INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Robustné PID-regulátory s obmedzeniami
Robust Constrained PID Control
DC0: Robust Design of PI_0 Controllers

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Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.

Contents

PI$_0$ controllers

- Generic and equivalent structures for static feedforward control with reconstruction and compensation input/output disturbances
- Fundamental properties
- Performance Portrait
- Optimal and robust tuning
- Robustness Charakteristics
- Nonmodelled dynamics influence
- Disturbance compensation

Dynamical Class 0 (DC0)

- DC0 includes all controllers that ideally have monotonic setpoint step responses both at the plant and at the controller outputs
- By speeding up dynamics of these transients they converge up to rectangular steps
- PI_0 controllers belong to the simplest controllers of DC0
- They are fundamental controllers: it means that in the nominal case (without nonmodelled dynamics) their output may be arbitrarily speeded up (up to rectangular steps at the controller, or plant outputs), i.e. they fulfill conditions:

Fundamental Controller of DC0

- For the closed loop poles

$$-\infty < \alpha_2 < \alpha_1 < 0$$

- the normalised setpoint responses corresponding to zero initial conditions and to a step *w*(*t*)

$$\bar{y}(\alpha_i, t) = y(\alpha_i, t) / w(t)$$

- satisfiy conditions

$$1 > \bar{y}(\alpha_2, t) > \bar{y}(\alpha_1, t) > 0\ ;\ \ \forall t > 0\ ;$$

$$\lim_{t \to \infty} \bar{y}(\alpha_i, t) = 1\ ;\ \ i = 1 \text{ or } 2$$

Fundamental Controllers of DC0

- Similar importance as the Mendelejev Periodic Table of elements in chemistry
- DC0 represents the first row of the Table of fundamental PID controllers and includes controllers that are fully linear

		Dominant dynamics							
Dynamic class	I-action	K	$Ke^{-T_d s}$	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}$	$\left[\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}\right]e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	PD	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

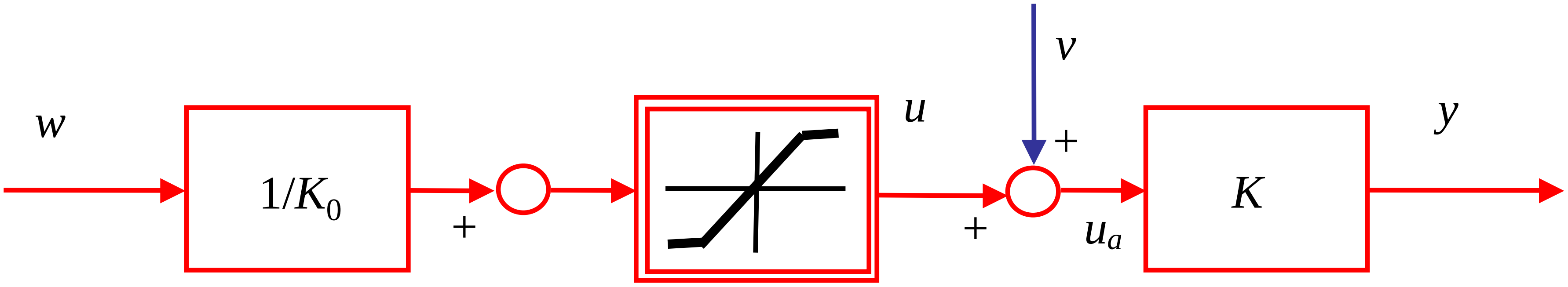
FF = static feedforward control

Pr = predictive (dead time) controllers

Fundamental Controllers of DC0

- The core structure for the setpoint following in DC0 is known as:

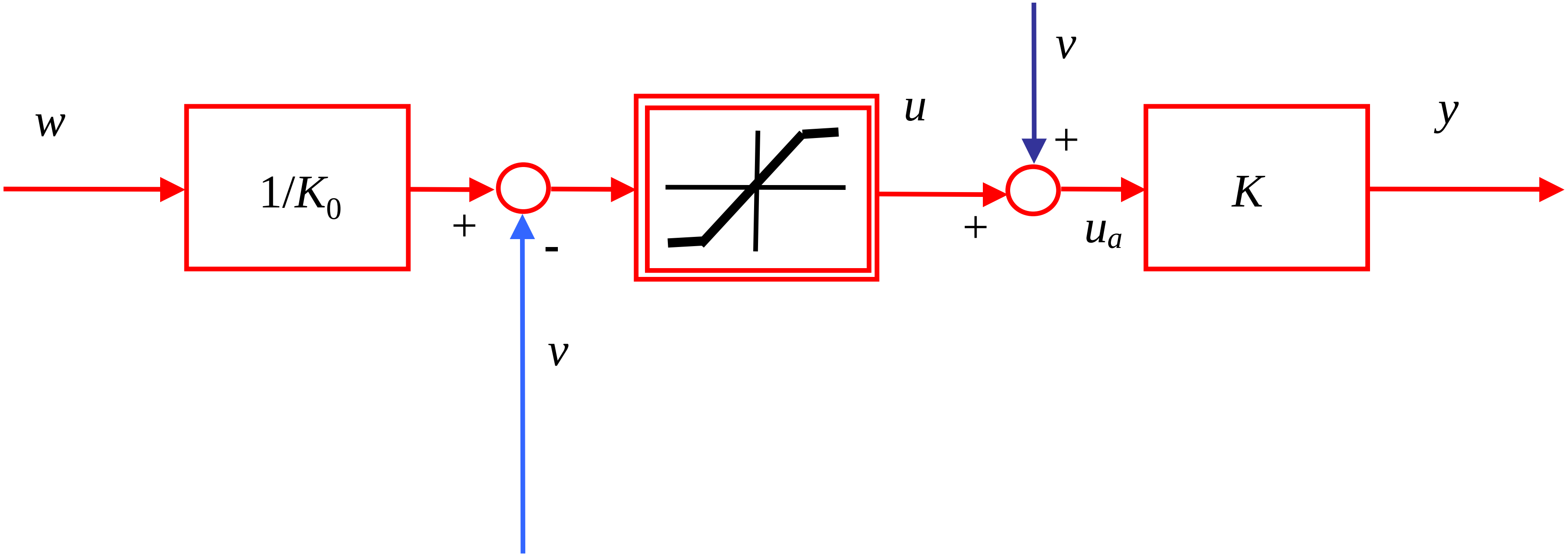
Static feedforward control



Fundamental Controllers of DC0

- A measurable input disturbance can be compensated by a counteracting signal at the controller output

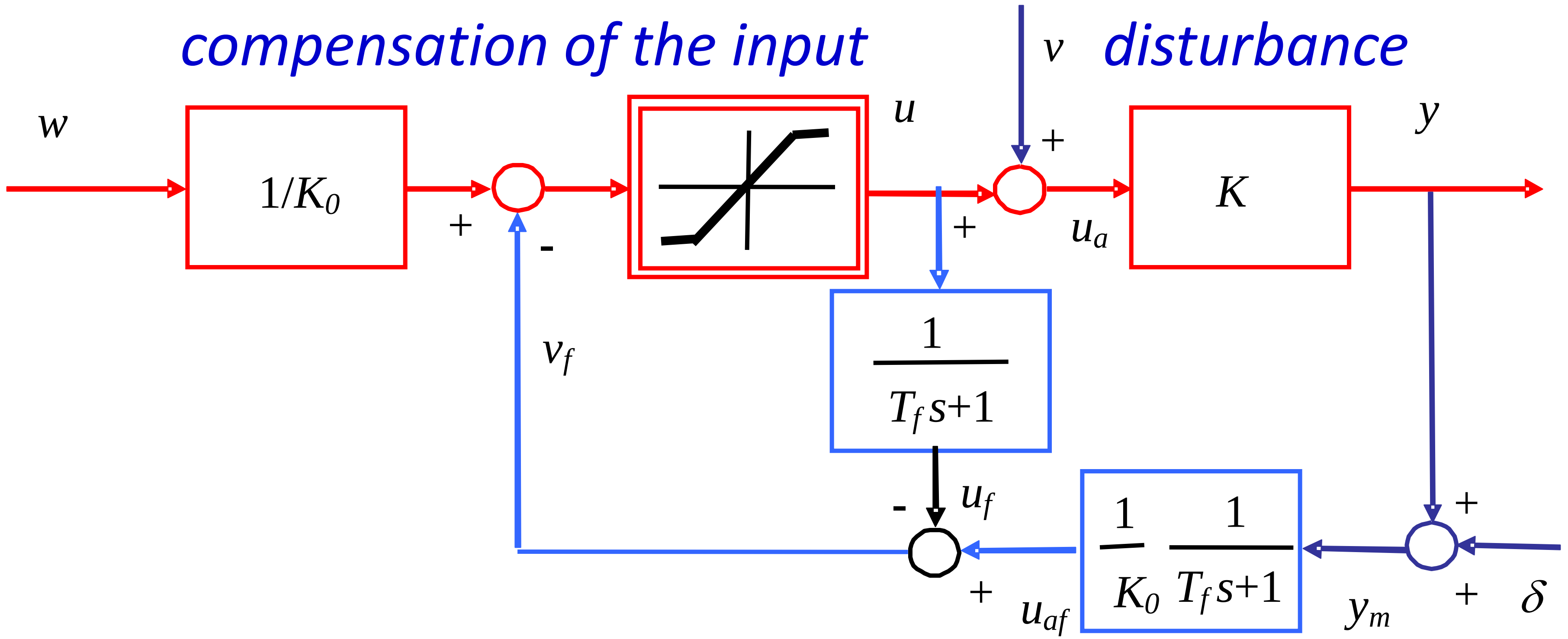
Static feedforward control with compensation of the input disturbance



l_0-controller - generic structure

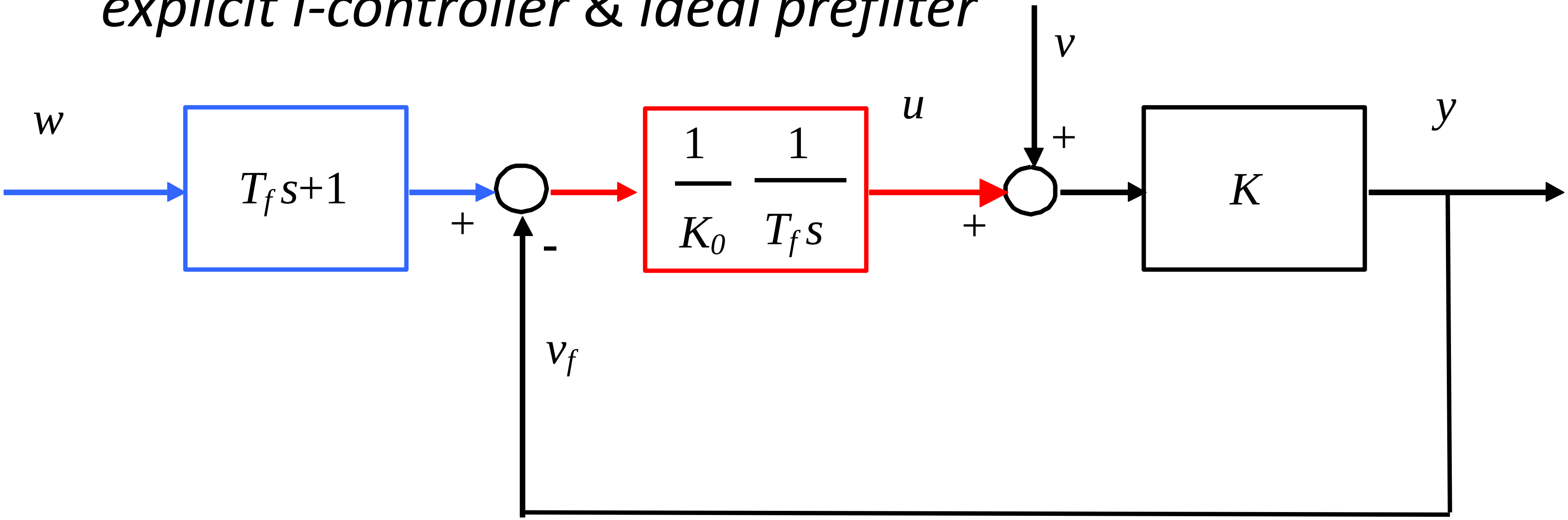
- A nonmeasurable input disturbance can be reconstructed by a **Disturbance Observer**

Static feedforward control with reconstruction and compensation of the input disturbance



I_0-controller – equivalent structure

- The generic structure with the Disturbance Observer may be transformed to an equivalent structure with *explicit I-controller & ideal prefilter*

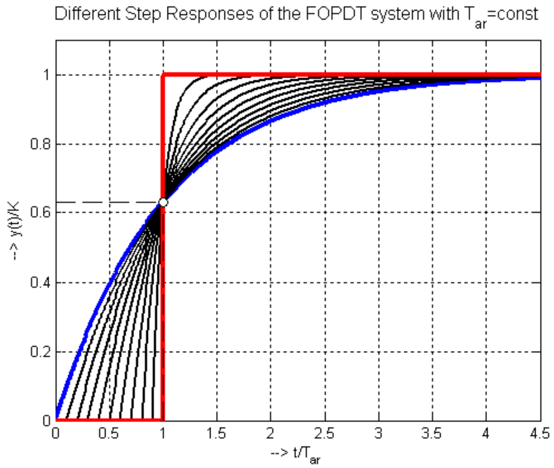


Omitting prefilter = filtered response continuous for *t* = 0

It will be denoted as **FI_0 controller = I controller**

PI_0 and PrI_0 Controllers

- I_0 controllers are based on the *simplest* plant approximation by a gain (memoryless plant)
- By approximating the plant dynamics by a time constant, or by a dead time it is possible to derive PI_0 and Predictive I_0 (PrI_0) controllers
- Both approximations corresponds to evaluating the *Average Residence Time* by measuring e.g. the integral area over the setpoint step response



PI_0 Controllers

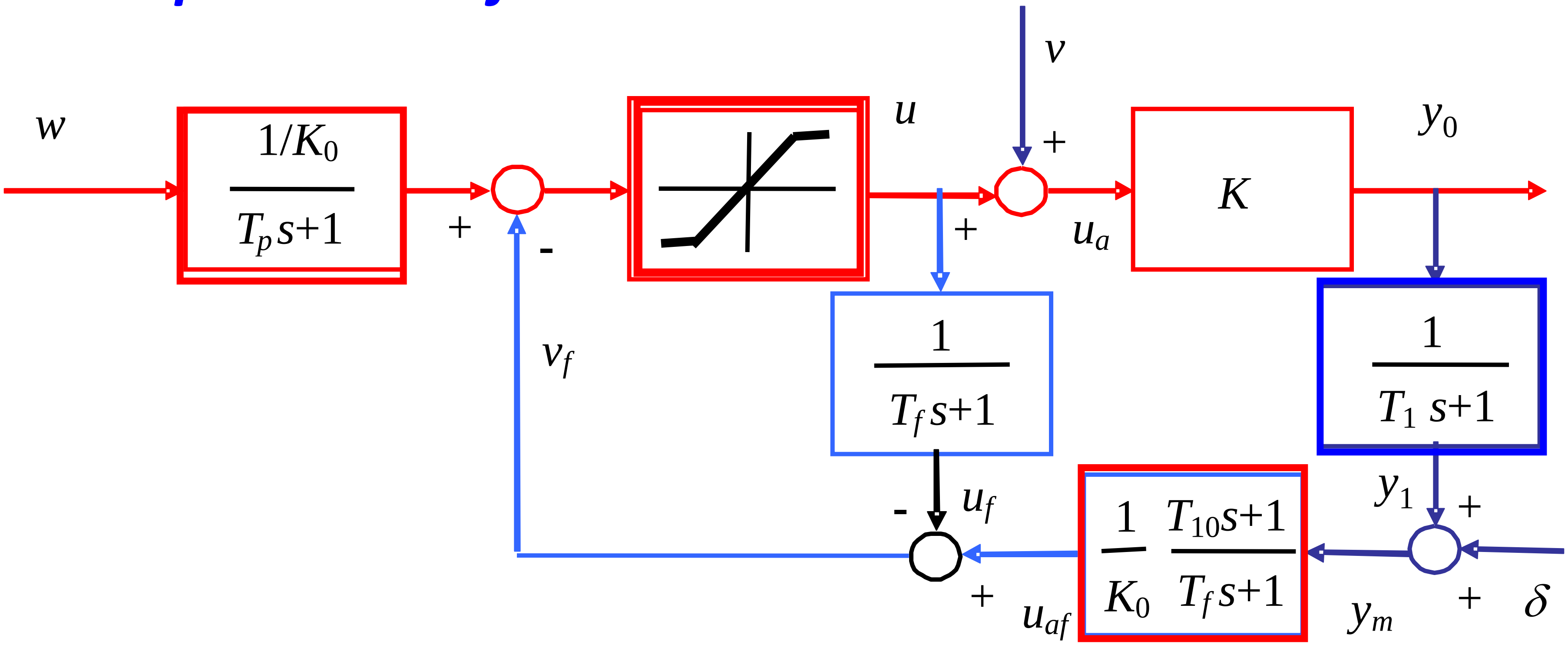
- Static feedforward control
- Extended possibly by prefilter (time constant T_p) to achieve continuous control changes after setpoint steps (increased robustness)
- Two different generic schemes for reconstruction and compensation of input, or output disturbances with disturbance filter T_f
- Compensation of dominant loop time constant
- Use of the parralel plant model (IMC like structure), or of the inverse plant model (Disturbance Observer structure)
- Monotonic signals at the plant input and output

PI$_0$-IM controller (with inverse model) generic fundamental structure

Static feedforward control ***+***

+reconstruction & compensation of input disturbance ***+***

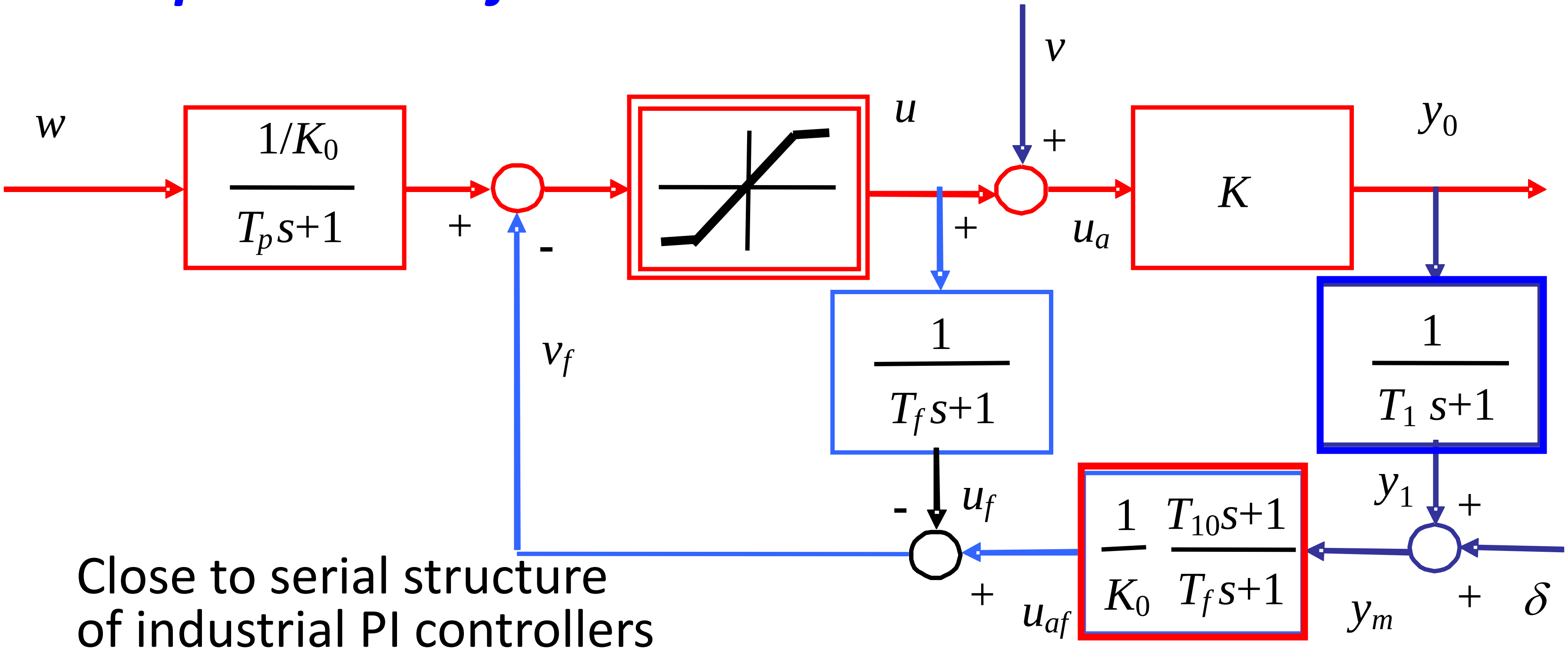
compensation of dominant time constant



Static feedforward control ***+***

+reconstruction & compensation of input disturbance ***+***

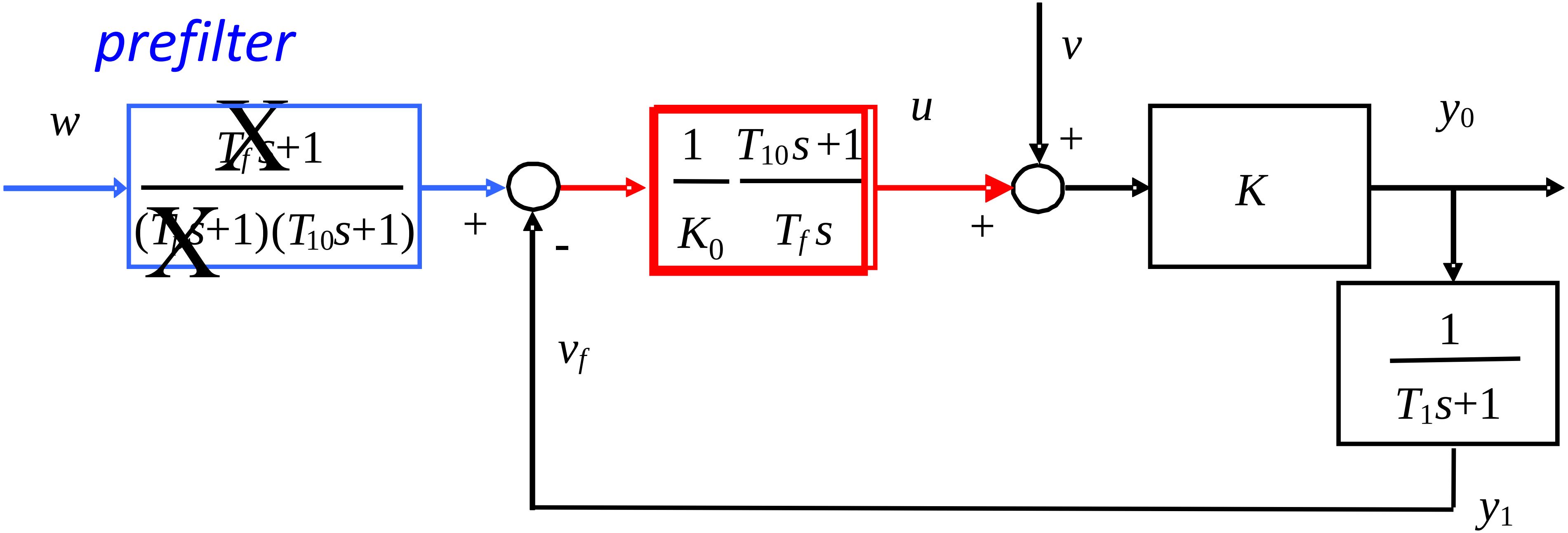
compensation of dominant time constant



Close to serial structure
of industrial PI controllers

PI_0-IM : Equivalent structure with prefilter

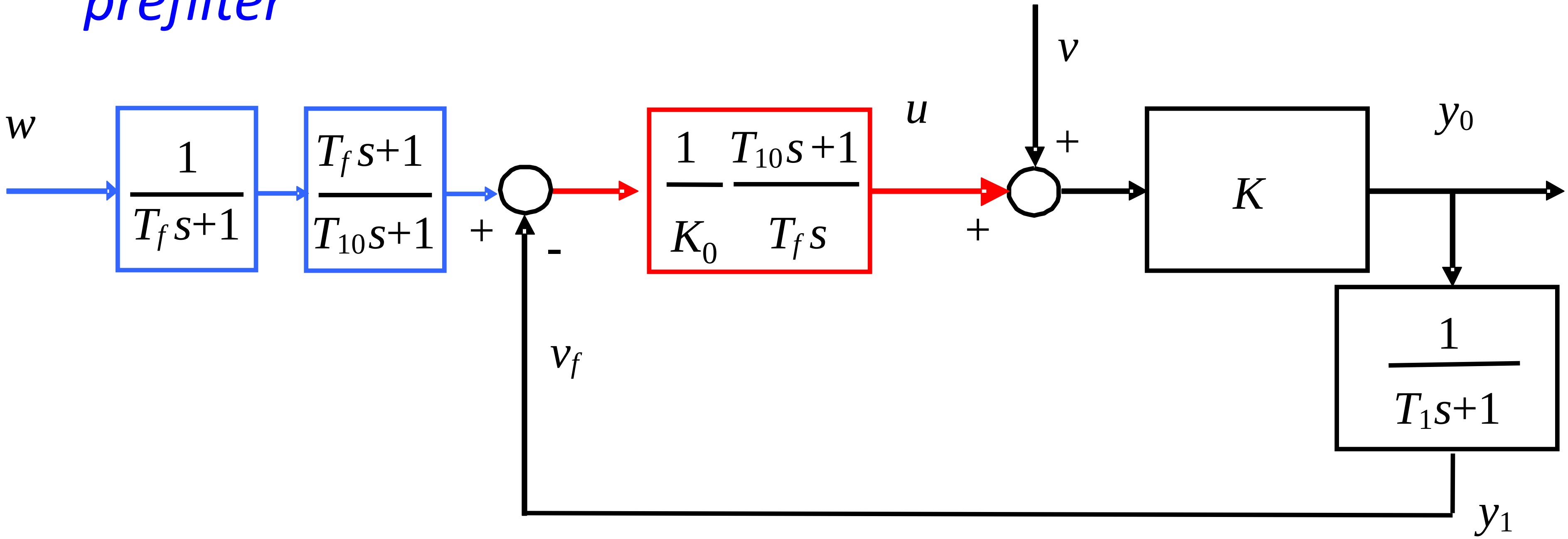
Controller with equivalent prefilter



– the most frequently used structure in literature

– $T_p=T_f$ => filtered control continuous for t = 0 – controller with error acting on I only

Controller with equivalent prefilter



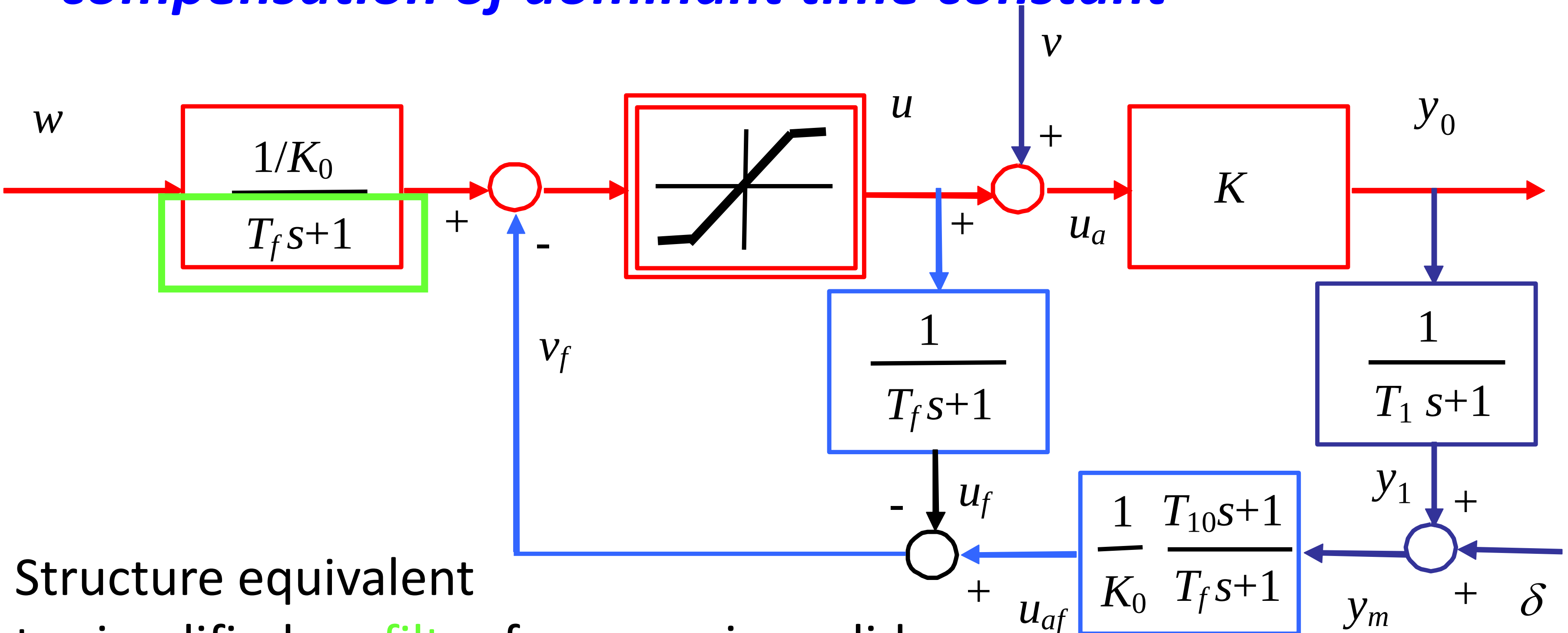
– ommitting prefilter numerator = use of prefilter with $T_p=T_f$ in the generic structure

PI_0-IM controller with prefilter generic fundamental structure

Static feedforward control +

+reconstruction & compensation of input disturbance +

compensation of dominant time constant



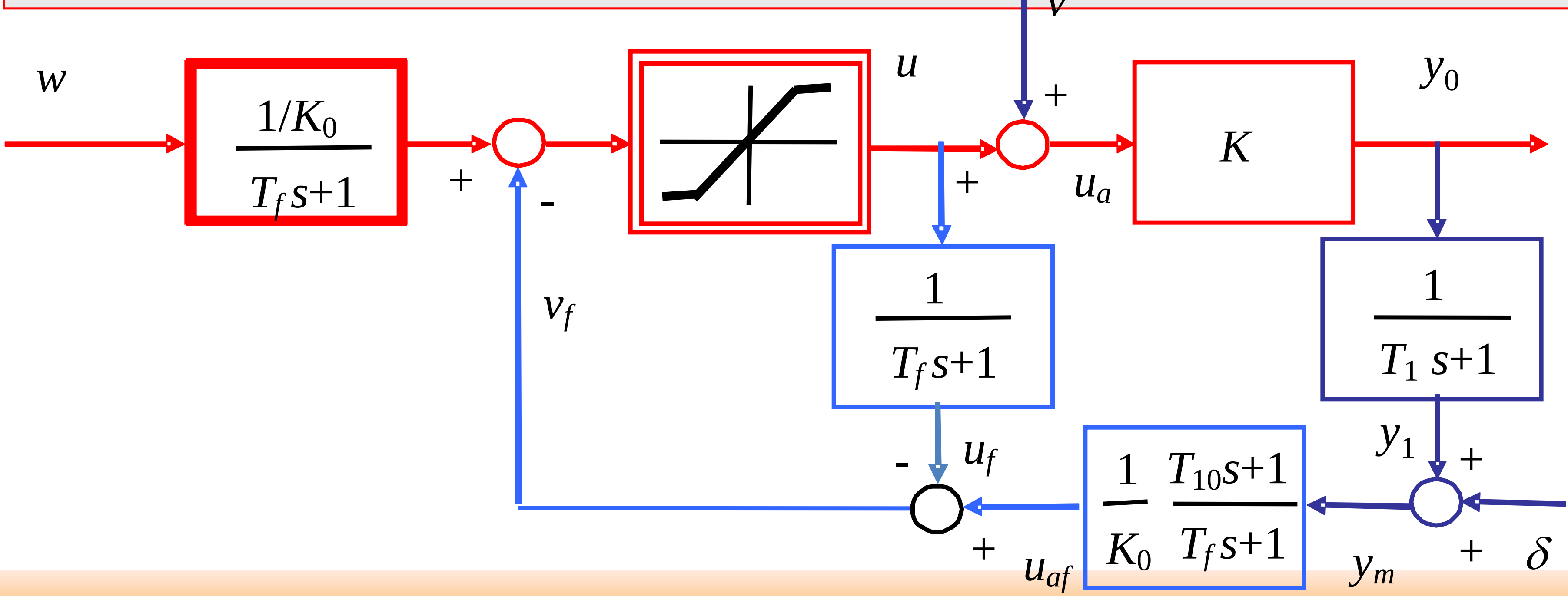
Structure equivalent
to simplified prefilter from previous slide

PI_0-IM controller: Output y_0 (+prefilter)

Prefilter $T_p = T_f$

$$F_{w0}(s) = \frac{Y_0(s)}{W(s)} = \frac{K(1+T_f s)(1+T_1 s)}{K_0 T_f T_1 s^2 + (K_0 T_f + K T_{10})s + K} \Rightarrow$$

$$F_{w0}(p) = \frac{(1+\tau_f p)(1+\tau_1 p)}{\kappa \tau_f \tau_1 p^2 + (\kappa \tau_f + 1)p + 1} \; ; \quad p = T_{10} s \; ; \; \tau_1 = \frac{T_1}{T_{10}} \; ; \; \tau_f = \frac{T_f}{T_{10}} \; ; \; \kappa = \frac{K_0}{K}$$

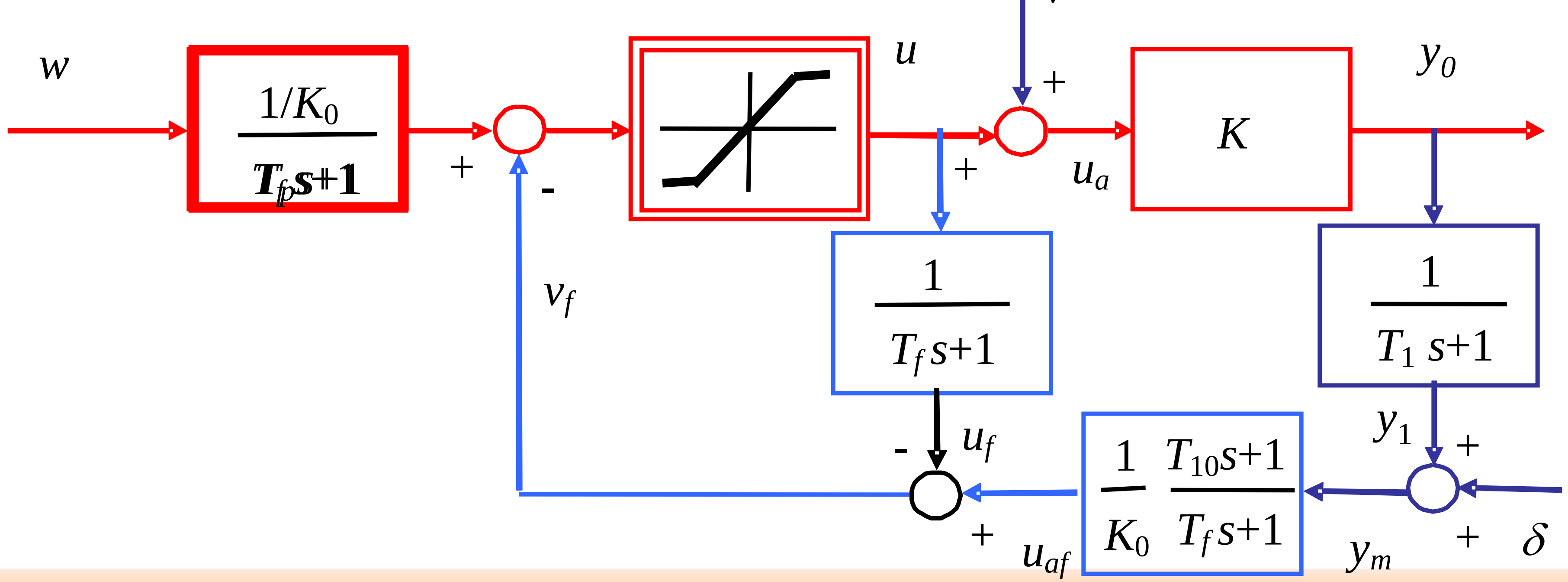


PI_0-IM controller: Output y_1 (+prefilter)

Prefilter $T_p = T_f$

$$F_{w1}(s) = \frac{Y_1(s)}{W(s)} = \frac{K(1+T_f s)}{(1+T_p s)\left[K_0 T_f T_1 s^2 + (K_0 T_f + K T_{10})s + K\right]} \Rightarrow$$

$$F_{w1}(p) = \frac{1}{\kappa \tau_f \tau_1 p^2 + (\kappa \tau_f + 1)p + 1} \; ; \quad p = T_{10} s \; ; \; \tau_1 = \frac{T_1}{T_{10}} \; ; \; \tau_f = \frac{T_f}{T_{10}} \; ; \; \kappa = \frac{K_0}{K}$$

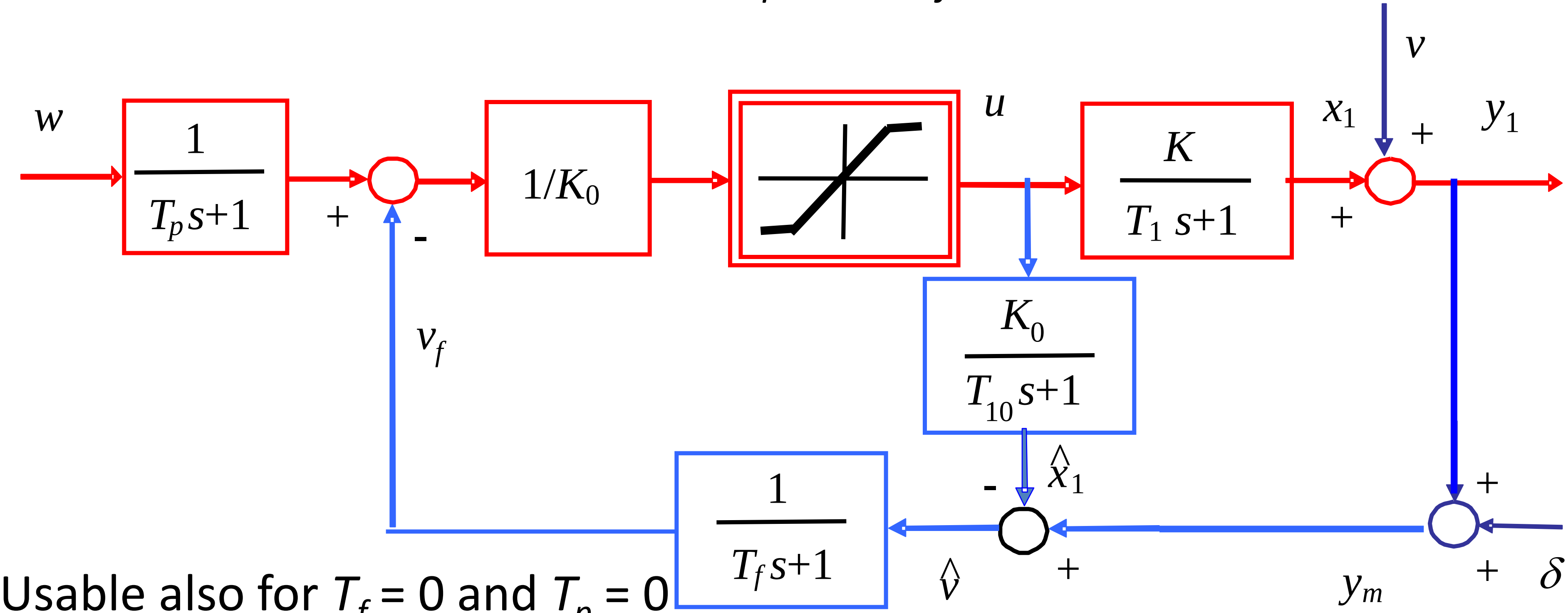


PI_0-PM controller (with paralel model)

Static feedforward control+

+reconstruction+compenzation of output disturbance -

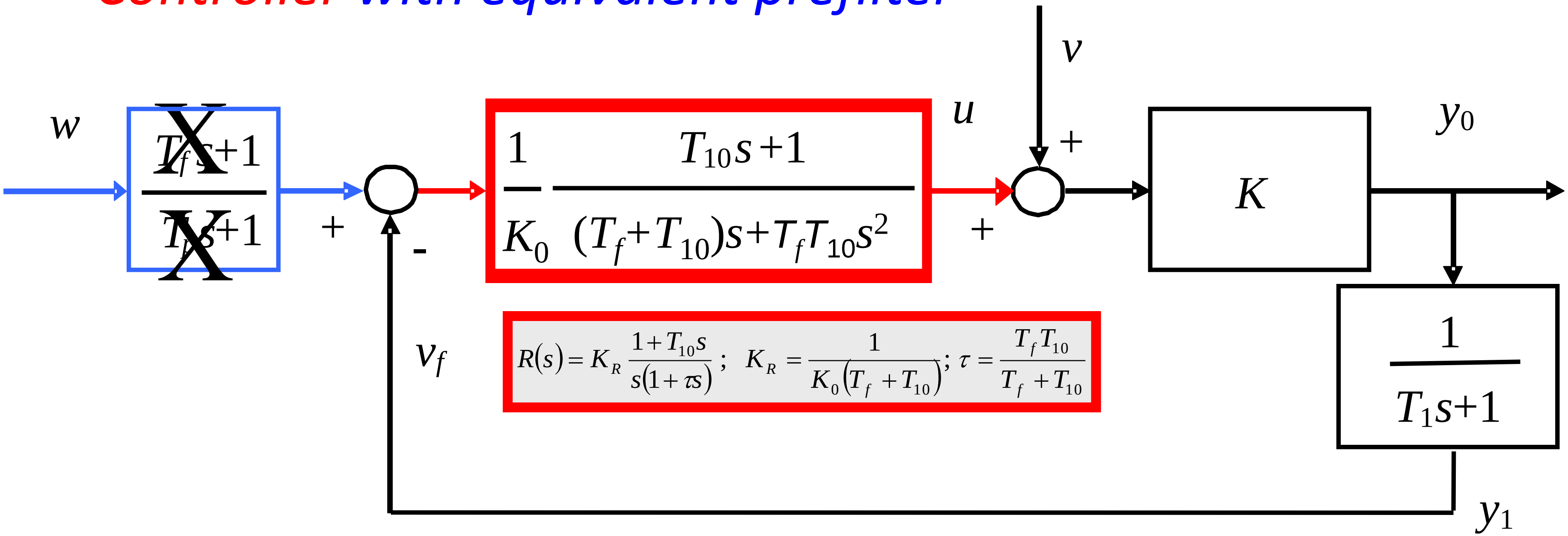
2DOF IMC structure with T_p and T_f



Usable also for $T_f = 0$ and $T_p = 0$

PI_0-PM : Equivalen structure

Controller with equivalent prefilter



– structure most frequently used in literature

– $T_p=T_f$ => filtered control continuous for t = 0 – controller with error acting on I only

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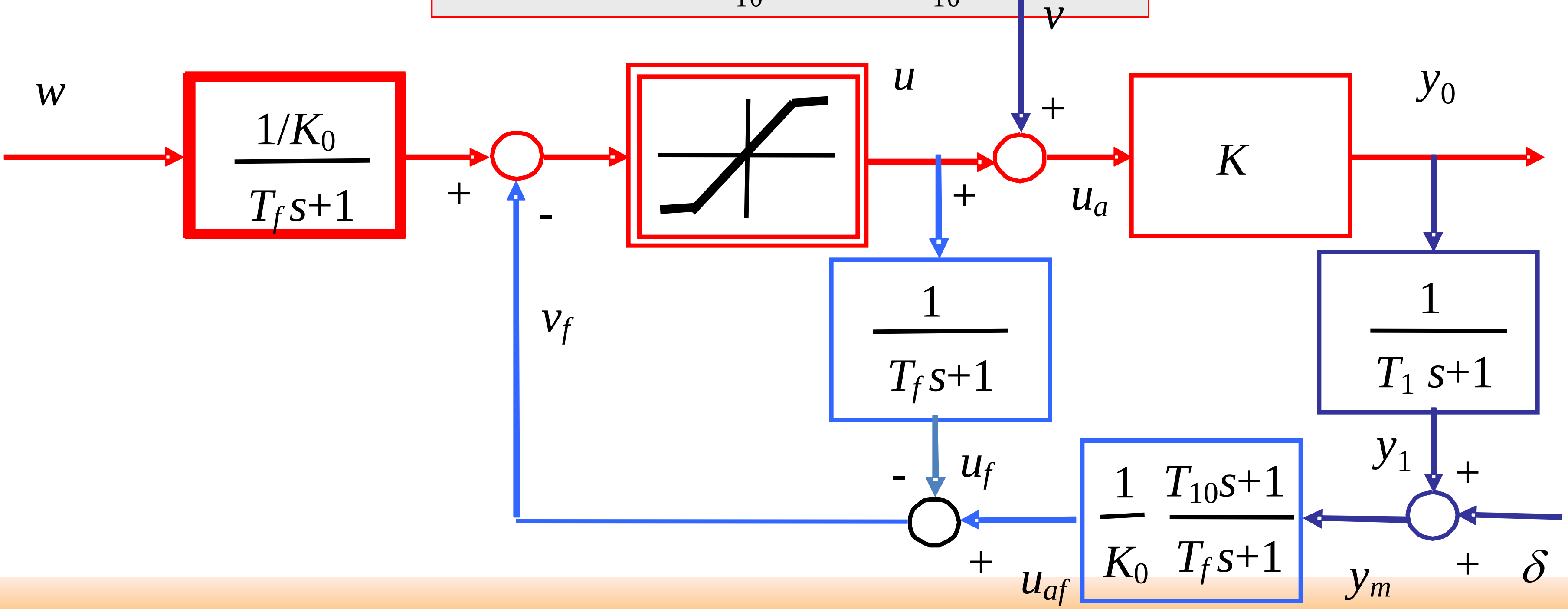
PP of the PI_0-IM controller (+prefilter):

$$F_{w0}(p)=\frac{1+\tau_1 p}{\kappa\tau_f\tau_1 p^2+(\kappa\tau_f+1)p+1}\;;\quad F_{w1}(p)=\frac{1}{\kappa\tau_f\tau_1 p^2+(\kappa\tau_f+1)p+1}$$

$$p=T_{10}s\;;\;\tau_1=\frac{T_1}{T_{10}}\;;\;\tau_f=\frac{T_f}{T_{10}}\;;\;\kappa=\frac{K_0}{K}$$

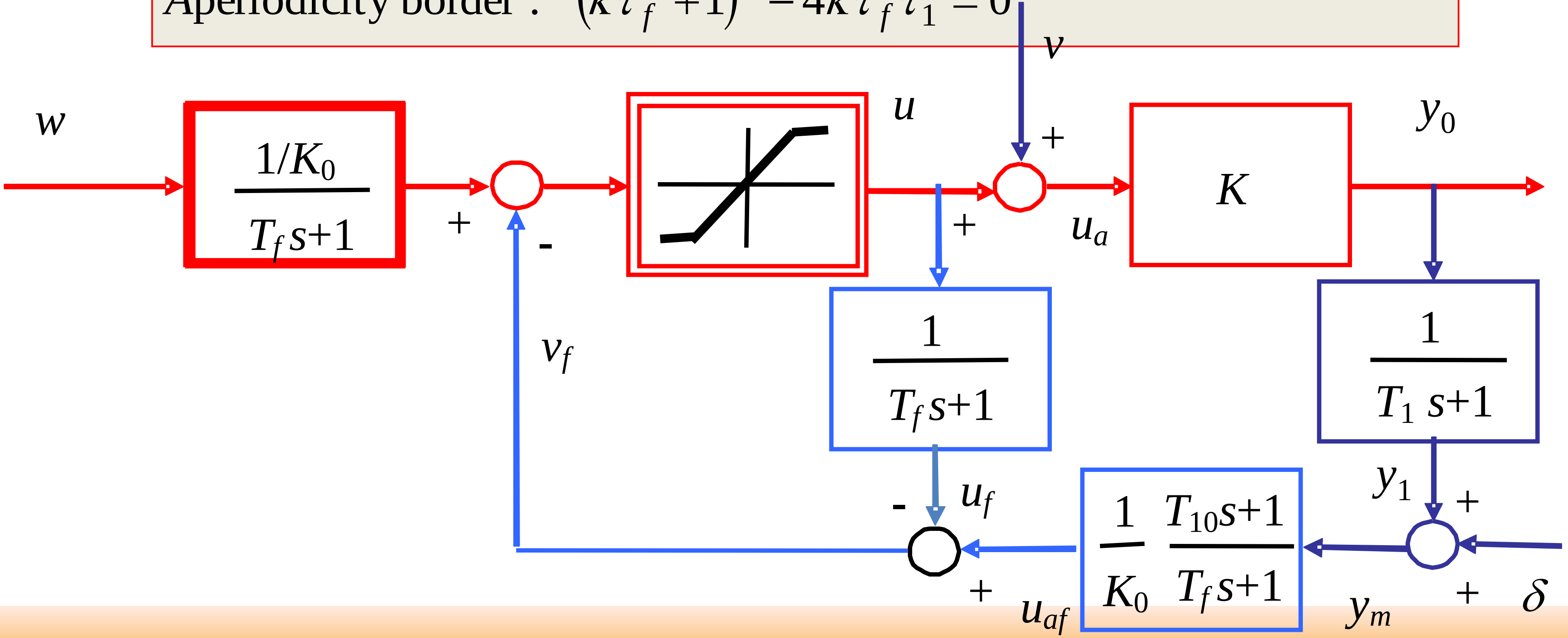


PP of the PI_0-IM controller (+prefilter):

Analytical tuning - *aperiodicity border*

$$\kappa\tau_f\tau_1 p^2 + (\kappa\tau_f + 1)p + 1 \Rightarrow p_{1,2} = \frac{-(\kappa\tau_f + 1) \pm \sqrt{(\kappa\tau_f + 1)^2 - 4\kappa\tau_f\tau_1}}{2\kappa\tau_f\tau_1}$$

$$\text{Aperiodicity border: } \quad (\kappa\tau_f + 1)^2 - 4\kappa\tau_f\tau_1 = 0$$



PP of the PI_0-IM controller (+prefilter):

Generating Performance Portrait – first two steps:

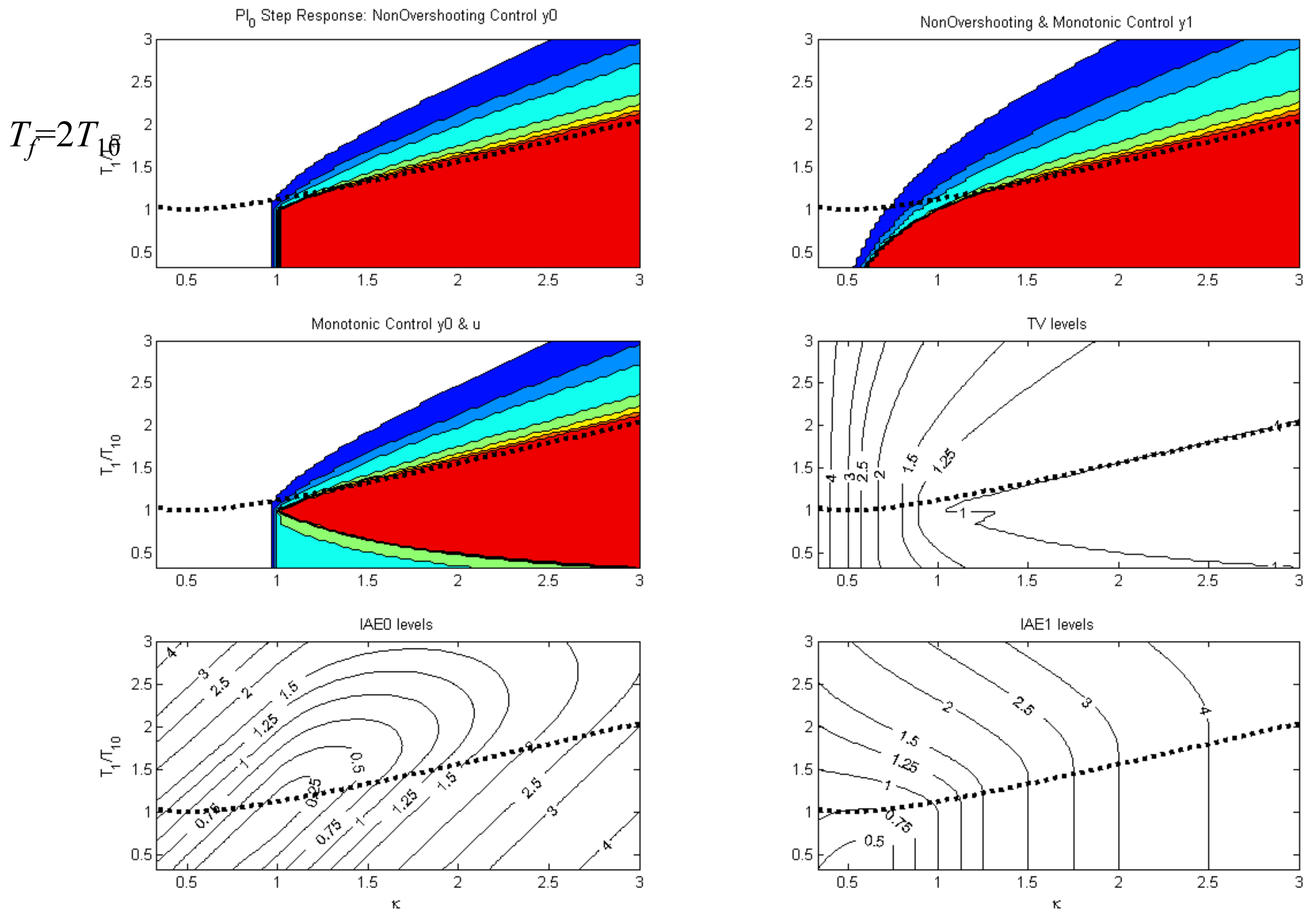
1. Mapping properties in 3D with variables (κ, τ_1, τ_f) – over grid of defined points

2. Visualisation of observed properties:

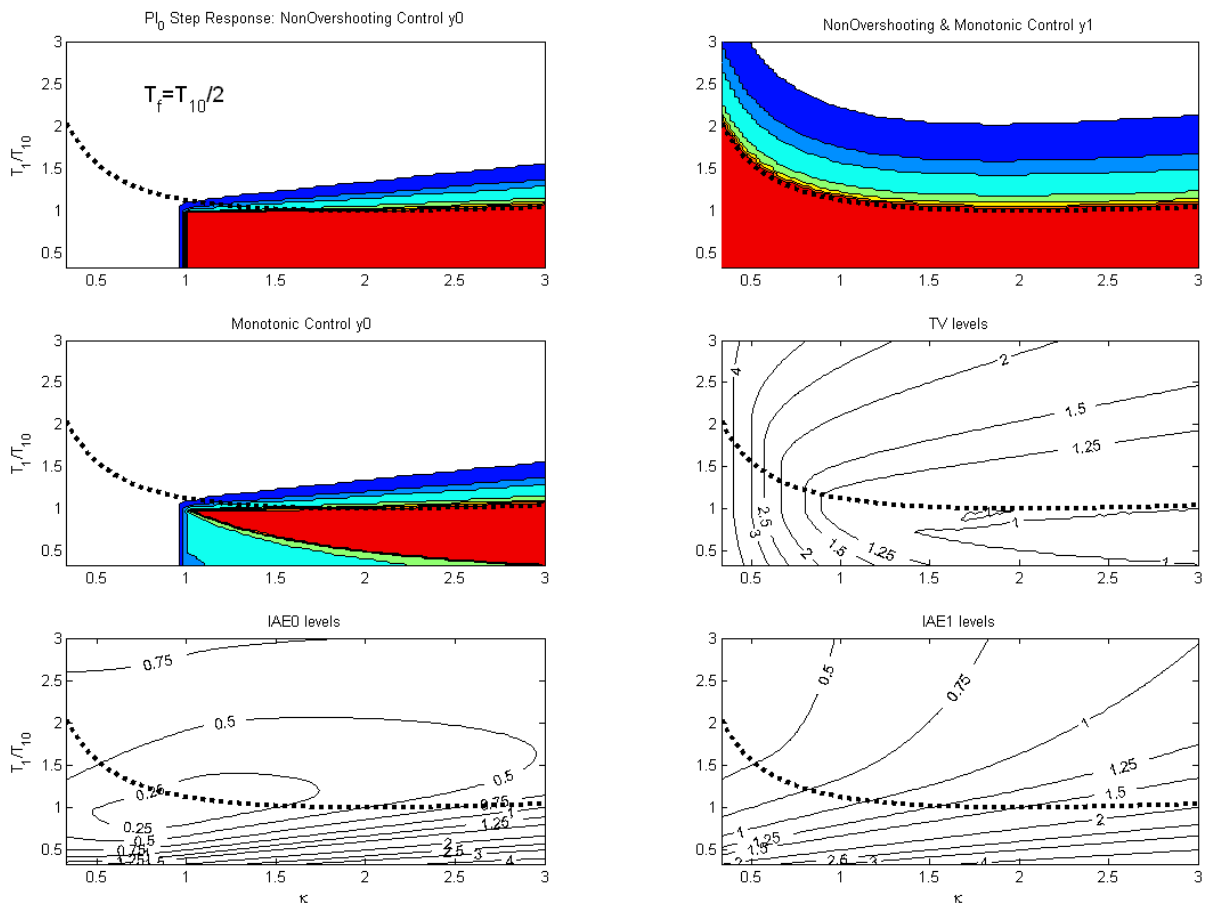
- TV or TV_0 values for plant input and output,
- maximal overshooting for outputs y_0 and y_1 ,
- deviations from monoticity for input u and outputs y_0 , y_1 ,
- IAE values for outputs y_0 and y_1 etc.

Colors used for denoting areas with amplitude deviations not exceeding defined values, integral deviations shown by contours in 2D planes, e.g. for τ_f=*const*.

Performance Portrait, T_f=2T_{10}



Performance Portrait, $T_f=T_{10}/2$



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Uncertainty sets of the PI_0 controller

Interval loop parameters

$$K \in \langle K_{\min}, K_{\max} \rangle \; ; \; c_K = K_{\max} / K_{\min} \geq 1 \; ; \quad T_1 \in \langle T_{1\min}, T_{1\max} \rangle \; ; \quad c_T = T_{1\max} / T_{1\min} \geq 1 \; ;$$

Uncertainty box UB in the plane of interval parameters (κ, τ_1) in a section through the 3D space (κ, τ_1, τ_f) for $\tau_f = const$

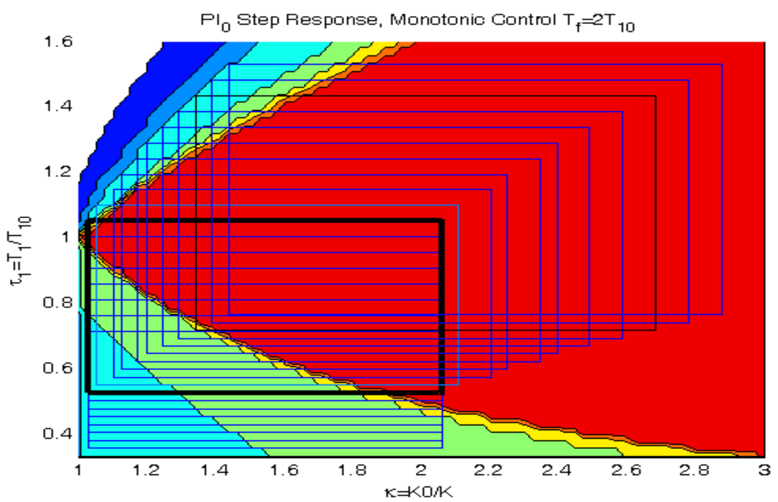
$$UB = \begin{bmatrix} \kappa_{\min} \tau_{1\max} & \kappa_{\max} \tau_{1\max} \\ \kappa_{\min}, \tau_{1\min} & \kappa_{\max} \tau_{1\min} \end{bmatrix} \; ; \quad \kappa = \frac{K_0}{K} \; ; \quad \tau_1 = \frac{T_1}{T_{10}} \; ; \quad \tau_f = \frac{T_f}{T_{10}}$$

For one interval parameter we get horizontal or vertical uncertainty line segment in the plane of parameters (κ, τ_1) in a section through the 3D space (κ, τ_1, τ_f) for $\tau_f = const$

Uncertainty sets of the PI_0 controller

Sweeping the parameter space for minimal mean IAE, ε=0.02

Sweeping in 3D space of parameters (κ,τ_1,τ_f)



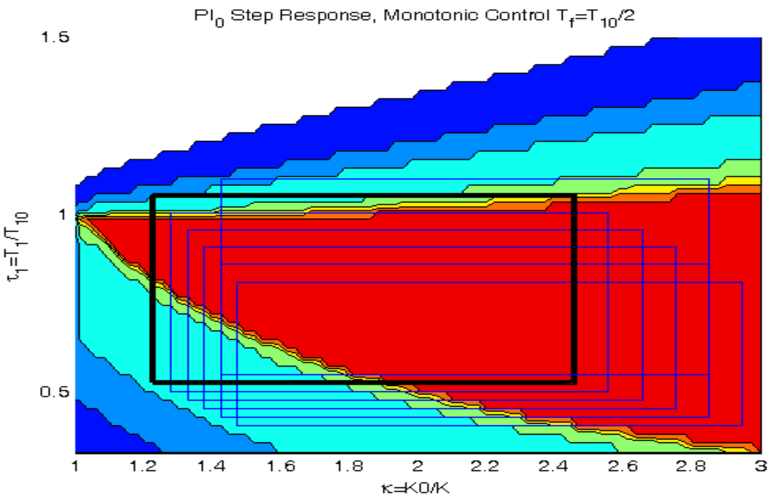
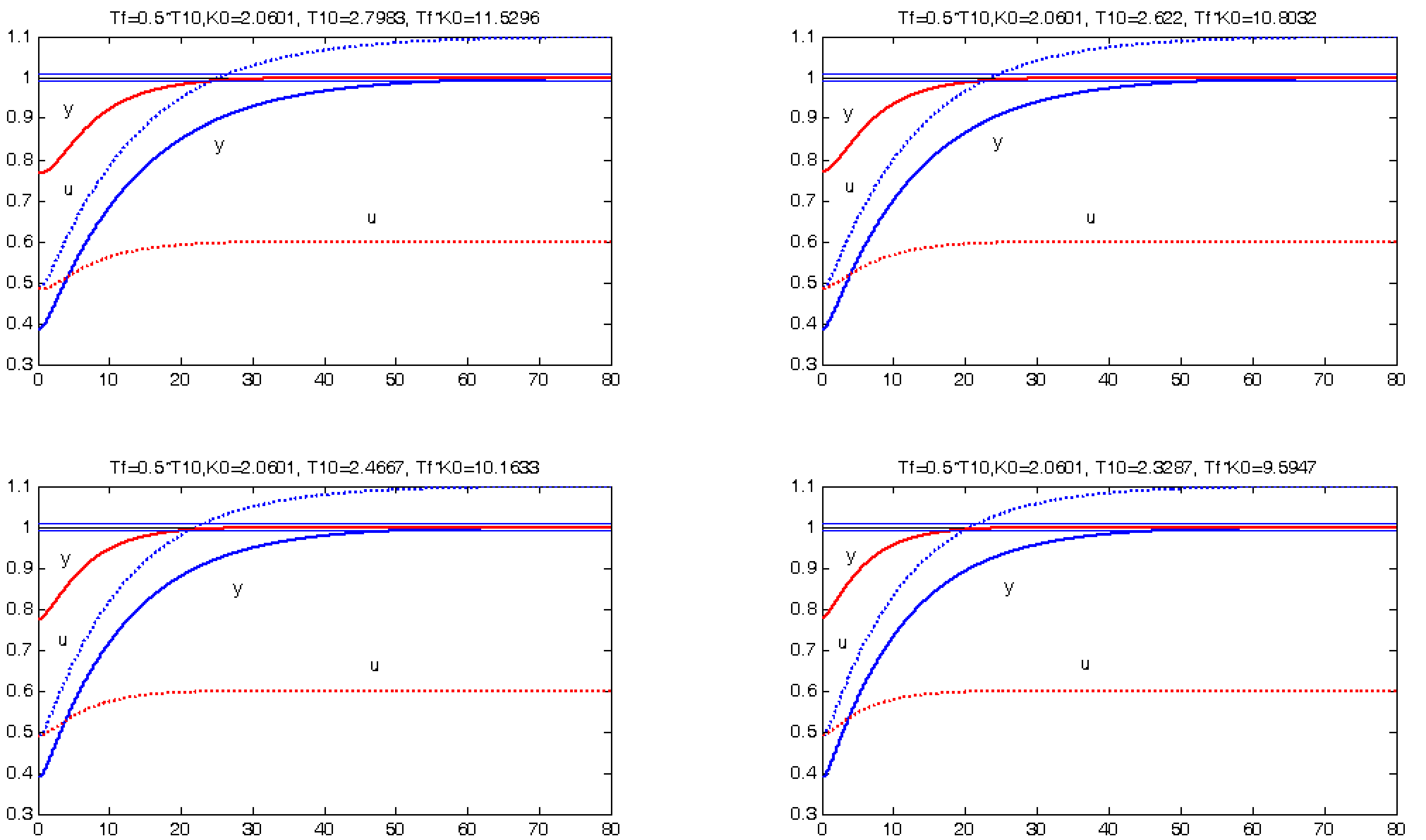


Illustration of possible sequence of found values for $T_f=const$

$K_0=2.108$; $T_{10}=1.819$; $IAE_{0min}=0.027$; $IAE_{0mean}=1.764$; $IAE_{0max}=5.04$

$K_0=2.461$; $T_{10}=1.897$; $IAE_{0min}=0.084$; $IAE_{0mean}=0.643$; $IAE_{0max}=1.22$

Verification of found optimal responses



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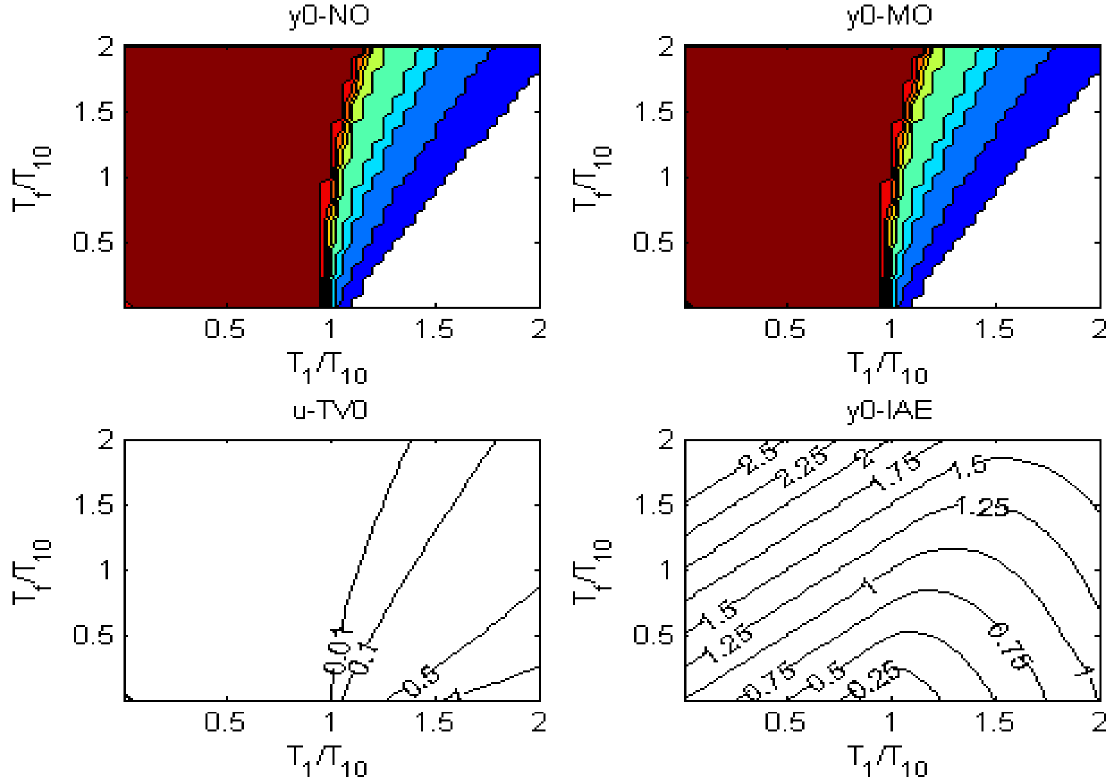
PI_0 – output y_0, 2D portrait, $K_0=K_{max}$

The fastest transients correspond to $T_f/T_{10} \rightarrow 0$ and $T_1/T_{10} \rightarrow 1$

Optimal tuning for ε=0: $T_f/T_{10} \rightarrow 0$ a $T_1=T_{1max}$

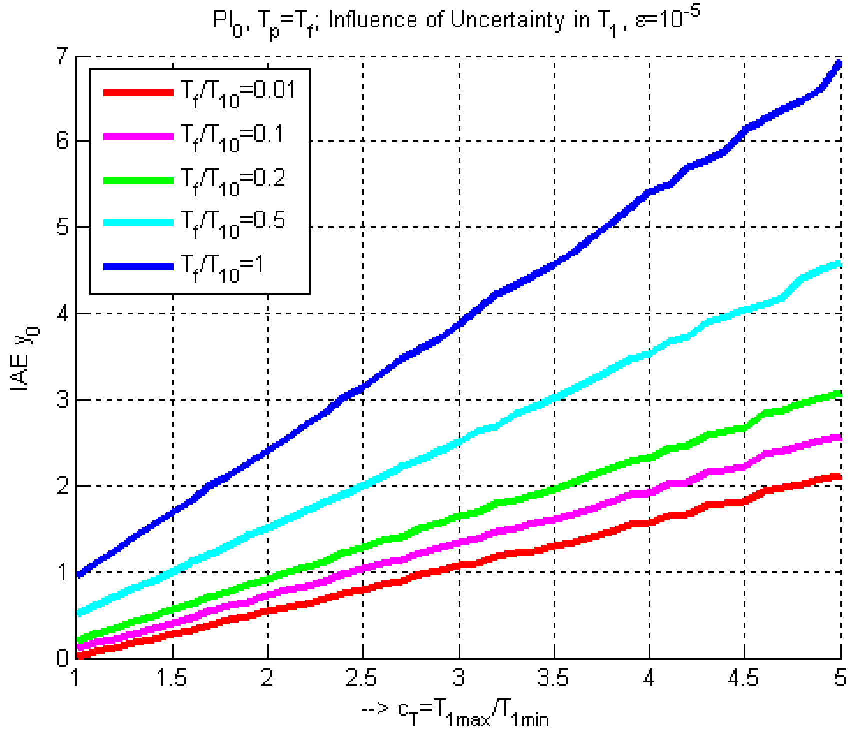
Min value of T_f depends on the disturbance response

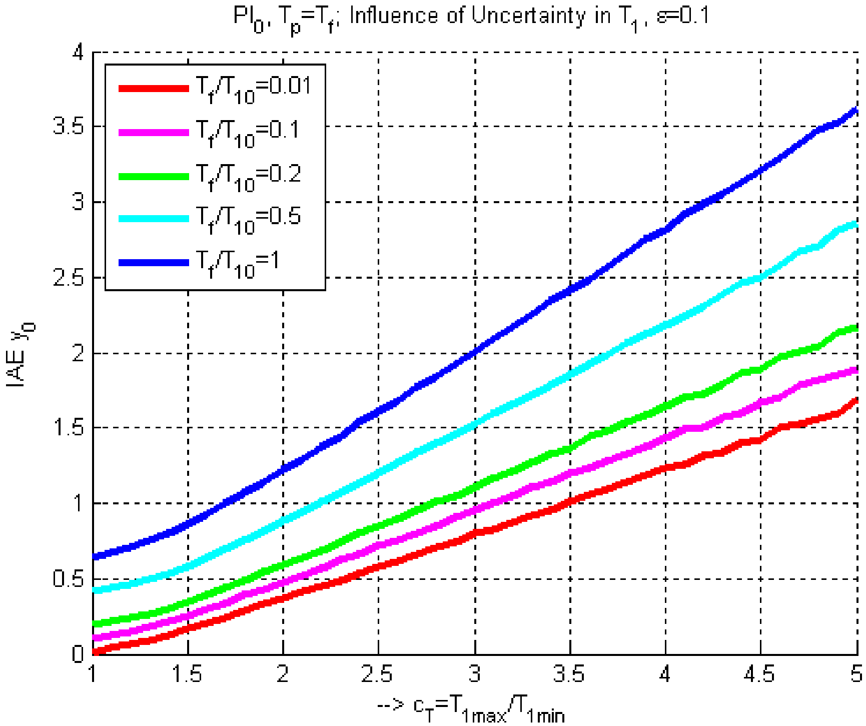


PI_0 - controller
influence of uncertainty in T_1

K=K_0, parameter T_f and different tolerated deviations

Sensitivity may be decreased by working with lower T_f and by accepting larger deviations (increasing ε)

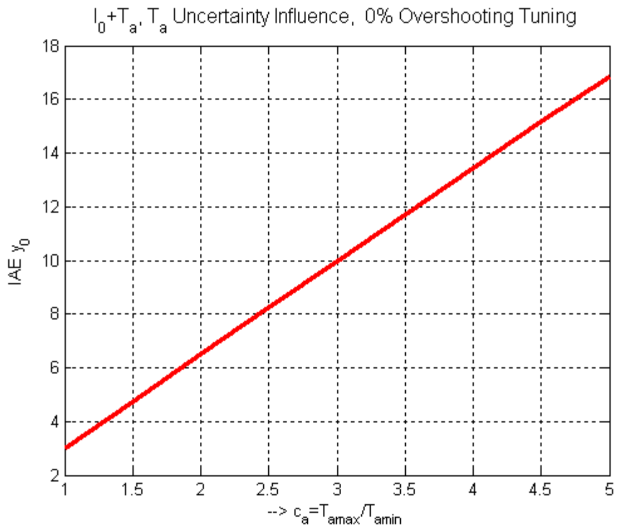


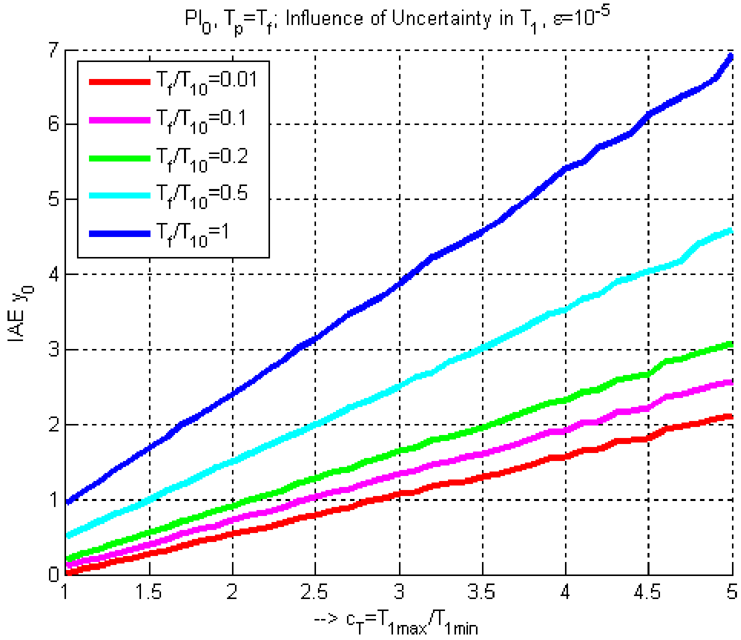


PI_0 versus I_0 controller
influence of the uncertainty in T_1

Decreased sensitivity on the uncertainty in determining the dominant time constant T_1 (for the I_0 denoted as T_a) is for the PI_0 controller well to see for MO transients ($\varepsilon=10^{-5}$)

Sensitivity decrease by decreasing T_f – always restricted due to non-modelled dynamics

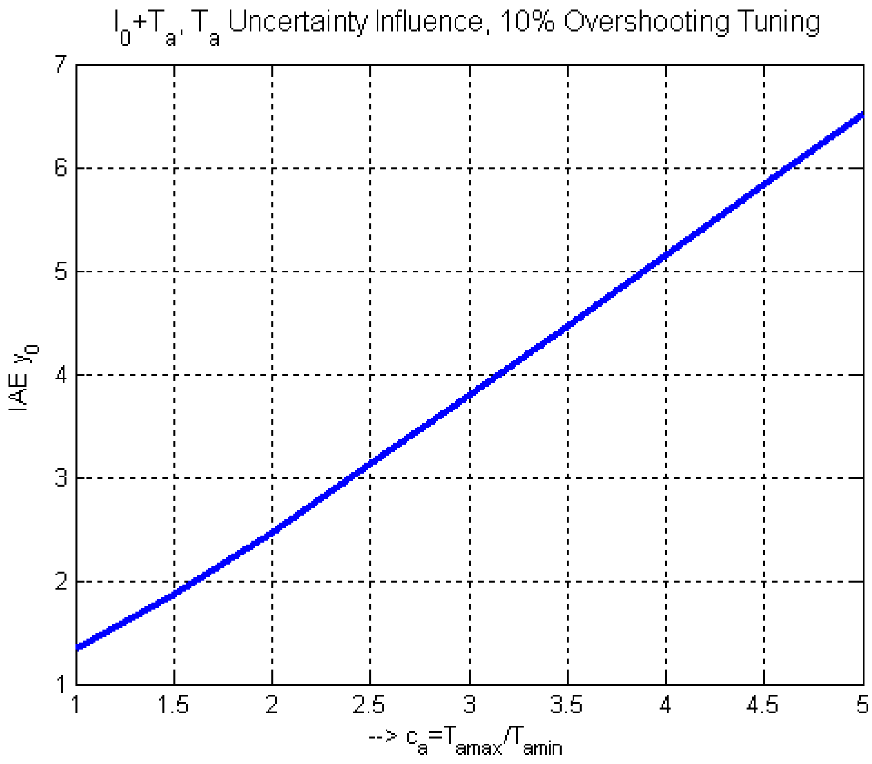


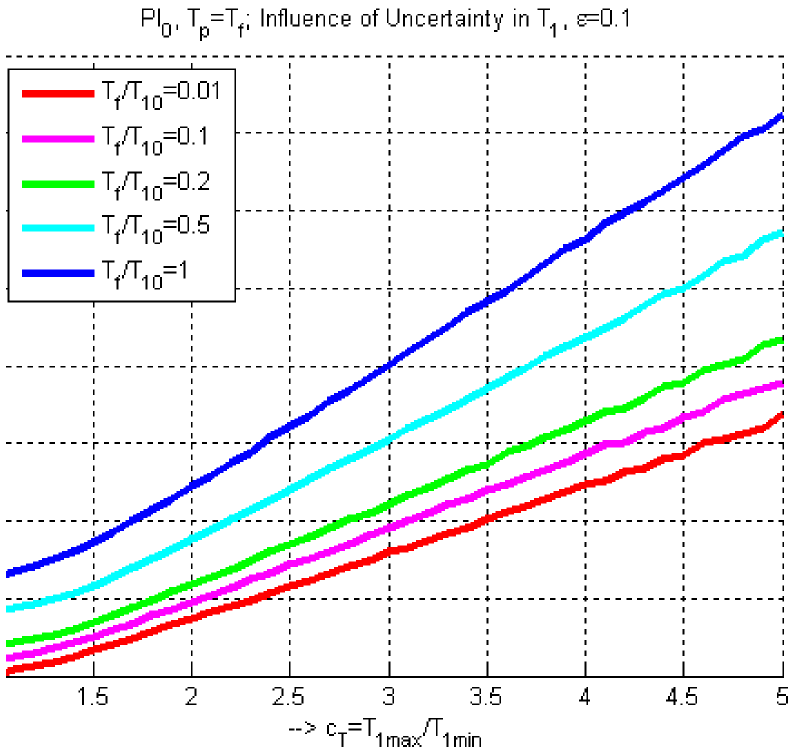


PI_0 or I_0 controller?
Influence of the uncertainty in T_1

Sensitivity on uncertainty may be decreased by increasing tolerated deviations (ε=0.1)

For PI_0 much more than for I_0

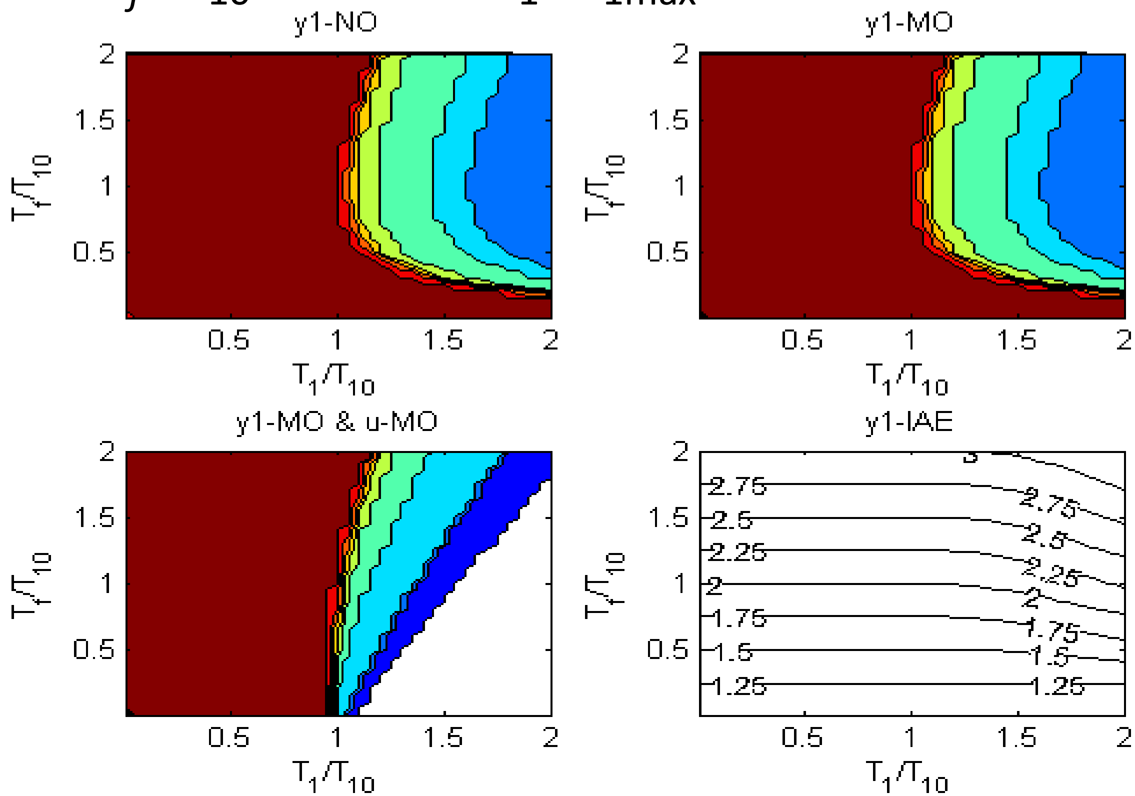




PI_0 – output y_1, 2D portrait, $K_0=K_{max}$

The fastest transients correspond to T_f/T_{10} ➔0, not strongly depending on T_1/T_{10}
Optimal tuning for ε=0: T_f/T_{10} ➔0 and $T_1=T_{1max}$

Min value of T_f depends on the disturbance response



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PI_0 – controller + FOPDT plant

Nominal for T_1/T_{10}= 1 a $K_0 = K_{max}$ T_f *is tuned as for I-controller*

Optimal tuning for interval parameter T_1 and output y_0 is solved in 4D (or in 3D for $K_0 = K_{max}$?) space

$$F_{w0}(s) = \frac{Y_0(s)}{W(s)} = \frac{K(1+T_1 s)e^{-T_d s}}{K_0 T_f T_1 s^2 + \left(K_0 T_f + K e^{-T_d s} T_{10}\right)s + K e^{-T_d s}}$$

$$\Rightarrow$$

$$F_{w0}(p) = \frac{(1+\tau_1 p)}{\kappa \tau_f \tau_1 p^2 e^{\tau_d p} + \left(\kappa \tau_f e^{\tau_d p} + 1\right)p + 1}$$

$$\tau_1 = \frac{T_1}{T_{10}}\ ; \quad \tau_f = \frac{T_f}{T_{10}}\ ; \quad \tau_d = \frac{T_d}{T_{10}}\ ; \quad p = T_{10} s\ ; \quad \kappa = \kappa_{\min} = \frac{K_0}{K_{\max}}\ ;$$

PI0 – controller + FOPDT plant

Nominal for T_1/T_{10}= 1 a $K_0 = K_{max}$ T_f *is tuned as for I-controller*

Optimal tuning for interval parameter T_1 and output y_0 is solved in 4D (or in 3D for $K_0 = K_{max}$?) space

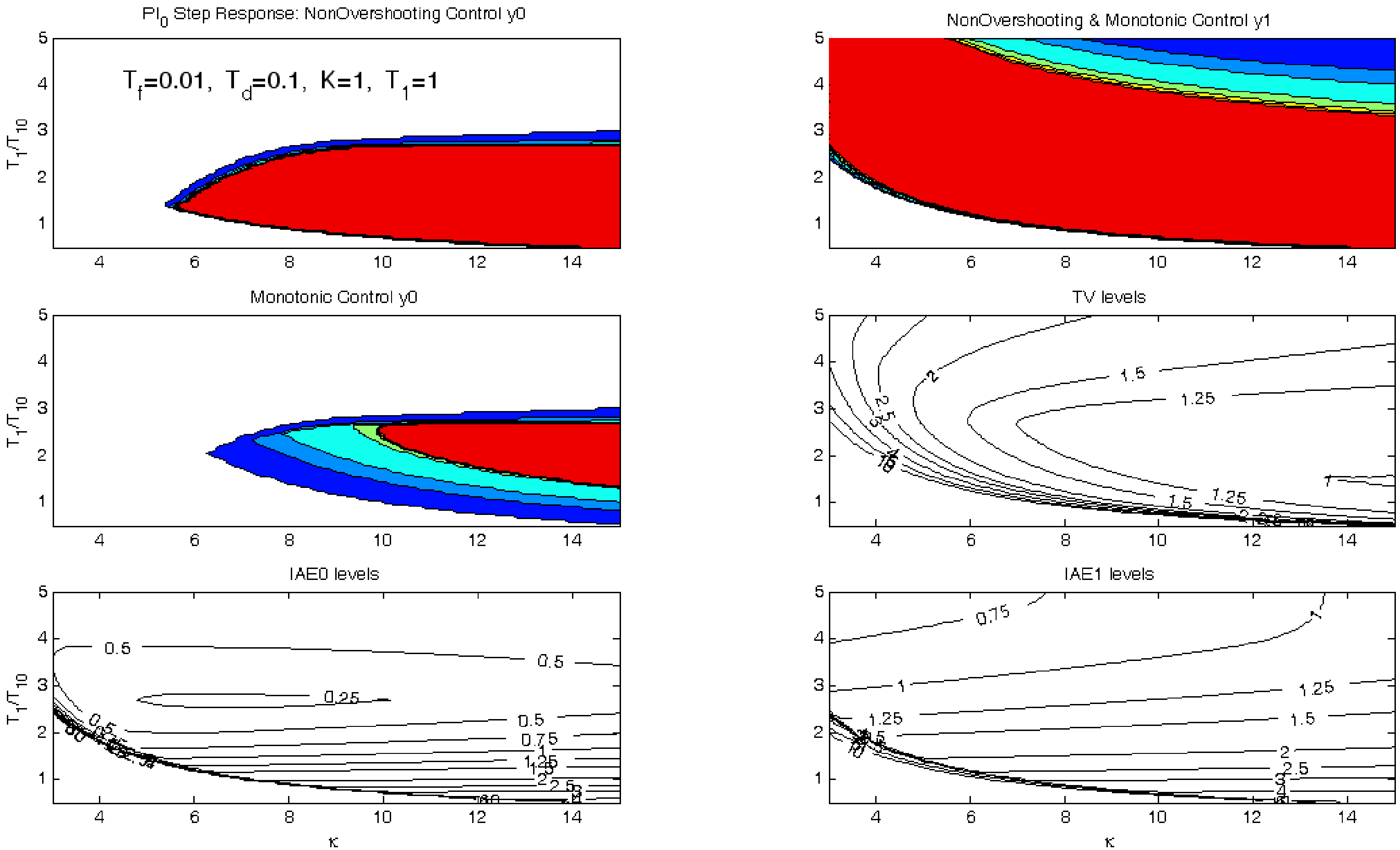
$$F_{w1}(s) = \frac{Y_1(s)}{W(s)} = \frac{Ke^{-T_d s}}{K_0 T_f T_1 s^2 + \left(K_0 T_f + Ke^{-T_d s} T_{10}\right)s + Ke^{-T_d s}}$$

$$\Rightarrow$$

$$F_{w1}(p) = \frac{1}{\kappa \tau_f \tau_1 p^2 e^{\tau_d p} + \left(\kappa \tau_f e^{\tau_d p} + 1\right)p + 1}$$

$$\tau_1 = \frac{T_1}{T_{10}}\ ; \quad \tau_f = \frac{T_f}{T_{10}}\ ; \quad \tau_d = \frac{T_d}{T_{10}}\ ; \quad p = T_{10}s\ ; \quad \kappa = \kappa_{\min} = \frac{K_0}{K_{\max}}$$

PI_0 – controller + dead time



Shift to larger values K_0 and T_{10}

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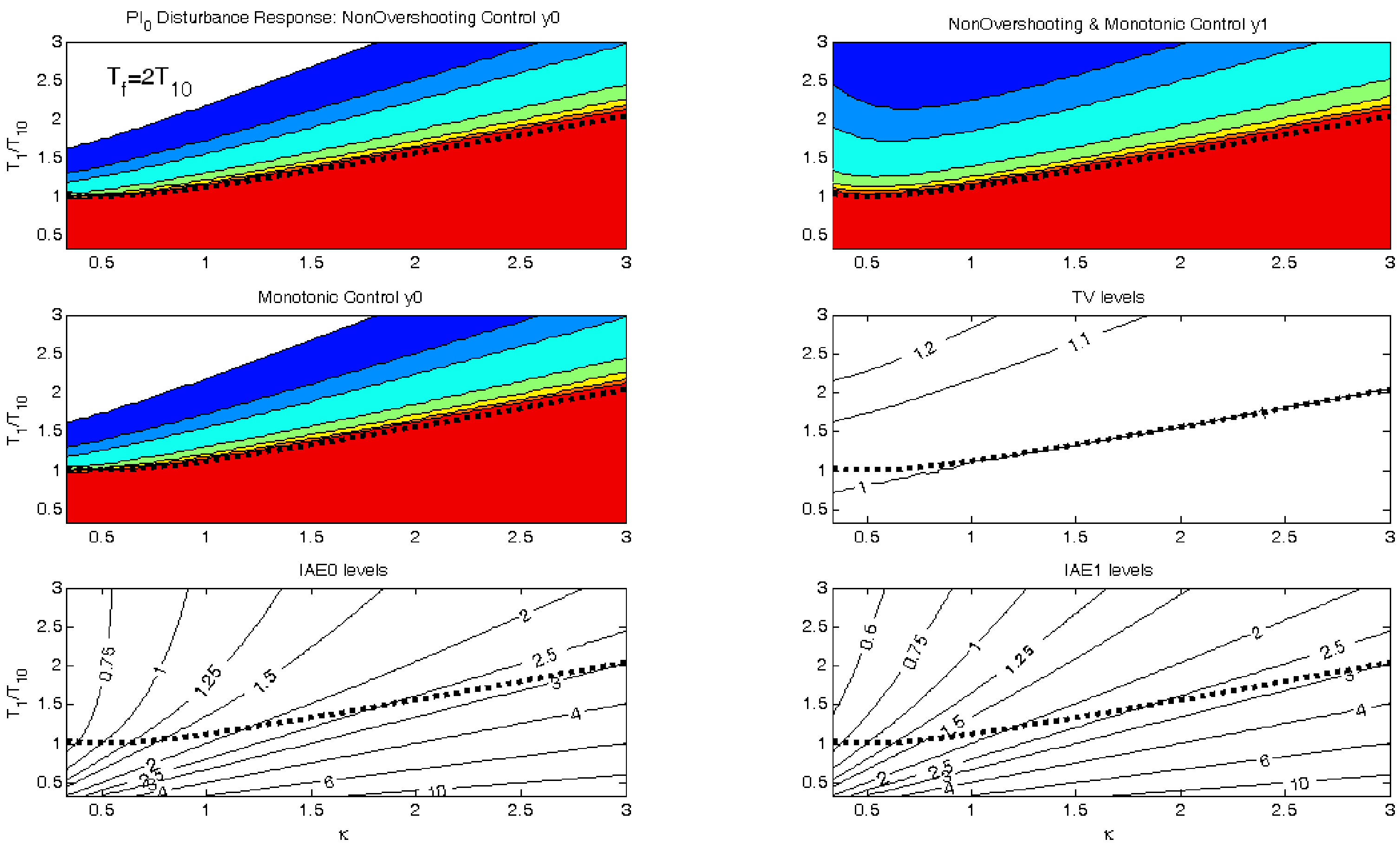
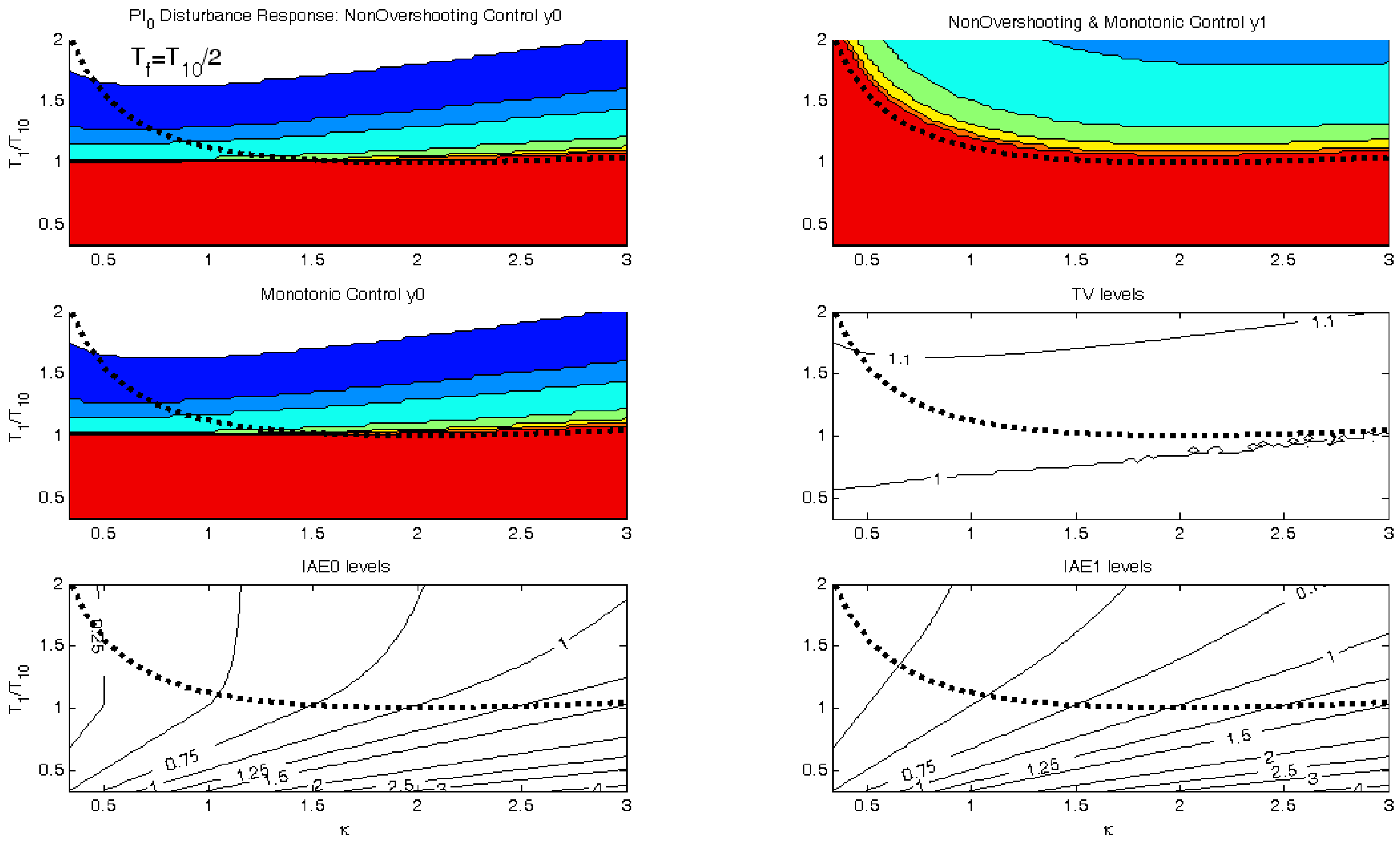
PI_0 – control: disturbance compensation



PI_0 – control: disturbance compensation



Conclusions

- There exist two types of PI_0 controllers guaranteeing monotonic transients at the plant input and output – these correspond to reconstruction & compensation of input and output disturbances
- Both may be derived from the static feedforward control extended by a prefilter and by observer based on the paralel model for output disturbances (IMC like structure) or inverse model for input disturbances

Conclusions

- Both structures show some important differences, e.g. for the IMC structure the DO time constant may be chosen as T_f=0, for the inverse model it must be T_f>0 (realization condition).
- Both structures may be studied by the Performance Portrait method.
- This may be used for optimal controller tuning both in the nominal as well as robust case

Conclusions

- It is to remember that different optimal tuning corresponds to the setpoint step and to the disturbance step
- The resulting tuning must balance these mostly contradictive requirements
- Setpoint steps may be modified by using prefilter – mostly with tuning $T_p = T_f$

Conclusions

- Higher quality requirements (lower tollerated deviations from ideal shapes) increase the sensitivity on parameter uncertainty
- Well tuned PI_0 controller guarantees lower sensitivity to parameter uncertainty than the simpler I_0!!!
- However, remember increased noise sensitivity that represents the main limitation in using PI control