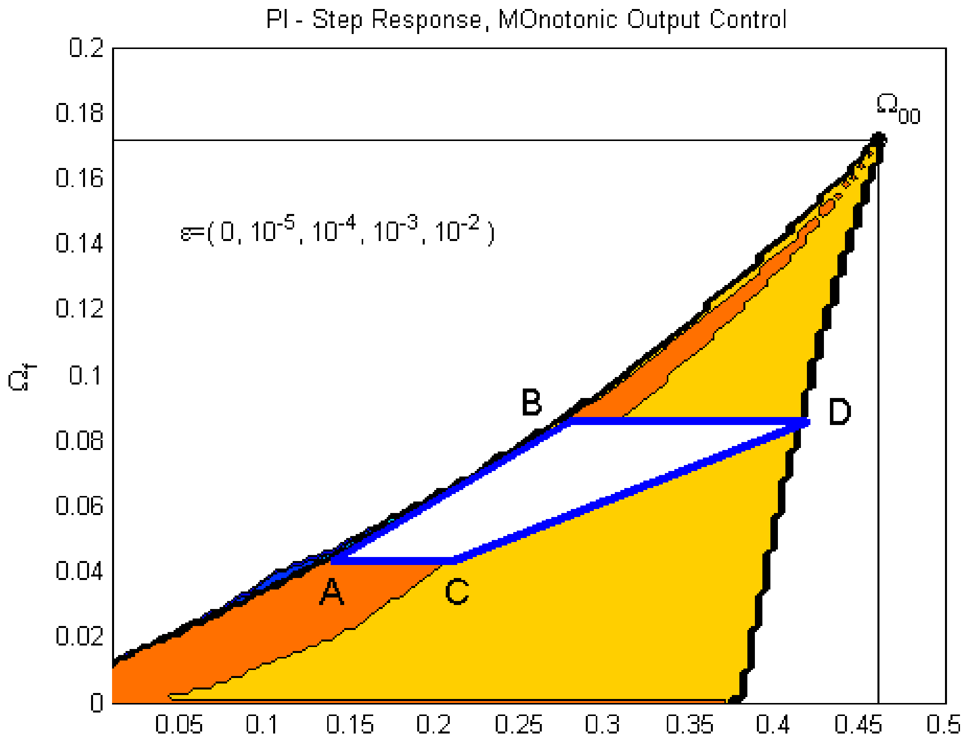
Uncertainty Set – y-NO

US may not include optimal point and its neighbourhood



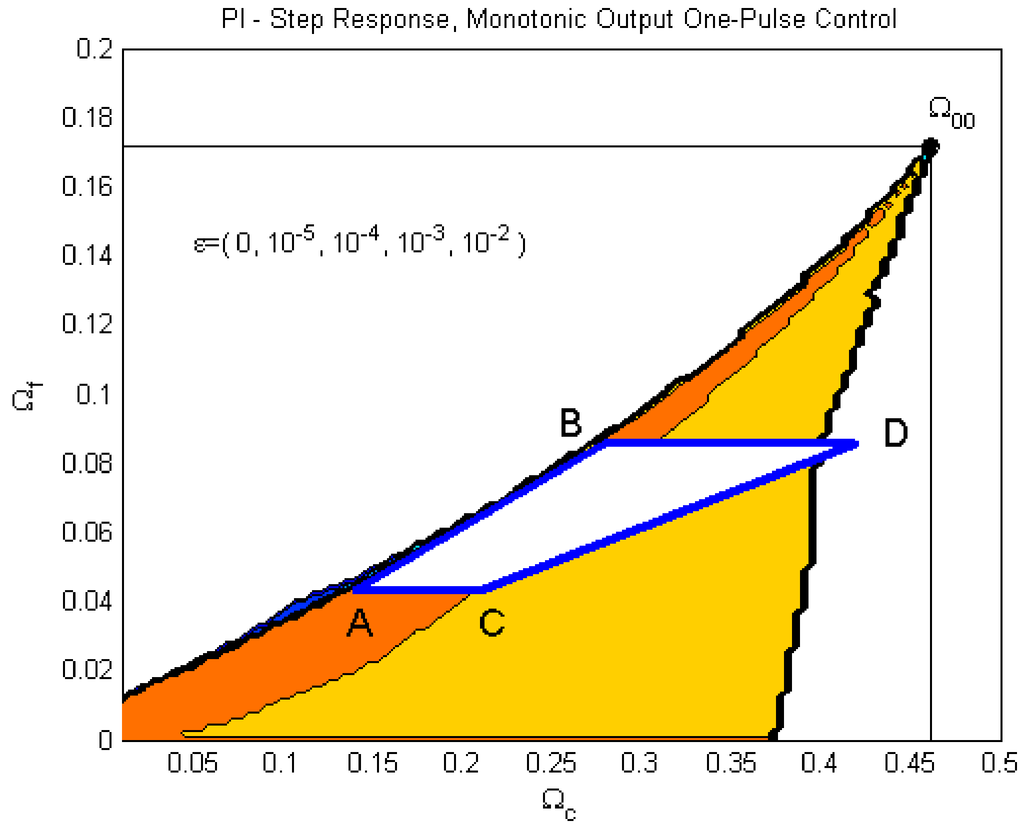
Uncertainty Set –y-MO

US may not include optimal point and its neighbourhood



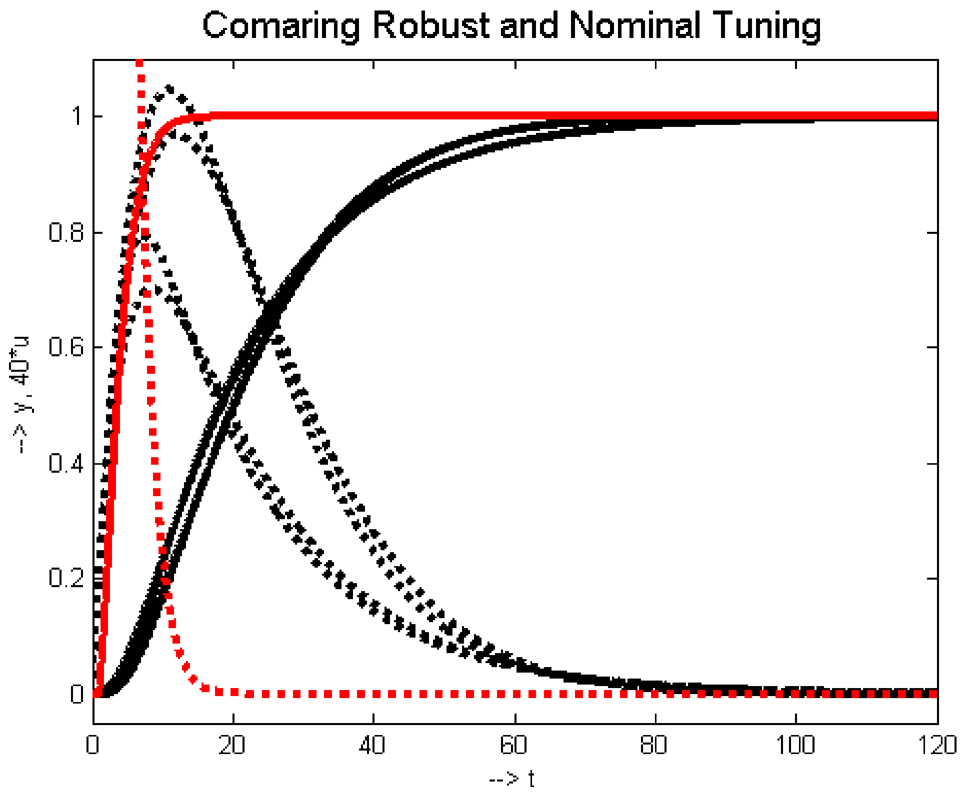
Uncertainty Set – u-1P

US may not include optimal point and its neighbourhood



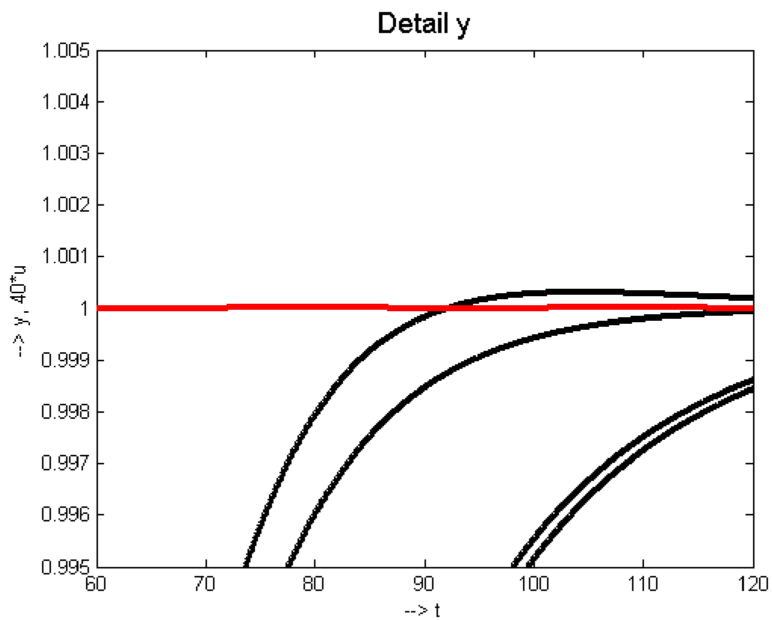
Corresponding Time Responses

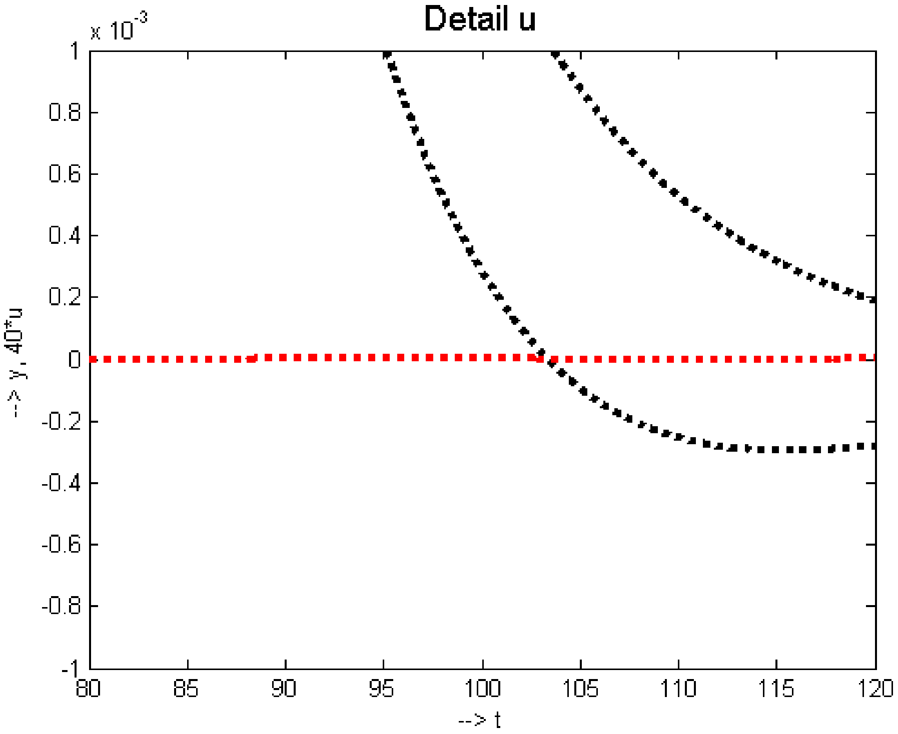
Robust tuning gives much more conservative tuning than the nominal optimal one



Corresponding Time Responses

Detail showing overshooting – one of the vertices of US was out of MO region





Sub-Conclusions

- Traditional parallel PI controller is not able to guarantee monotonic setpoint step responses for integral plant
- Monotic responses require use of the 2DOF PI controller (setpoint weighting or prefilter)
- Both solutions are equivalent from the output point of view, not at the level of P and I-action

Sub-Conclusions

- When using setpoint weighting the value of I-action does not represent negative disturbance
- The excessive integration of I action is compensated by intentionally „distorted“ P action
- Zero disturbance is compensated by the counter-acting P and I action (too complicated)

Sub-Conclusions

- Performance Portrait represents interesting method for controller evaluation and tuning
- It enables both nominal and robust tuning of PI controllers guaranteeing specified performance given by tolerated deviations from ideal NO, MO & 1P responses with minimal IAE, TV0 and TV1 values
- It also enables evaluation of constraints influence

Sub-Conclusions

- For parallel PI controller it is not possible to locate ULS or US into areas of y-MO & u-1P control close to the optimal working point with minimal IAE values – the corresponding areas are too narrow
- Due to this, robust transients are reasonably slower than the nominal ones
- It will be interesting to compare the above achieved results with other approaches based on disturbance observer.

Objectives 2

To explain:

- principle of robust tuning of the PI controller in 3D by using the performance portrait method,
- basic differences in tuning achieved with the PP method in comparing with the traditional methods of robust tuning

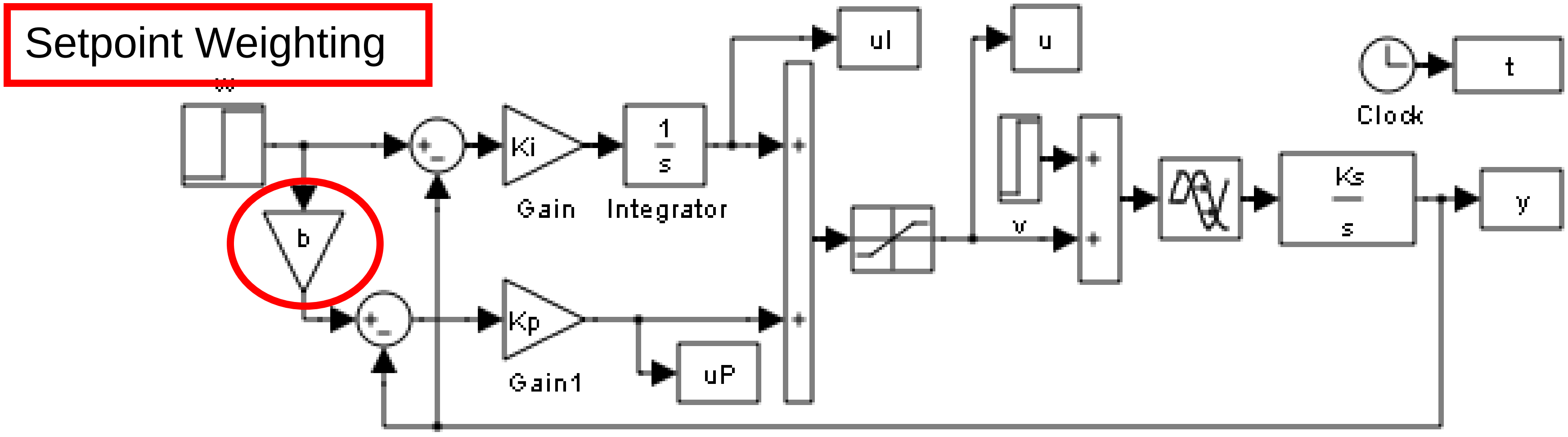
$$F(s)=\frac{K_s}{s}e^{-T_d s}$$

$$K_s \in \langle K_{s\min}, K_{s\max}\rangle;\ T_d \in \langle T_{d\min}, T_{d\max}\rangle$$

Aim

– to achieve MOnotonic (MO) transients at the plant output

– to achieve 1P transients at the plant input with specified tolerable deviations and low IAE



2DOF PI – Prefilter

Aim

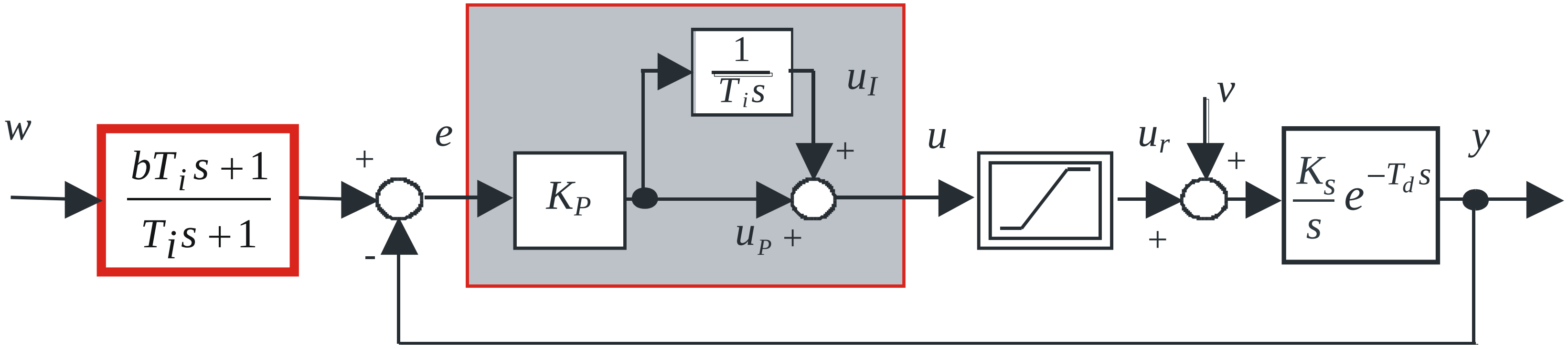
– to achieve MOnotonic (MO) transients

at the plant output

– to achieve 1P transients at the plant input

with specified tolerable deviations and low IAE

$$R(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$



Comparative Analysis

- Vítečková-Víteček, 2008 – Tripple Real Dominant Pole (TRDP)
- Skogestad, 2003 Simple/Skogestad IMC (SIMC) modified by prefilter
- Hägglund-Åström, 2002 Approximative Ms-constrained Integral Gain Op-timization (AMIGO) modified by prefilter
- Åström-Panagopoulos-Hägglund, 1998 Nonconvex Optimisation (NCON) – cannot be modified to give monotonic responses

Closed Loop TF – Normed Variables

Closed loop transfer functions for normed variables

$$F_w(s) = \frac{Y(s)}{W(s)} = \frac{K_s K_c (1 + bT_i s)}{s^2 T_i e^{T_d s} + K_P K_s (T_i s + 1)}$$

$$p = T_d s; \;\; K = K_s K_c T_d \;\; ; \;\; \tau_i = T_i / T_d \;\; ; b$$

$$\Downarrow$$

$$F_w(p) = \frac{K(b\tau_i p + 1)}{p^2 e^p + K(\tau_i p + 1)} = \frac{B(p)}{A(p)}$$

Performance Portrait

3D performance portrait mapped for the setpoint step responses over 27x27x21 points

$$K \in \langle 0.1, 1.4 \rangle; \tau_i \in \langle 3.5, 15.5 \rangle; b \in \langle 0.1 \rangle$$

Subsequently swept for:

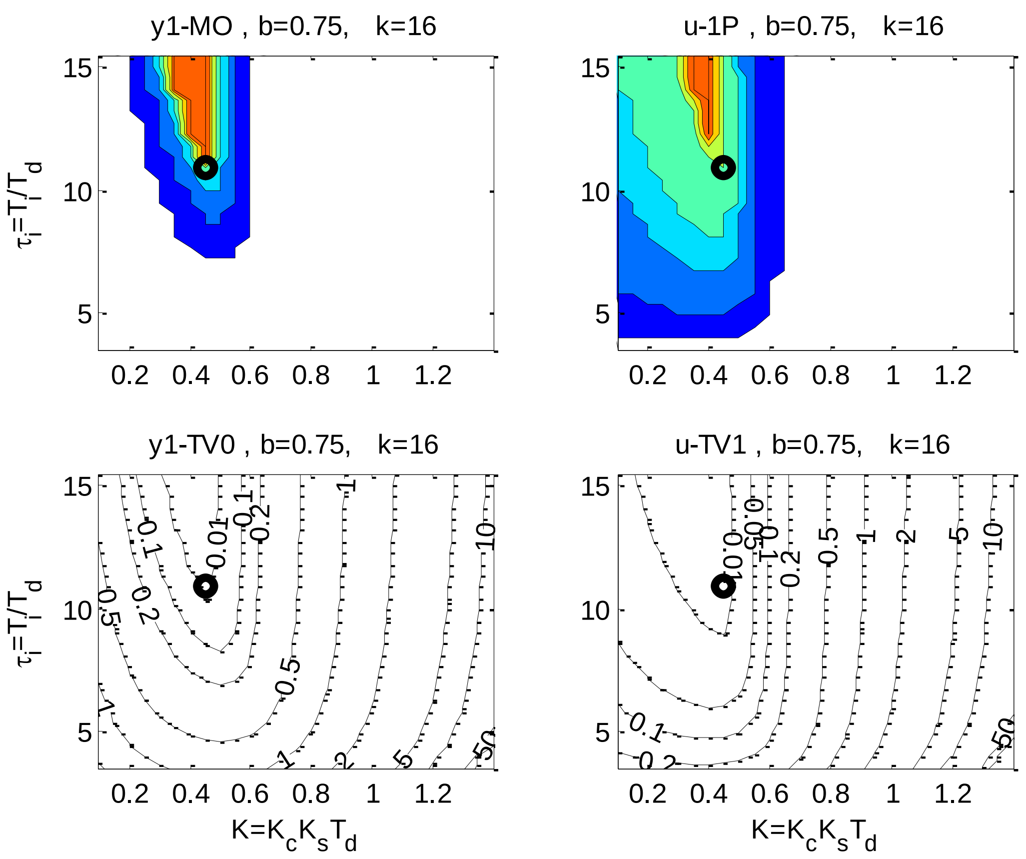
- Min IAE
- Max $K_i = K_c / T_i$

and used for:

- Nominal tuning
- Robust tuning

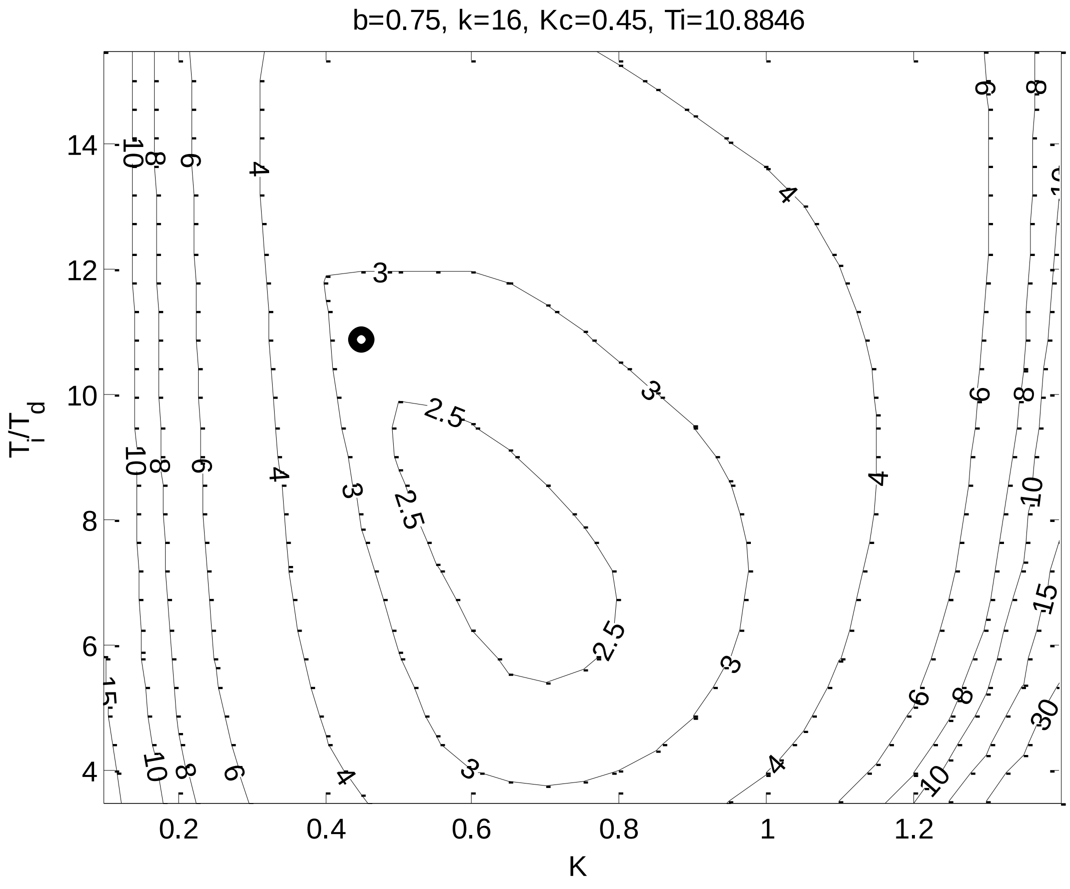
Min IAE - Performance Portrait

Amplitude and integral measures give equivalent results - just optimal layer shown



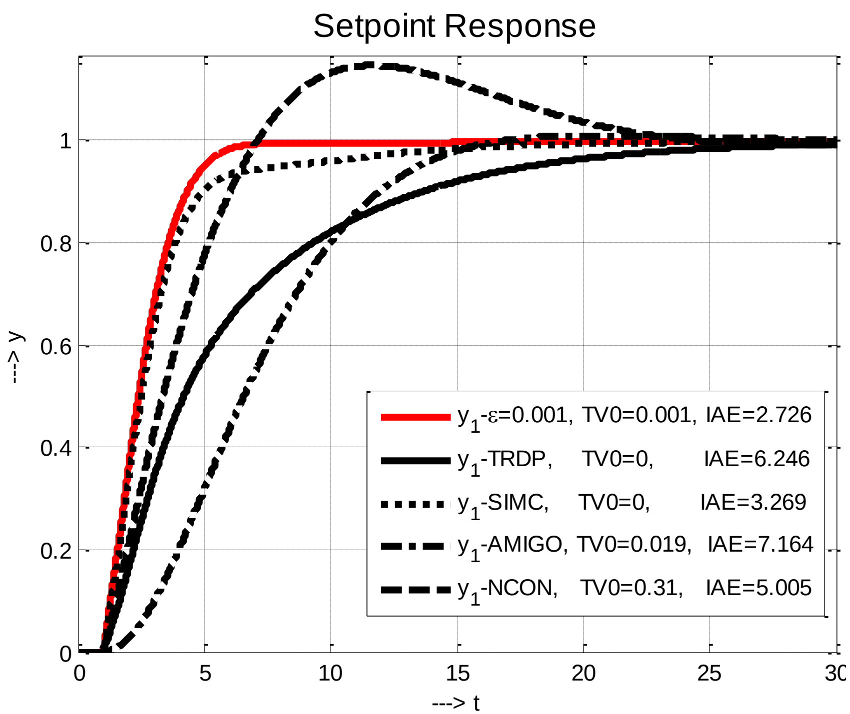
Min IAE - Performance Portrait

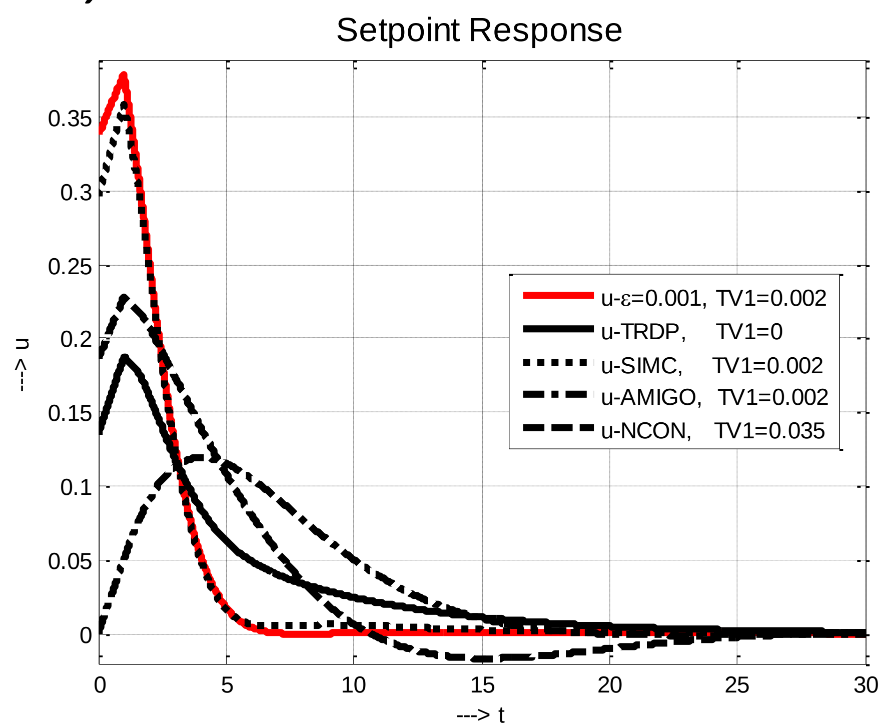
Absolute minimum of IAE is out of MO and 1P area



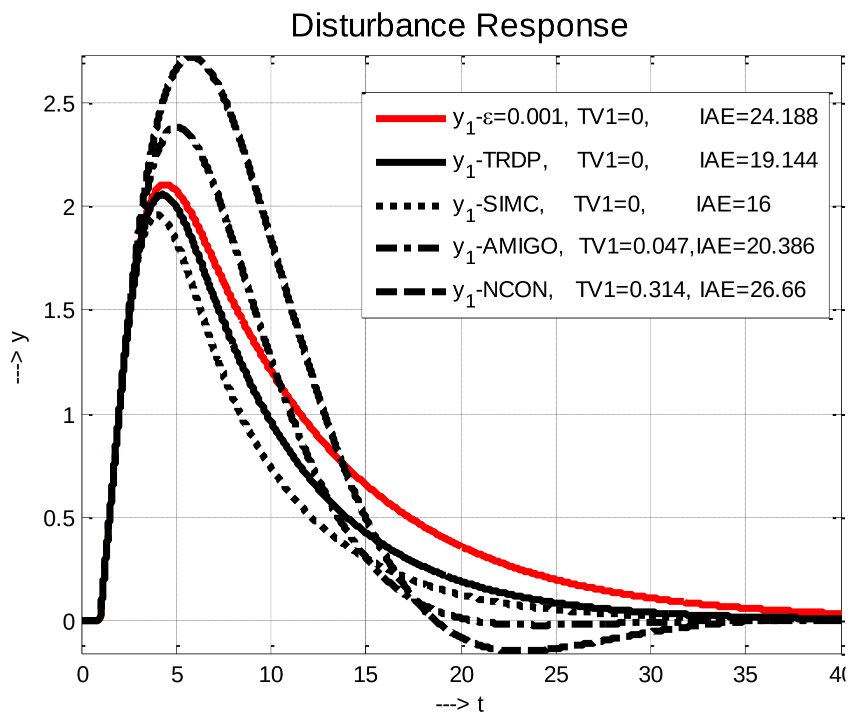
Min IAE – Setpoint Steps

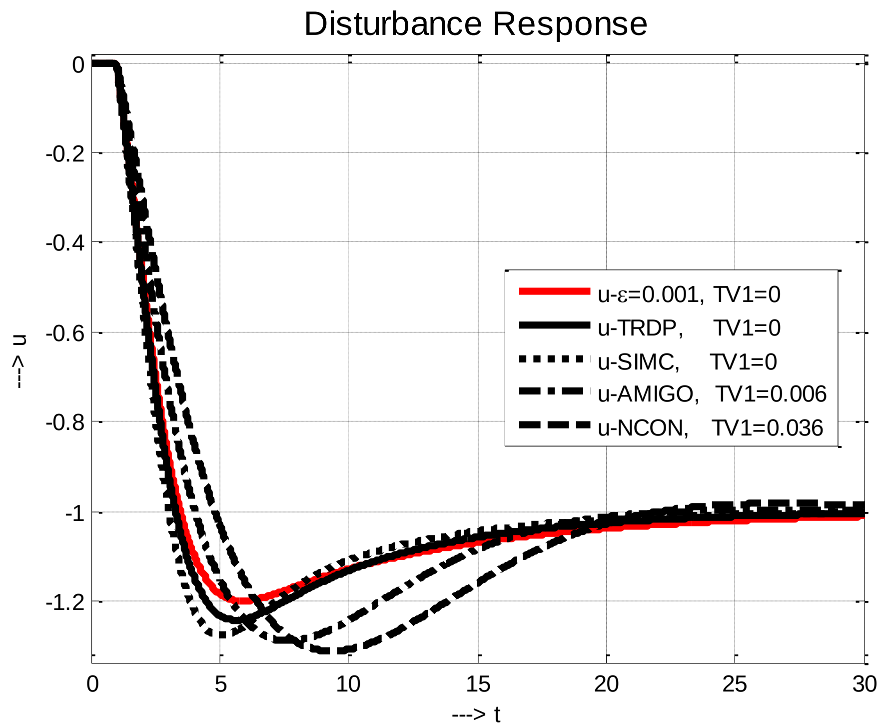
Optimization tending to $T_i \to \infty$, $b \to 1$





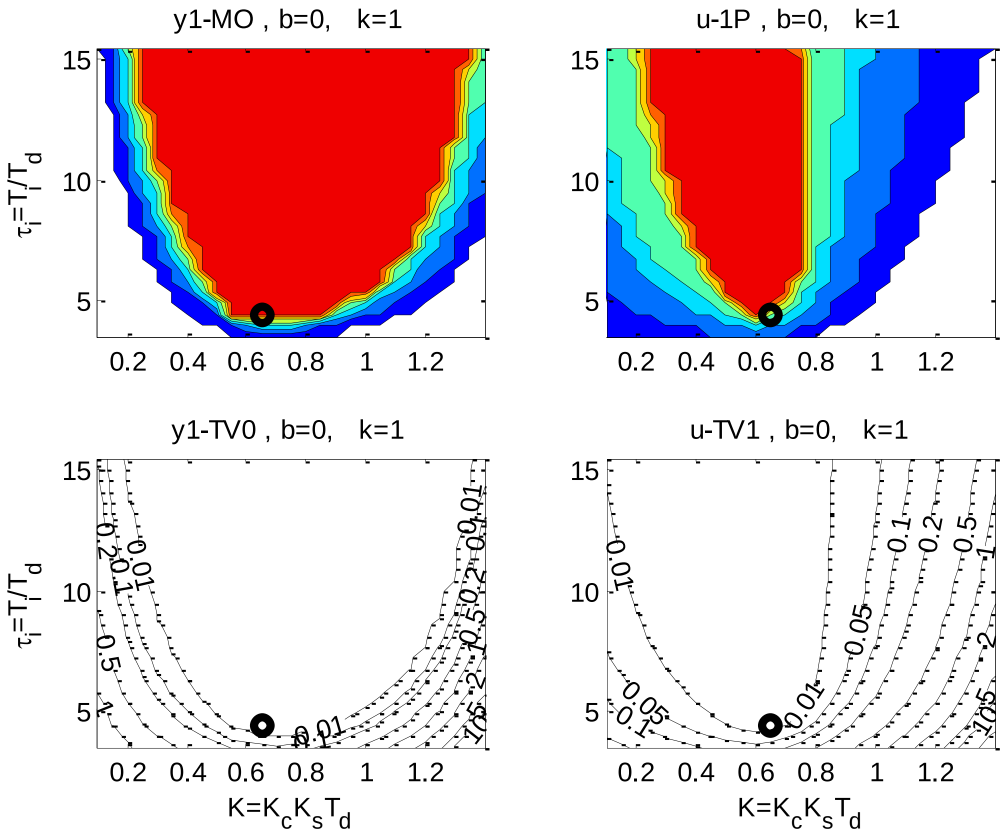
It has sense just for IAE composed from the setpoint as well as disturbance responses





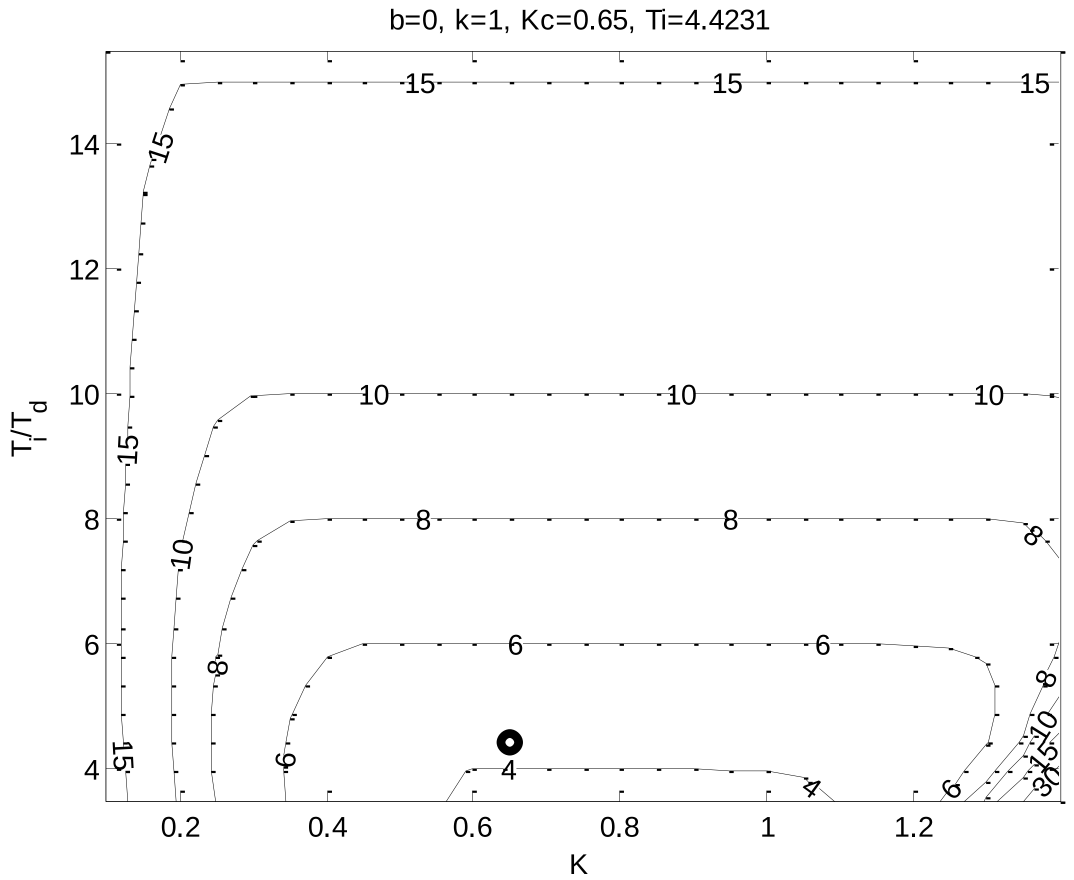
Max K_i - Performance Portrait

Amplitude and integral measures give equivalent results - just optimal layer shown



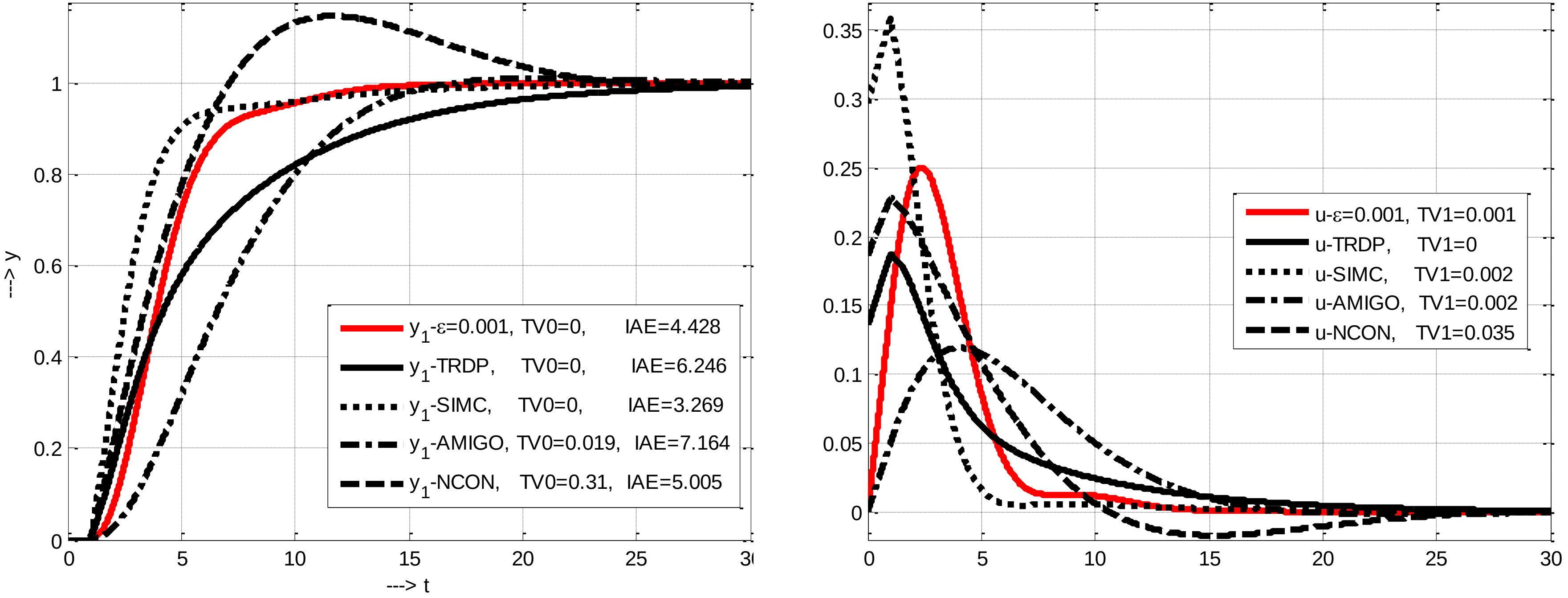
Max K_i - Performance Portrait

Minimum achieved for b=0; It is enough to continue in 2D



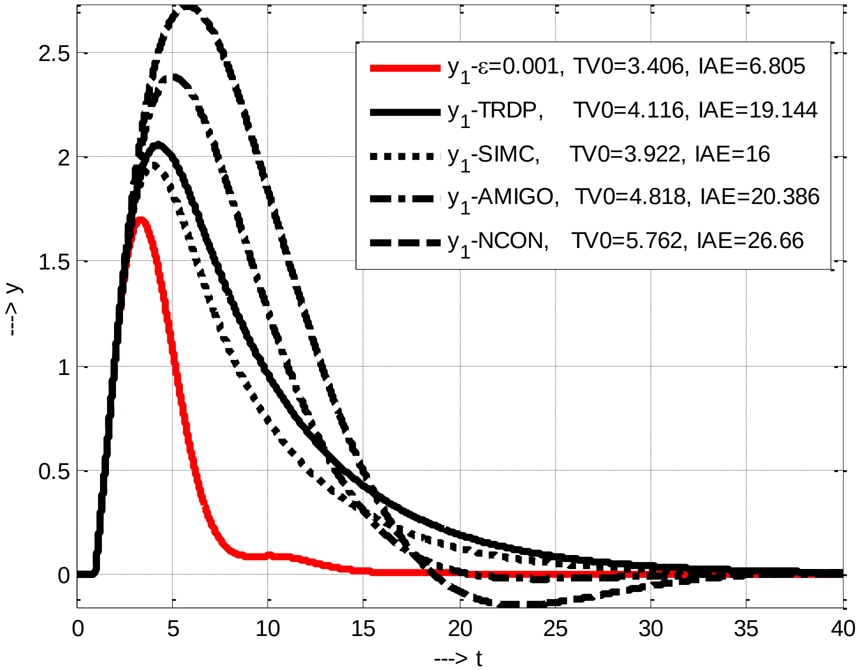
Max K_i - Setpoint Steps

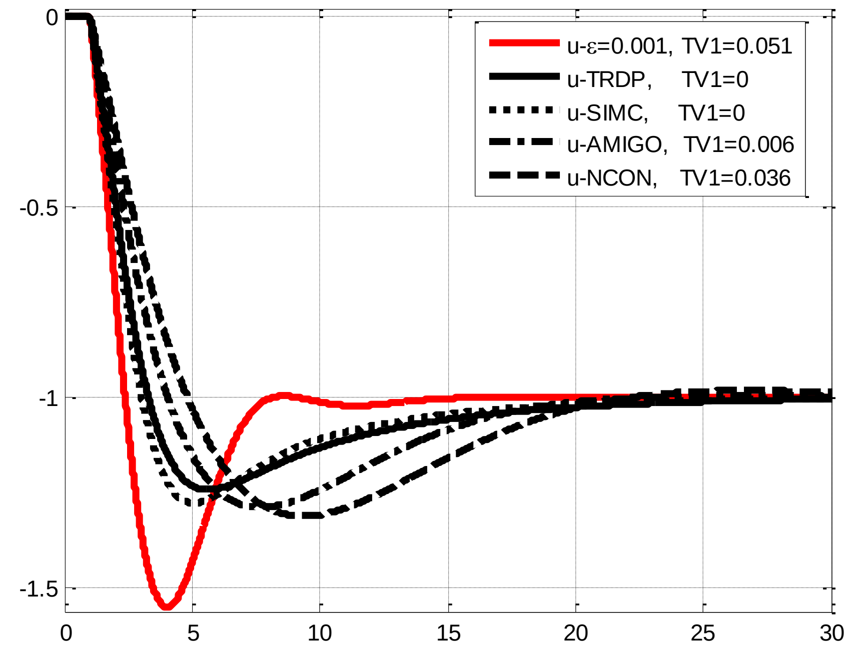
Now, the PP method does not give the absolutely best setpoint response (this was not required)



Max K_i - Disturbance Steps

But the disturbance response is clearly the best – although the design uses just PP derived from the setpoint steps – possibility for improvement





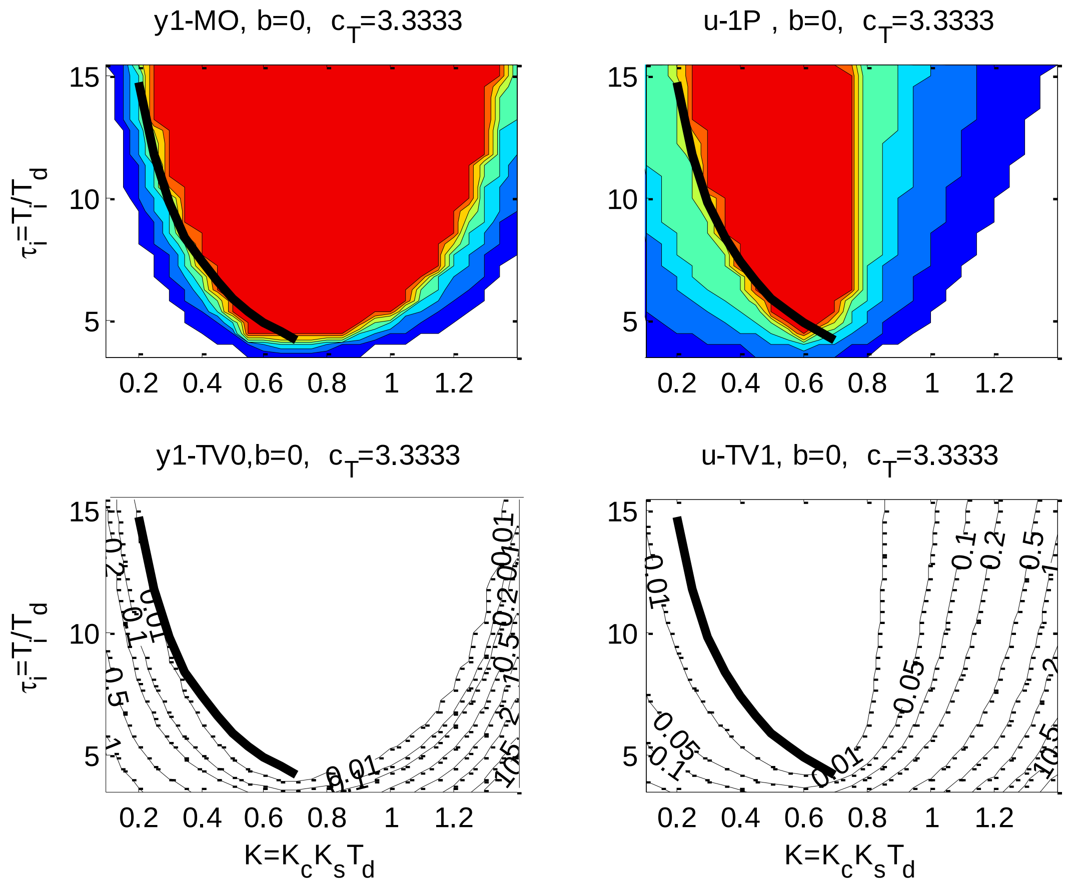
Max K_i - Performance Portrait

Dead Time Uncertainty $T_{d\,\min} = 0.3;\; T_{d\,\max} = 1.0$

Parabolic

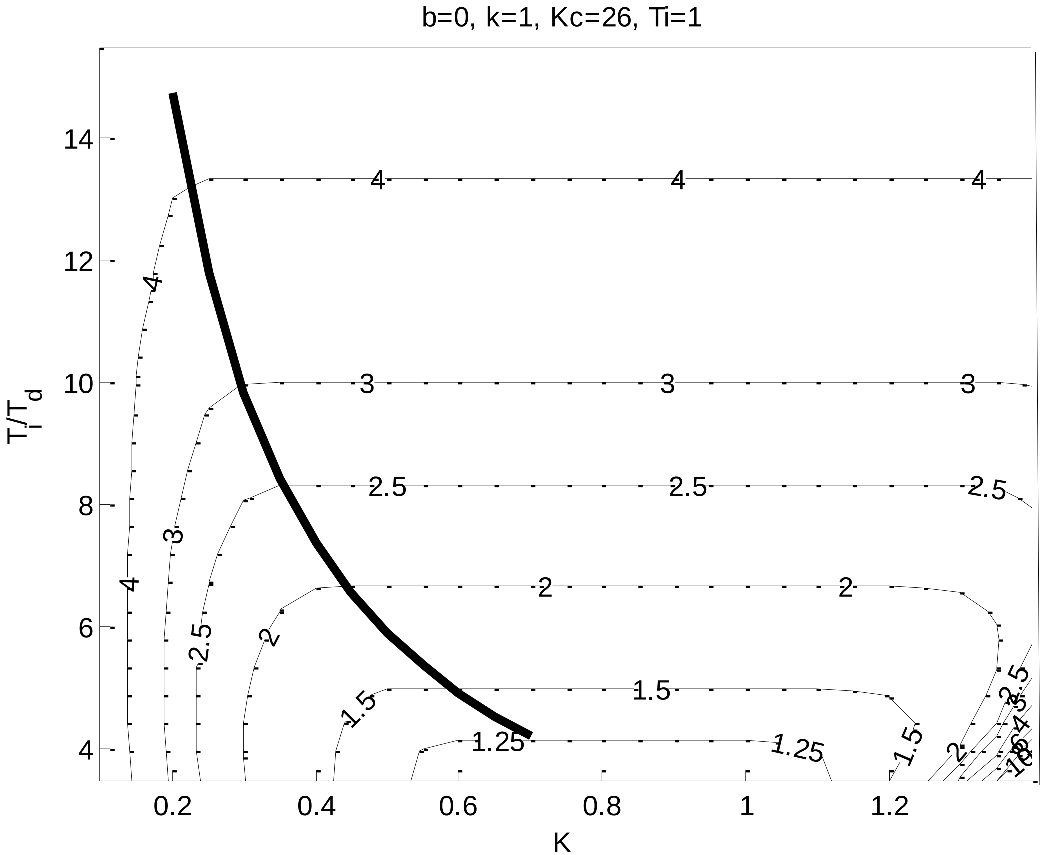
Uncertainty

Set

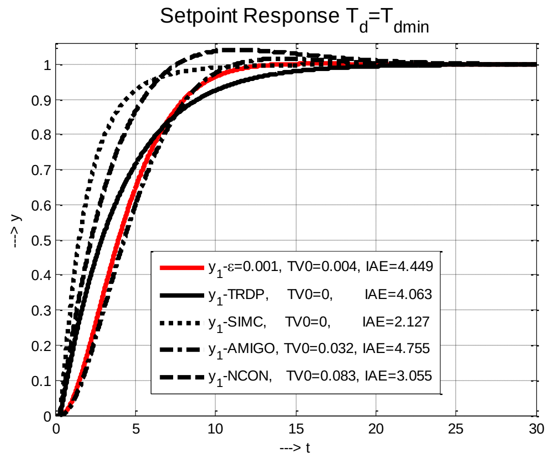


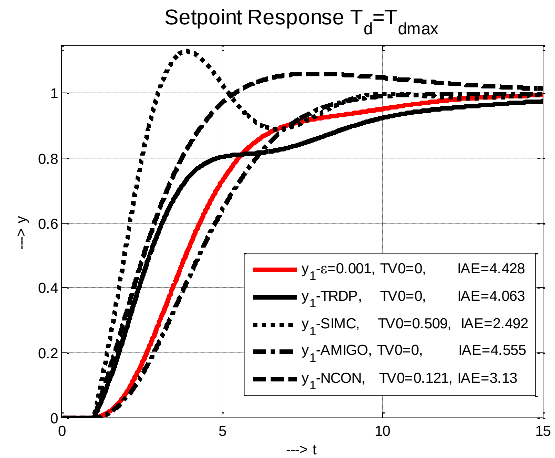
Max K_i - Performance Portrait

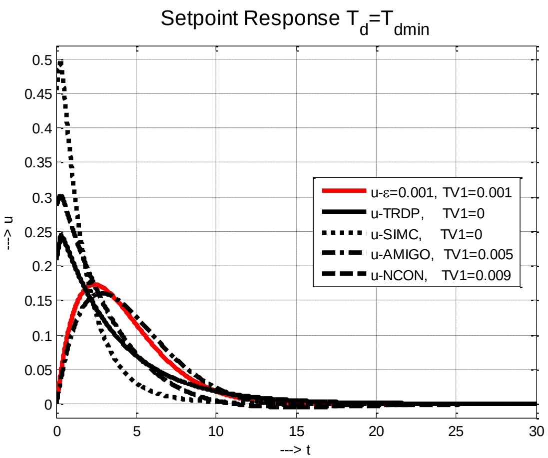
Minimum achieved for *b*=0; It is enough to continue in 2D

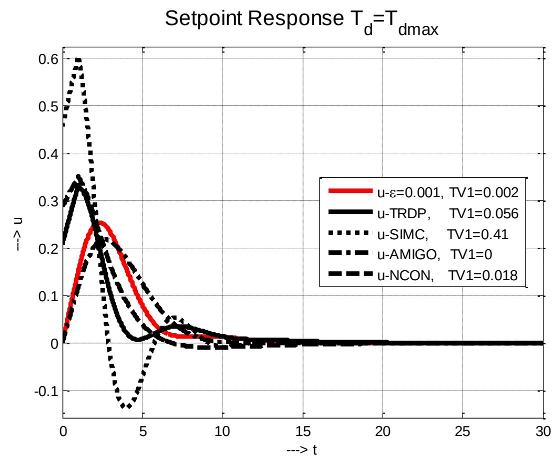


Max K_i - Setpoint Steps



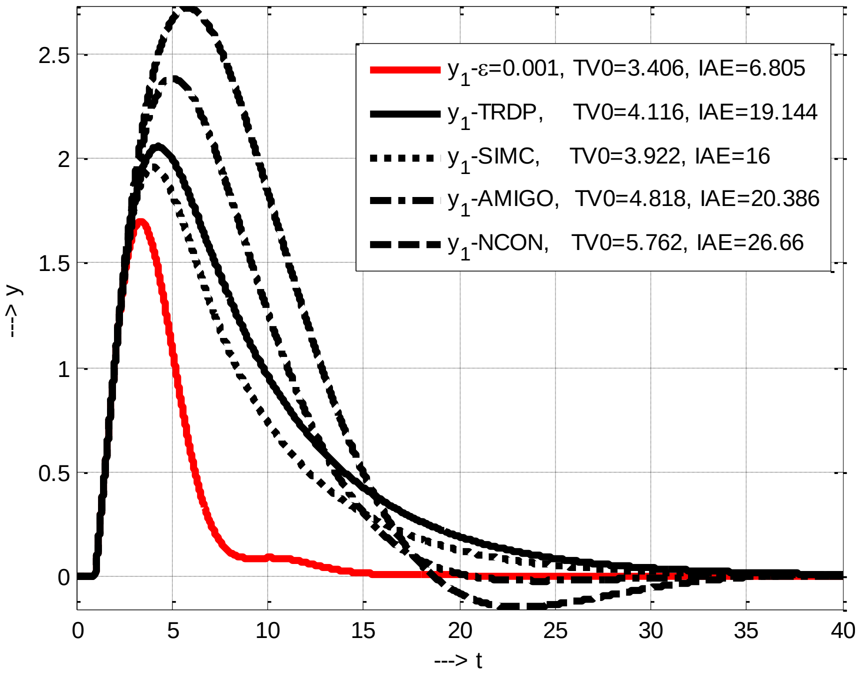


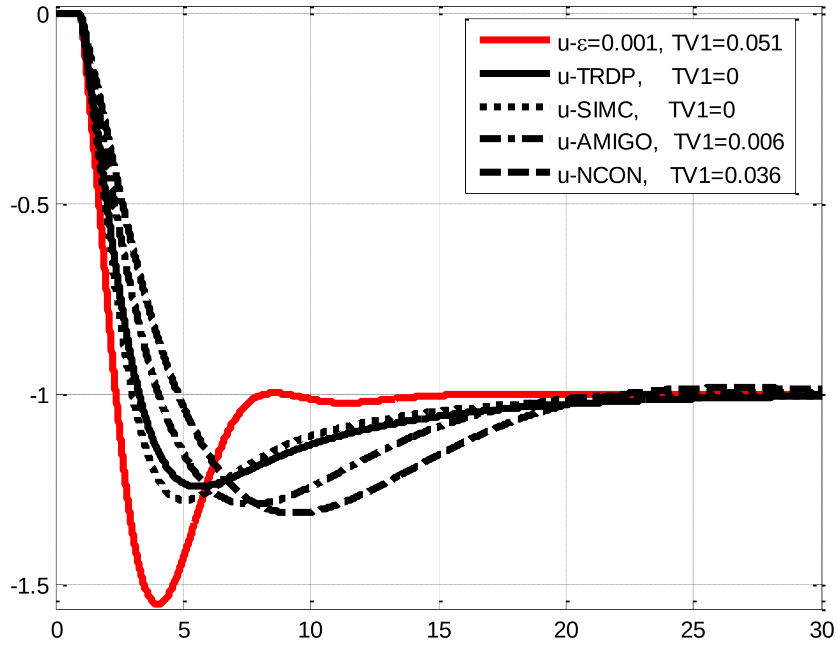




Max K_i - Disturbance Steps

But the disturbance response is clearly the best – although the design uses just PP derived from the setpoint steps – possibility for improvement



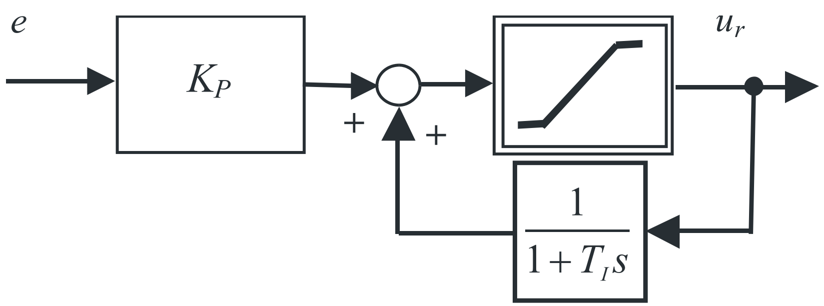


Conclusions I.

- All tested traditional methods give results that may be interesting for some applications
- But, depending on given uncertainty, for some applications the offered tuning may be too conservative
- For other ones too aggressive
- The PP method gives result exactly matching the plant uncertainty and given specifications

Conclusions II.

- Neither the anti-windup modification according to Åström-Hägglund, 1995 and use of prefilter guarantee monotonic responses in case of constraints



- Reason – the pole-zero cancellation between prefilter and closed loop does not hold exactly in constrained case
- Despite we know to tune this controller, it is not appropriate for constrained control