INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

# Robustné PID-regulátory s obmedzeniami
# Robust Constrained PID Control
## Robust Constrained P controller Design

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# Contents

# Dynamical class „1“

- First Order Plant
- Rate of the changes proportional:
  - to the control signal
  - to the output signal
- Examples:
  - velocity control
  - temperature control
  - level control
  - pressure control

$$\frac{dy}{dt} = K_S (u - v) - ay$$

$u =$ control signal

$v =$ input disturbanc e

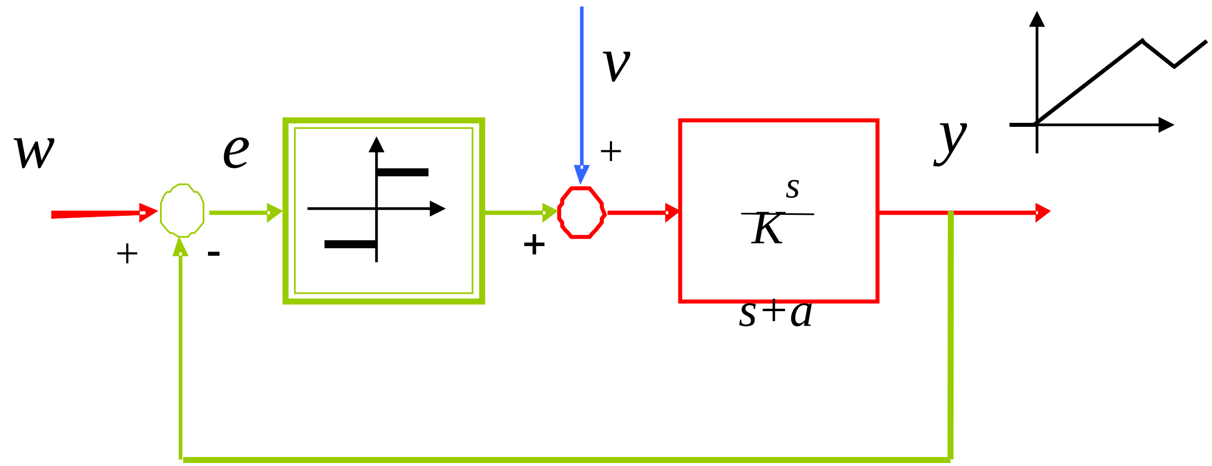
$y =$ plant output

$K_s, a =$ plant parameters

# Contents

- First Order Systems
- Relay Minimum Time Control
- Linear Pole Assignment Control
- Minimum Time Pole Assignment Control
- Controller Tuning – Nonmodelled Dynamics
- Conclusions

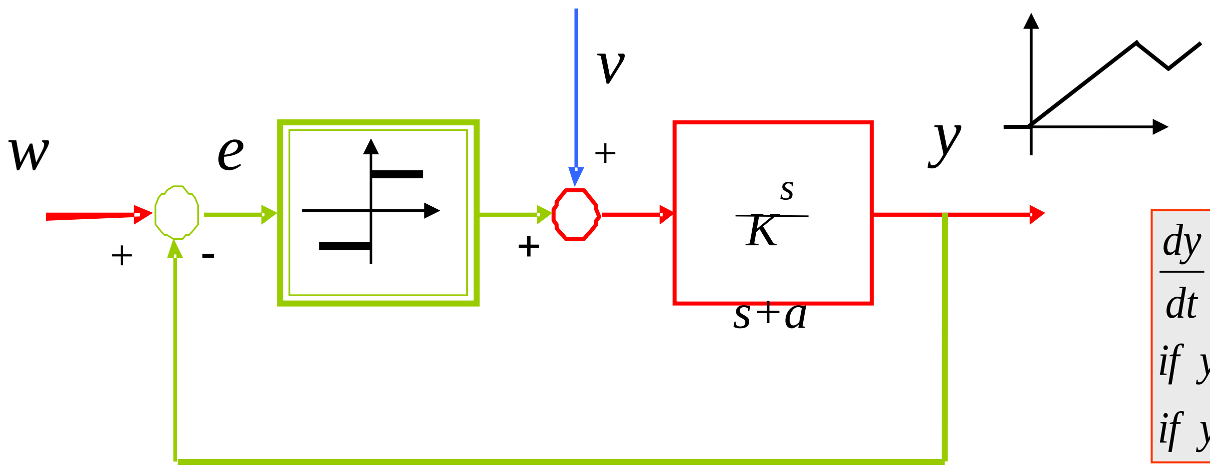
# Dynamical Class "1"
## Minimum time relay controller




$$\frac{dy}{dt} = K_S(u - v) - ay$$
$$if\ \ y > w\,then\ u = U_{min}$$
$$if\ \ y < w\,then\ u = U_{max}$$

- The fastest possible error decrease corresponds to limit control values
- Relay, On-Off, Bang-Bang Control
- Oscillations around desired state *w=const*

# Dynamical Class “1”
## Minimum time relay controller




$$\frac{dy}{dt} = K_S\left(u - v\right) - ay$$

$$\text{if } y > w \text{ then } u = U_{min}$$

$$\text{if } y < w \text{ then } u = U_{max}$$

- Frequency of Oscillations depends on the Nonmodelled Dynamics, Hystheresis, or on the Sampling Period
- Nonsymmetrical Dynamics = > permanent control error

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# Dynamical Class “1”
## Linear pole assignment

- Regular (Exponential) Error Decrease over **Limited Proportional Band**

$$\frac{de}{dt} = \alpha e \; ; \quad \alpha < 0$$

- Control error

$$e = w - y$$

- Plant

$$\frac{dy}{dt} = K_S \left(u - v\right) - ay$$

$$u = \text{control signal}$$

$$v = \text{disturbanc e}$$

$$y = \text{plant output}$$

$$K_s , a = \text{plant parameters}$$

- *w=const* – piecewise constant setpoint value

# Dynamical Class "1"
## Linear pole assignment

- Regular (Exponential) Error Dec
  over **Limited Proportional Ban**

$$\frac{de}{dt} = \alpha e \ ; \ \ \alpha < 0$$

- Control error

$$e = w - y$$

- Control algorithm

$$u = K_R e + u_w - v \ ;$$

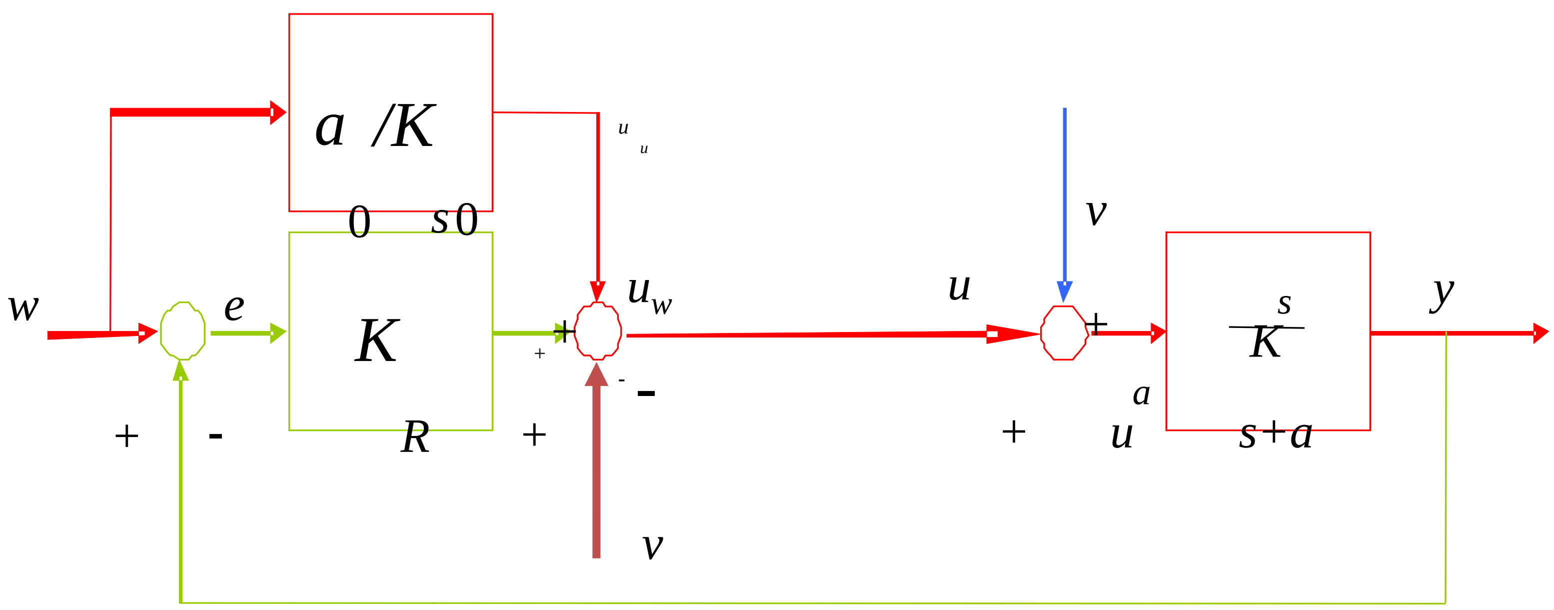
- Controller gain

$$K_R = -\frac{\alpha + a}{K_s} \ ;$$

- Static feedforward control

$$u_w = \frac{aw}{K_s}$$

# Dynamical Class “1”
## Linear pole assignment P-controller




- Control algorithm & scheme

# Contents

- First Order Systems
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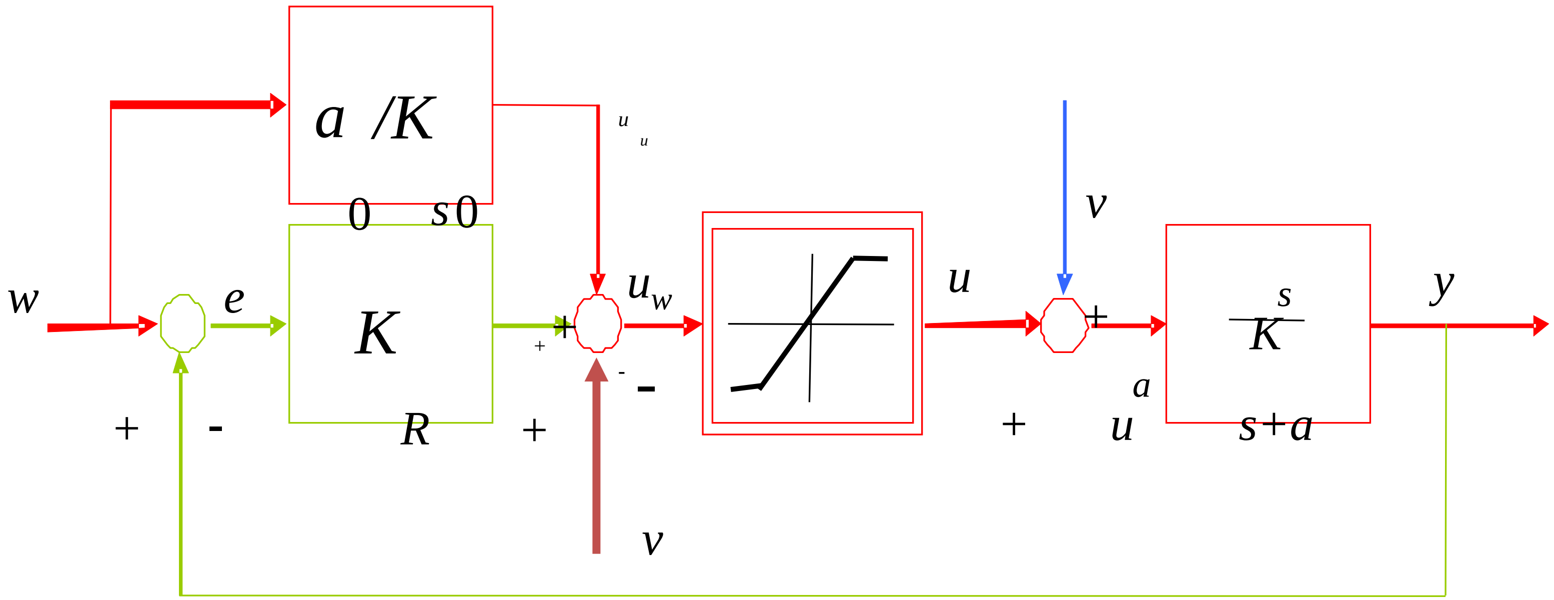
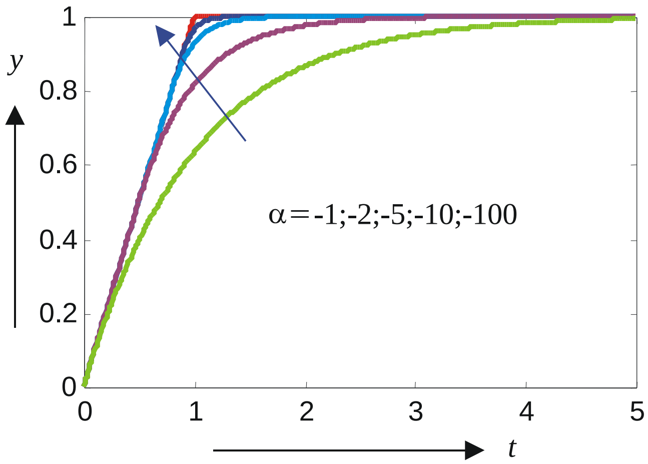
- Linear Pole Assignment Control
- Minimum Time Pole Assignment Control
- Controller Tuning – Nonmodelled Dynamics
- Conclusions

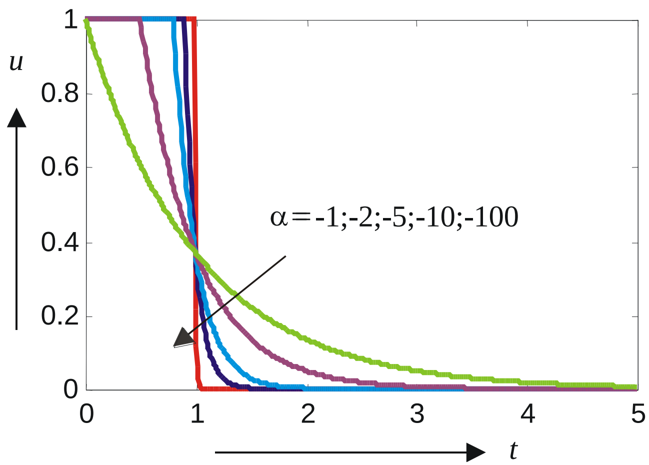
# Dynamical Class "1"
## Minimum time pole assignment P-controller




- Control algorithm & scheme
- Large control error – equivalent to the minimum time control (limit control signal values)
- Small control error – equivalent to the pole assignment control (proportional control)

# Dynamical Class "1"
## Minimum time pole assignment P-controller

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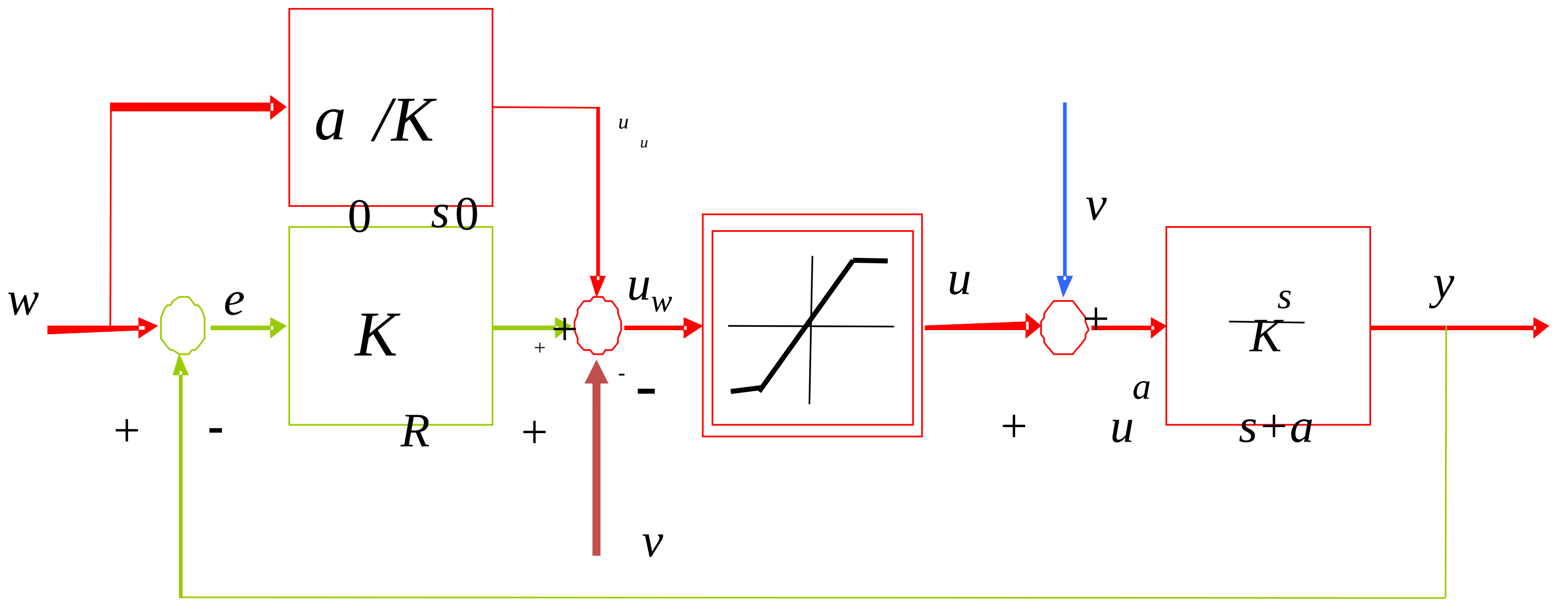
# Dynamical Class “1”
## Minimum time pole assignment P-controller




- Admissible inputs

$$K\left(U_{\min}+v\right)\le w\le K\left(U_{\max}+v\right)\ \text{pre}\,K=K_s/a>0$$
$$K\left(U_{\max}+v\right)\le w\le K\left(U_{\min}+v\right)\ \text{pre}\,K=K_s/a<0$$

- Admissible initial states

$$\left[K_s(U_{\min}+v)-ay_0\right]\left[K_s(U_{\max}+v)-ay_0\right]<0$$

# Sub-Conclusions I.

- Minimum Time Control (MTC) – the fastest possible approach towards the required state
- Oscillations around the required state
- No free parameter to modify the dynamics of transients
- Pole Assignment Control (PAC) – dynamics modified by the closed loop poles
- Transients are speeded up by shifting the real part of the closed loop pole to the left (more negative values)

# Sub-Conclusions II.

- By shifting $\alpha \to -\infty$ dynamics of the PAC approaches dynamics of the MTC (dominated by constraints)
- The setpoint values have to respect given constraints, plant gain and acting disturbances, or, conversely,
- The actuator has to be dimensioned with respect to extent of the setpoint values and disturbances
- Unstable systems are not controllable in any initial state

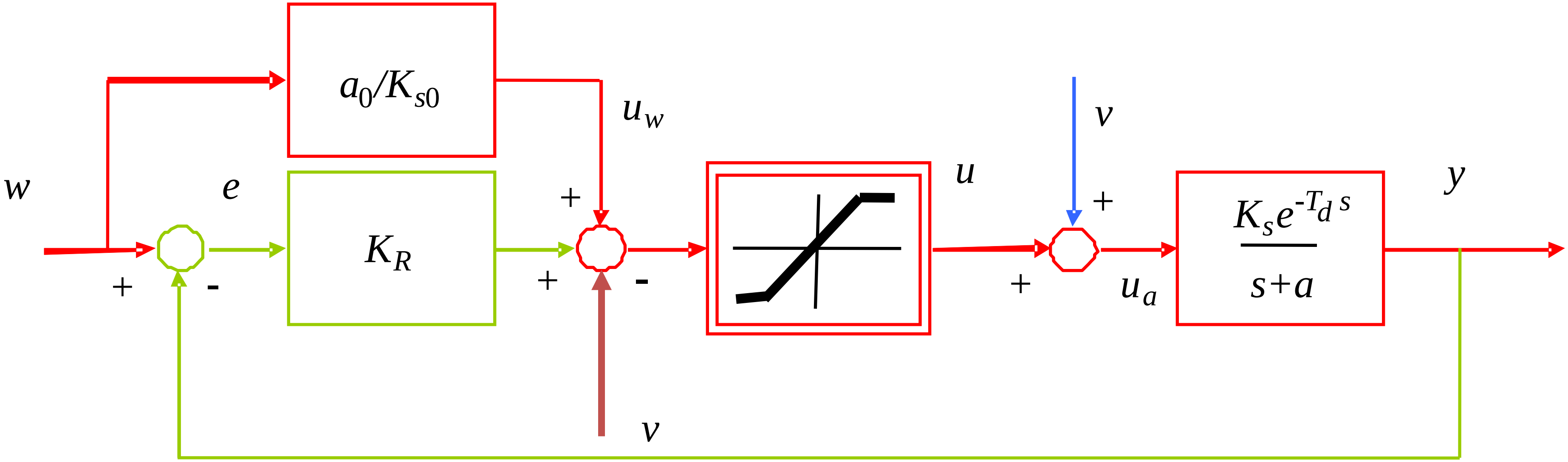
# Contents

# Controller Tuning

- Identification of the parameters of the dominant 1st order dynamics
- Approximation of the non-modeled dynamics
- Influence of relatively non-dominant non-modeled dynamics may be respected by choice of the controller gain
- When the non-modeled dynamics has essential influence – some its part has to be respected by the controller structure

# P-controller

## P-Controller + static feedforward + $1^{st}$ order plant $+T_d$




$$K_R = -(a_0+\alpha)/K_{s0} = (\Omega_r - a_0)/K_{s0} \;;\; \Omega_r = 1/T_r = -\alpha$$

- $T_d \ll 1/|a|$; $T_r$= required settling time

$$F_w(s) = \frac{Y(s)}{W(s)} = \left(1+\frac{a_o}{K_{s0}K_R}\right)\frac{K_R F(s)}{1+K_R F(s)} = \frac{\alpha K_s e^{-T_d s}}{K_s e^{-T_d s}(a_0+\alpha) - K_{s0}(s+a)}$$

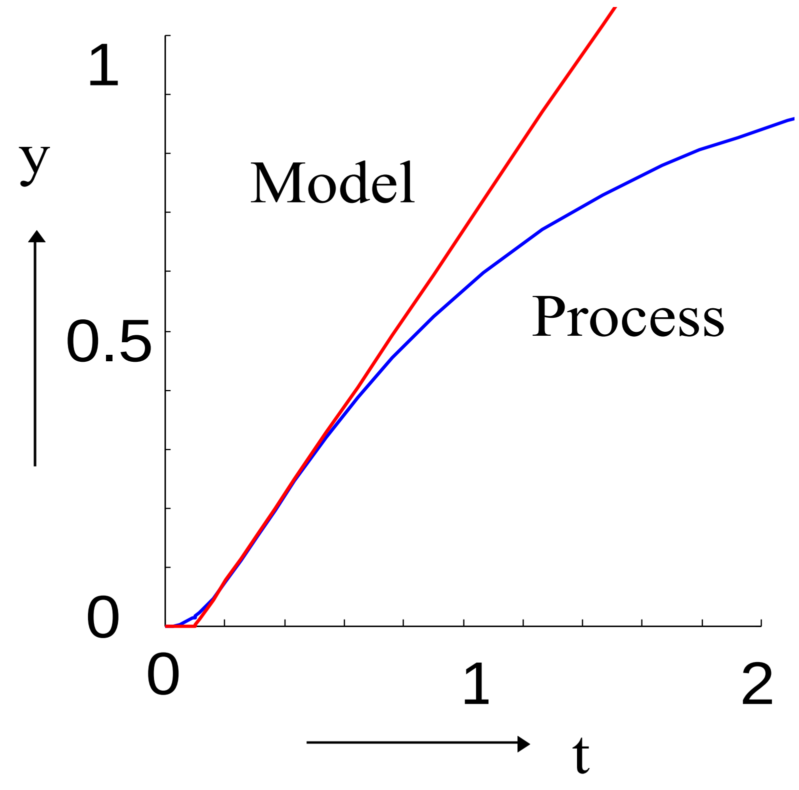
# Controller Tuning

- Ziegler and Nichols 1942 $I_1$+ time delay
- Double Real Dominant Pole (Oldenbourg & Sartorius, 1944): P controller

# Example 1: $I_1T_d$-Approximation

$$S(s) = \frac{1}{(s+1)(0.1s+1)(0.05s+1)}$$

$$\hat{S}(s) = 0.75\frac{1}{s}e^{-0.1s}$$



# P-controller: $I_1T_d$-plant appr.

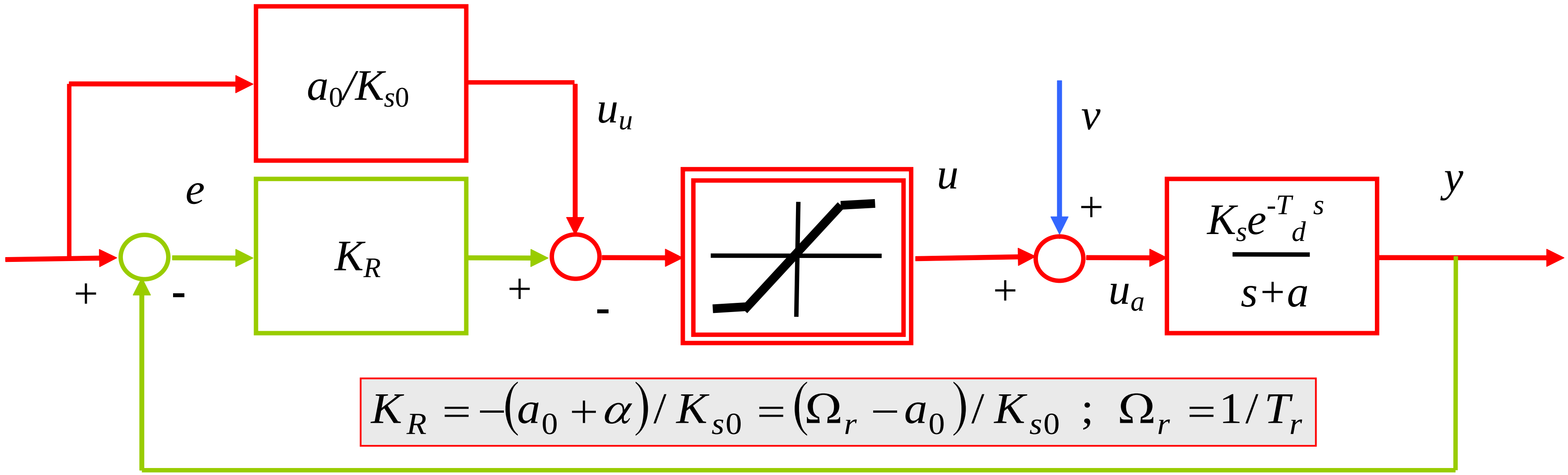
| | | $a=0$ | $a\neq 0$ |
|---|---|---|---|
| $T_d \le T$ | $K=K_RK_sT$ | $\dfrac{T}{\left(\sqrt{T_d}+\sqrt{T}\right)^2}$ | $e^{aT_d}\left(e^{aT}-1\right)\left(1+\sqrt{\dfrac{1-e^{-aT}}{1-e^{-aT_d}}}\right)^2$ |
| | $z_{opt}$ | $\dfrac{\sqrt{T_d}}{\sqrt{T_d}+\sqrt{T}}$ | $\dfrac{e^{-aT}}{1+\sqrt{\dfrac{1-e^{-aT}}{1-e^{-aT_d}}}}$ |
| $T_d = nT$ | $K=K_RK_sT$ | $\dfrac{n^n}{(n+1)^{n+1}}$ | $aT\dfrac{e^{-aT_d}}{e^{-aT}-1}\dfrac{n^n}{(n+1)^{n+1}}$ |
| | $z_{opt}$ | $\dfrac{n}{n+1}$ | $\dfrac{n}{n+1}e^{-aT}$ |
| $T\to 0$ | $K_R$ | $\dfrac{1}{K_sT_de}$ | $\dfrac{e^{-aT_d}}{K_sT_de}$ |
| | $s_{opt}$ | $-\dfrac{1}{T_d}$ | $-a-\dfrac{1}{T_d}$ |

# P-controller: $I_1T_1$-plant appr.

| | | $a=0$ | $a\neq 0$ |
|---|---|---|---|
| $T \neq 0$ | $K = K_R K_s T$ | $\dfrac{1-D}{\left(1+\sqrt{\tau}\right)^2}$ | Complicated expression |
| | $z_{opt}$ | $\dfrac{D-\tau+(1-D)\sqrt{\tau}}{1-\tau}$ | Complicated expression |
| $T \to 0$ | $K_R$ | $\dfrac{1}{4K_s T_1}$ | $\dfrac{1}{K_s}\dfrac{(T_1 a - 1)^2}{4T_1}$ |
| | $s_{opt}$ | $-\dfrac{1}{2T_1}$ | $-\dfrac{T_1 a + 1}{2T_1}$ |
| $D = e^{-T/T_1}$ ; $\tau = \dfrac{T_1(1-D)}{T}$ | | | |

# P-controller

## P-Controller + static feedforward + 1$^{st}$ order plant +$T_d$




$$K_R = -(a_0+\alpha)/K_{s0} = (\Omega_r - a_0)/K_{s0} \;;\; \Omega_r = 1/T_r$$

$$F_w(s) = \frac{Y(s)}{W(s)} = \left(1+\frac{a_o}{K_{s0}K_R}\right)\frac{K_R F(s)}{1+K_R F(s)} = \frac{\alpha K_s e^{-T_d s}}{K_s e^{-T_d s}(a_0+\alpha) - K_{s0}(s+a)}$$

$$p = T_d s \;;\; K = K_R K_s T_d \;;\; A = aT_d \;;\; K_v = A_0/K_0 = a_0/K_{s0}$$

$$F_w(p) = \frac{Y(p)}{W(p)} = (1+K_v)\frac{Ke^{-p}}{p+A+Ke^{-p}}$$

# Analytical construction of the PP

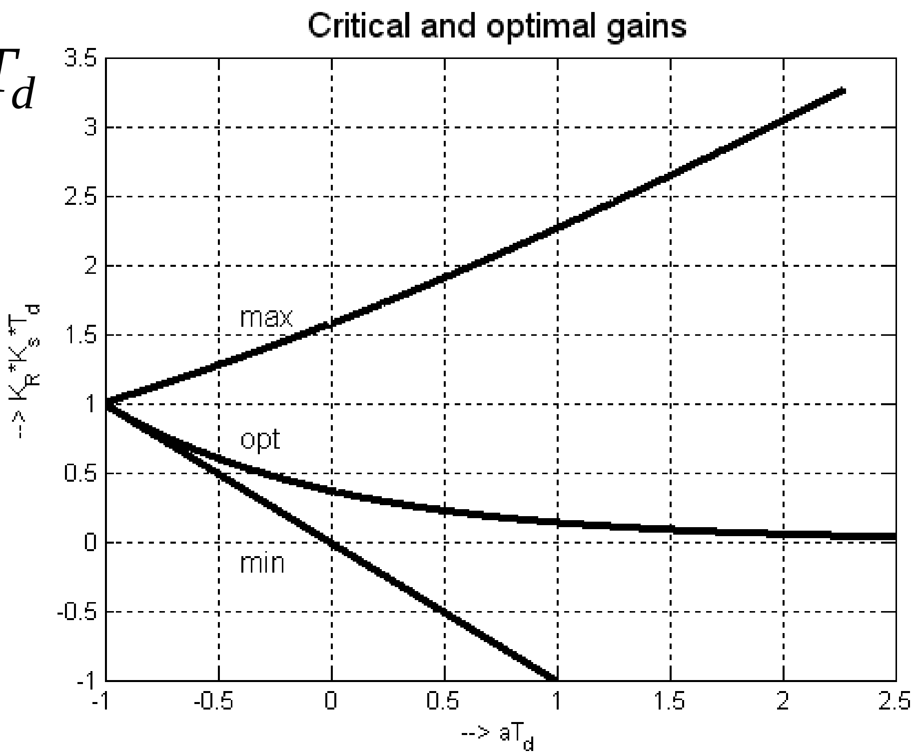
- Critical and optimal gains (aperiodicity border)

$$K_{min} < K < K_{max} \;;\; K = K_R K_s T_d$$

$$K_{max} = \frac{\tau_d}{\sin\tau_d} \;; A = -\frac{\tau_d \cos\tau_d}{\sin\tau_d} \;;$$
$$\tau_d \in (0, \pi/2) \cup (\pi/2, \pi)$$

$$K_{min} = -A = -aT_d$$

$$K_{opt} = e^{-(1+aT_d)}$$

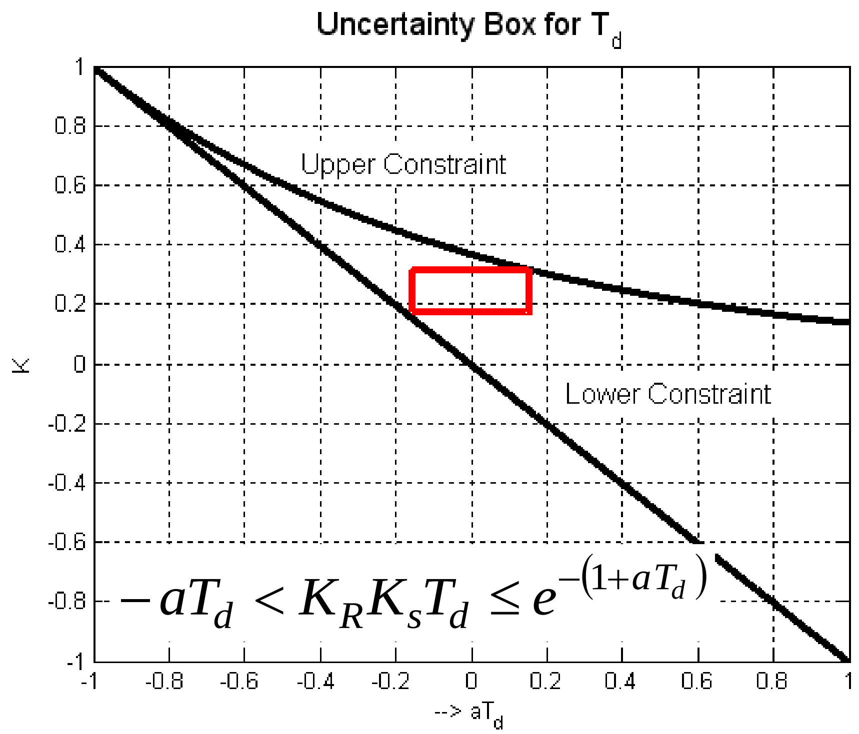


# Analytical construction of the PP

- Uncertainty Box

$$B(1,1) = \left(a_{\min} T_{d,\max}, K_R K_{s,\max} T_{d,\max}\right);$$
$$B(1,2) = \left(a_{\max} T_{d,\max}, K_R K_{s,\max} T_{d,\max}\right);$$
$$B(2,1) = \left(a_{\min} T_{d,\max}, K_R K_{s,\min} T_{d,\max}\right);$$
$$B(2,2) = \left(a_{\max} T_{d,\max}, K_R K_{s,\min} T_{d,\max}\right);$$

$$UB = \begin{bmatrix} B(1,1) & B(1,2)) \\ B(2,1) & B(2,2) \end{bmatrix};$$



# Example 1 Robust P- controller

- Plant with interval parameters

$$a \in \langle -0.9, 0.9 \rangle \; ; \; K_s \in \langle 0.7, 1.3 \rangle \; ; \; T_d \in \langle 0.1, 0.17 \rangle$$

- Possible parameterization of the UB

$$K_R = 1.4285$$

- or

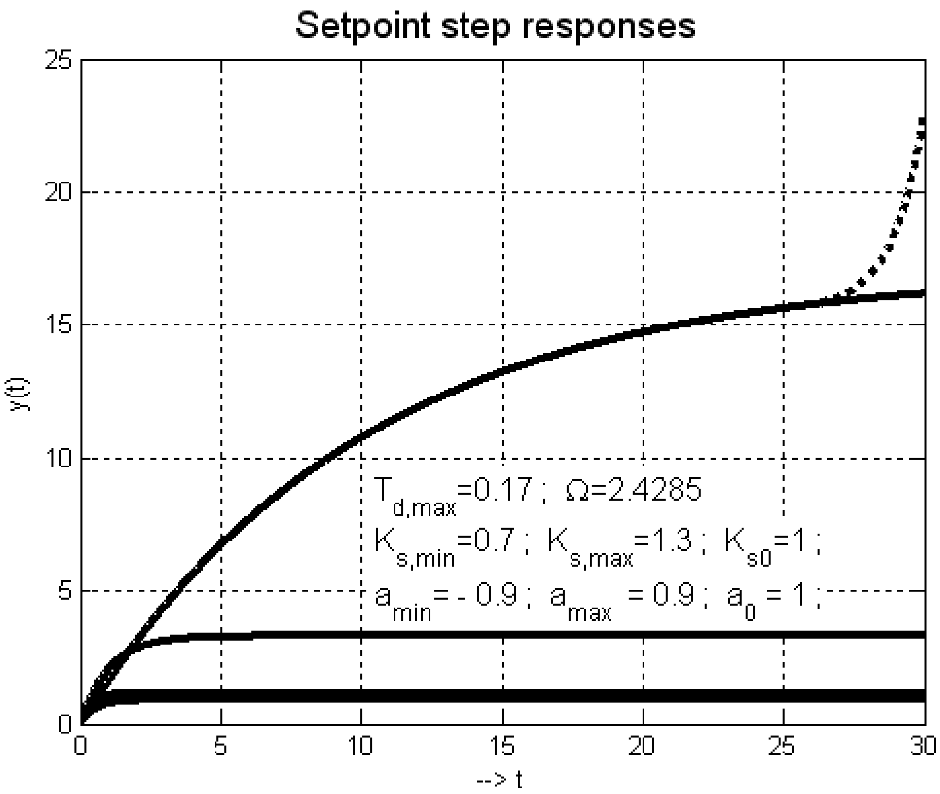
$$\boxed{K_{Ropt} = \frac{e^{-(1+a_0 T_d)}}{K_{s0} T_d}}$$

$$K_{s0} = 5.5 \; ; \; a_0 = 0 \; ;$$

$$K_{s0} = 1 \; ; \; a_0 = 1$$

# Example 1 Robust P- controller

- Paradox – monotonic transients, but with large steady-state error (effect similar to overshooting)



# Generating PP by simulations

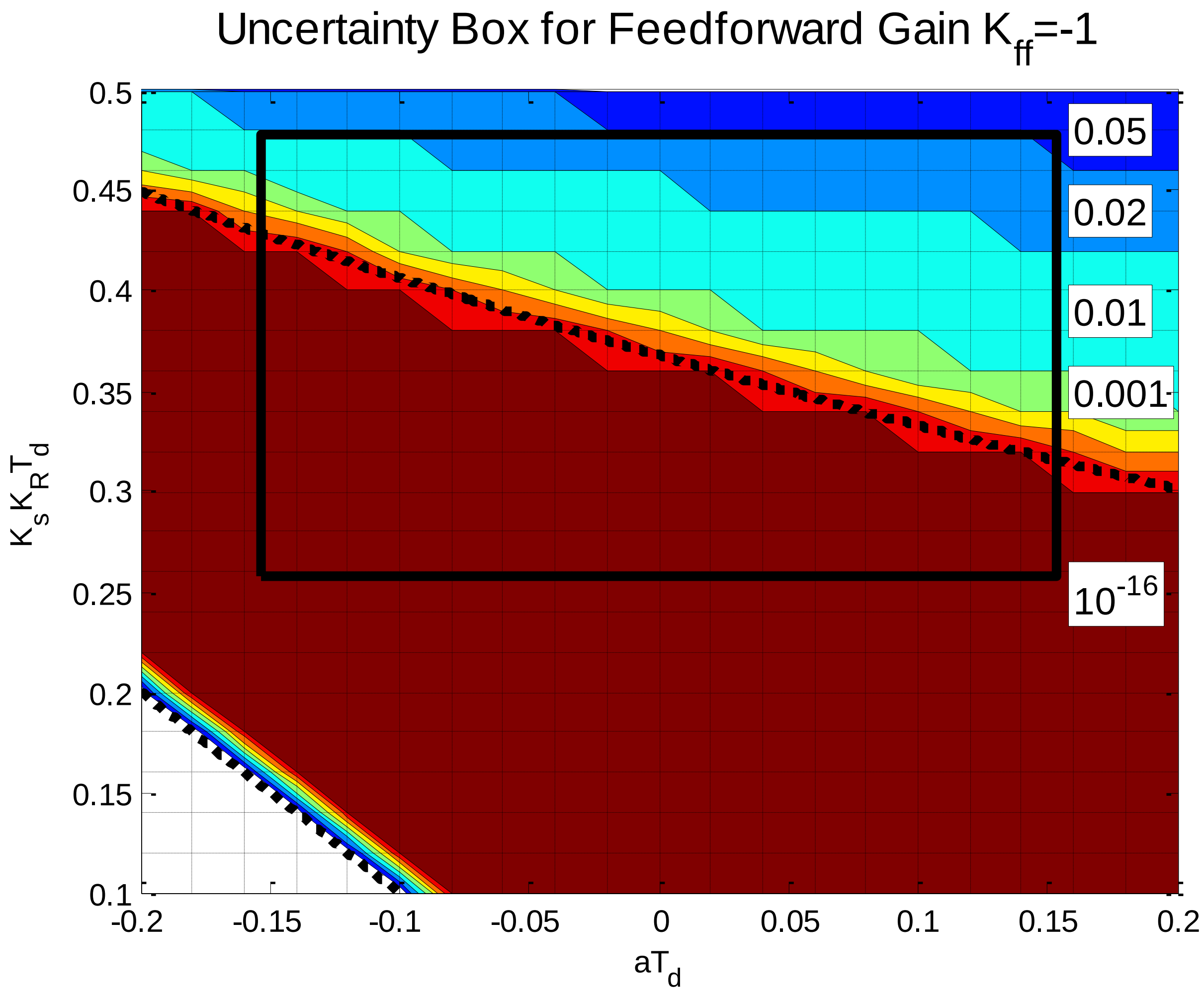
- Simulations in 3D $K, A, K_v$

$$p = T_d s \; ; \; K = K_R K_s T_d \; ; \; A = a T_d \; ; \; K_v = A_0 / K_0 = a_0 / K_{s0}$$

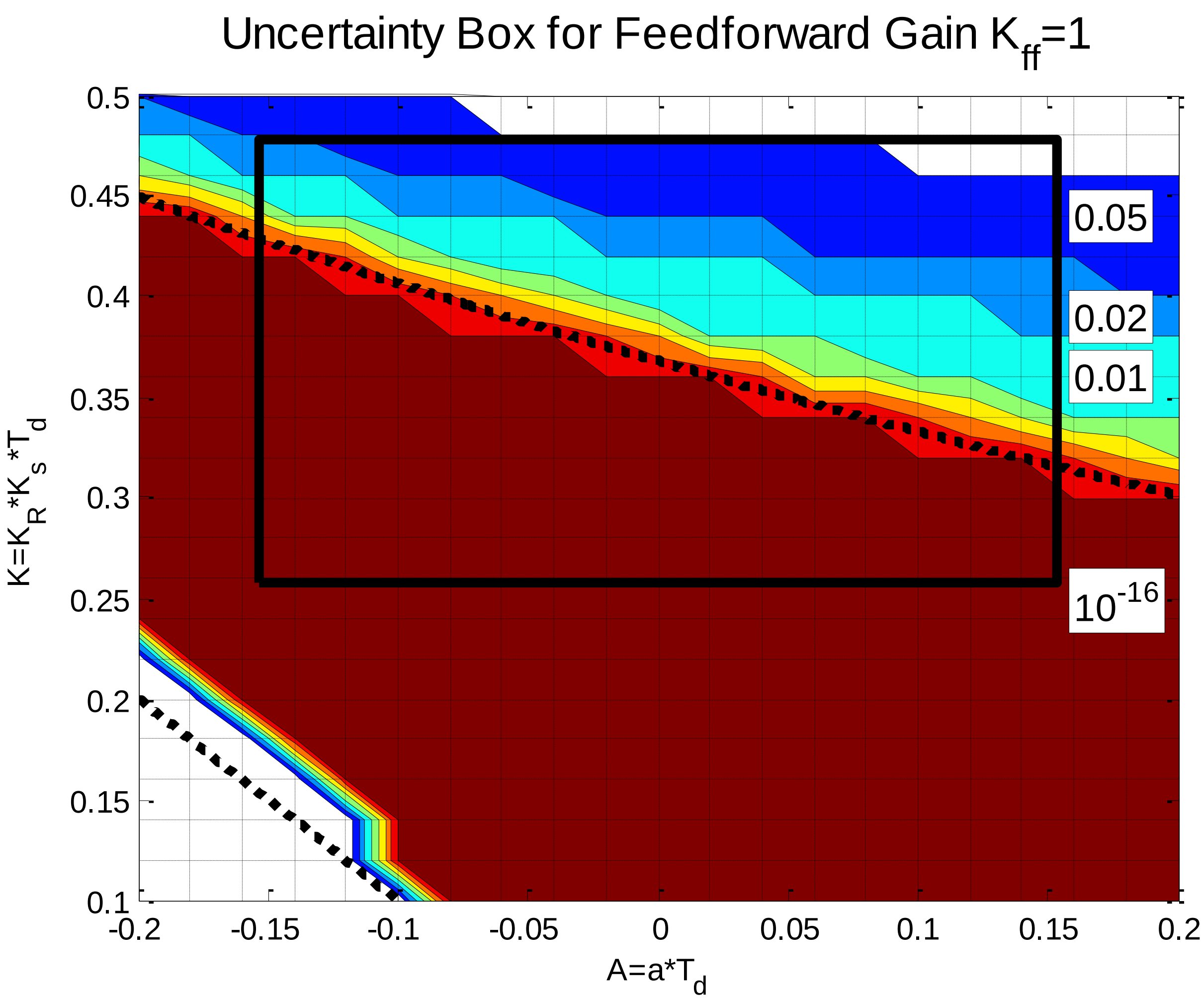
$$F_w(p) = \frac{Y(p)}{W(p)} = (1 + K_v) \frac{K e^{-p}}{p + A + K e^{-p}}$$

- Mapping the PP with different tolerances for deviations from strict monotonicity and non-overshooting

# Generating PP by simulations



# Generating PP by simulations



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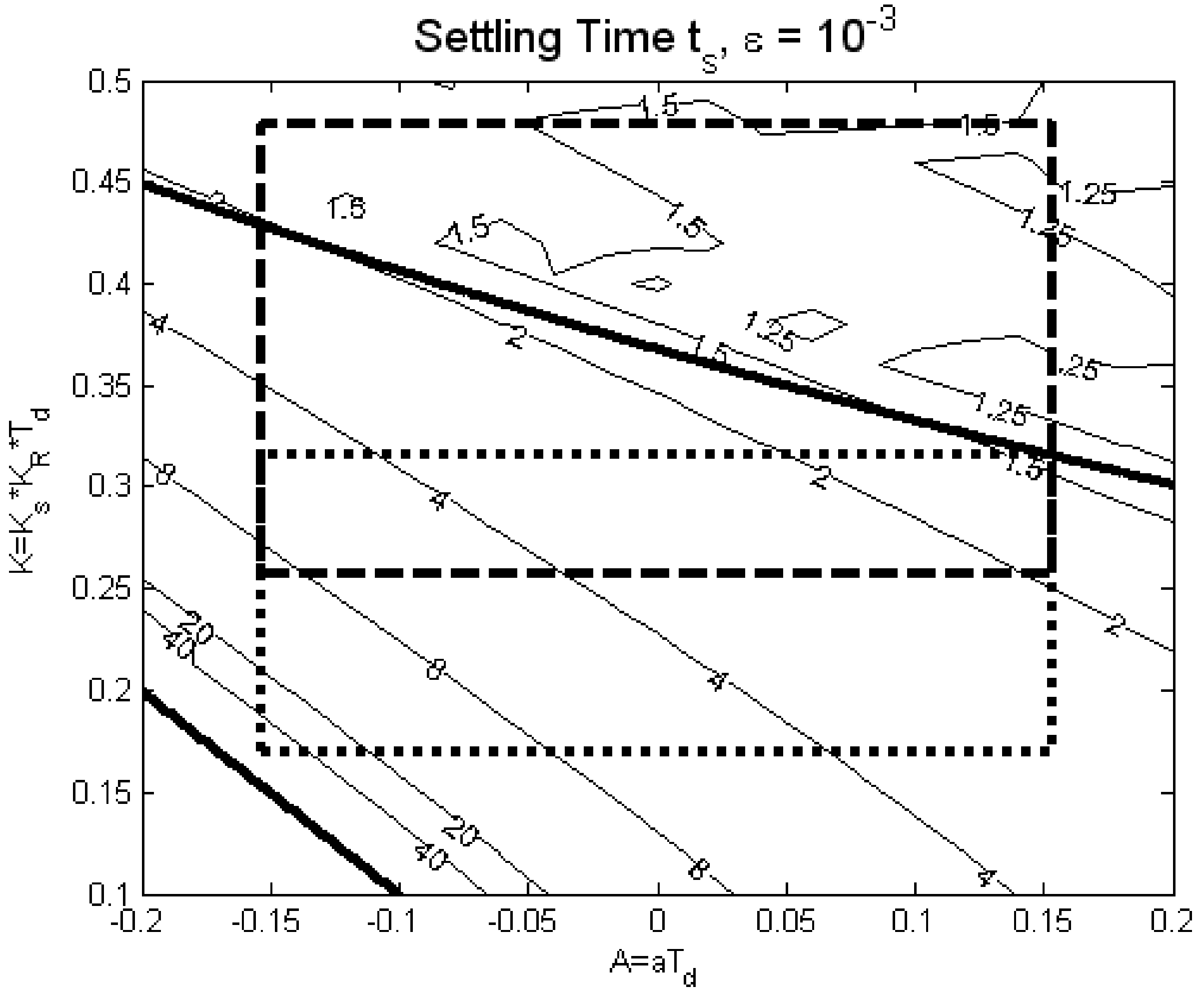

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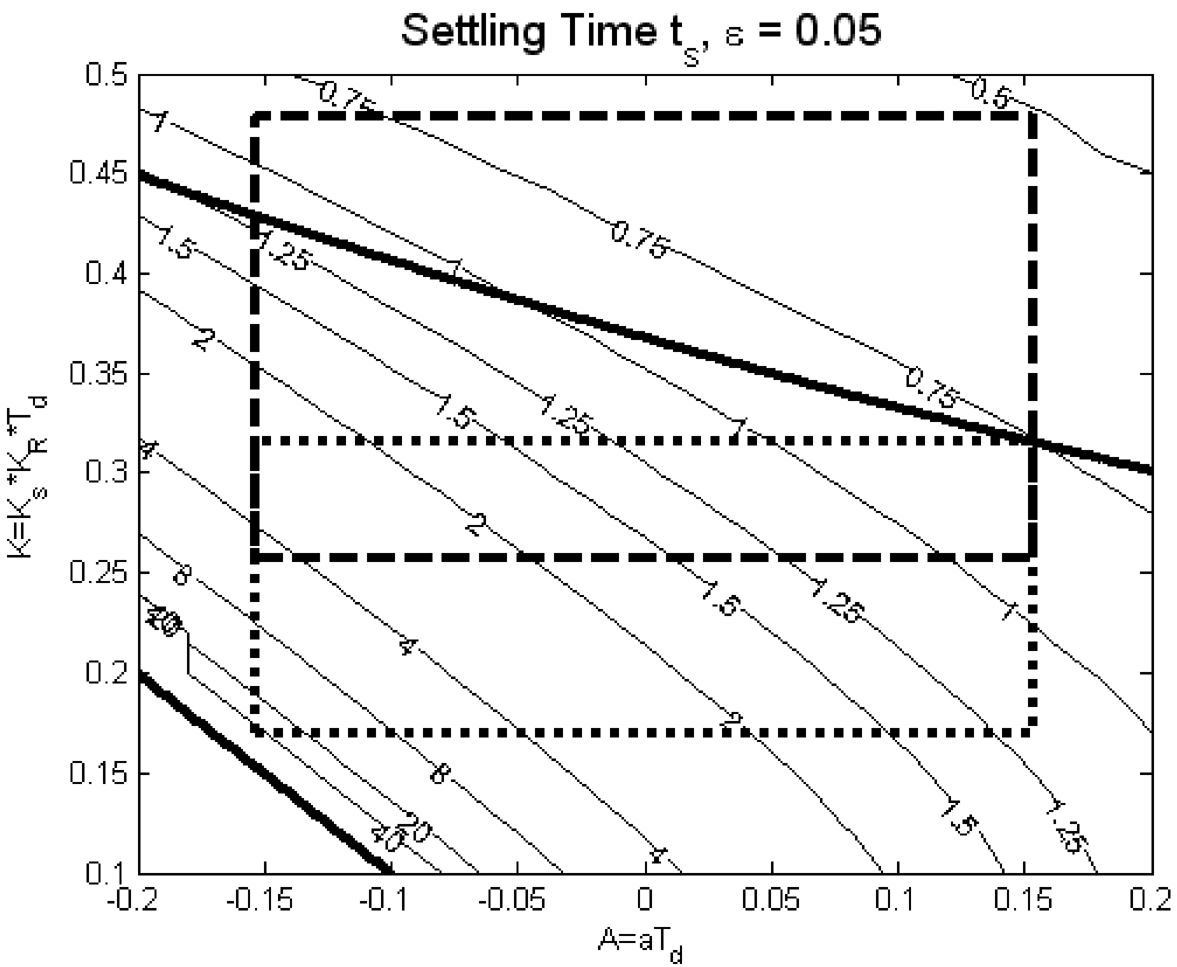

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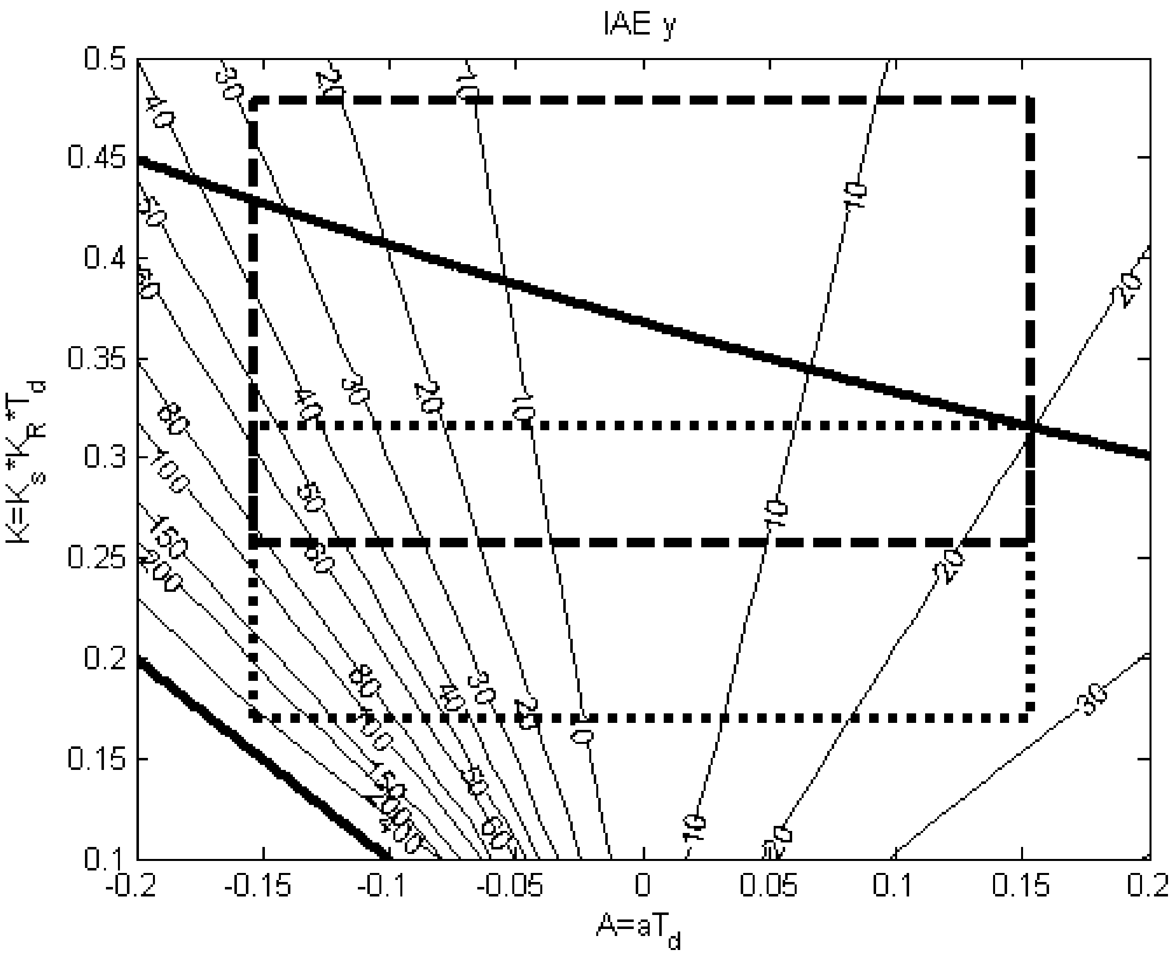

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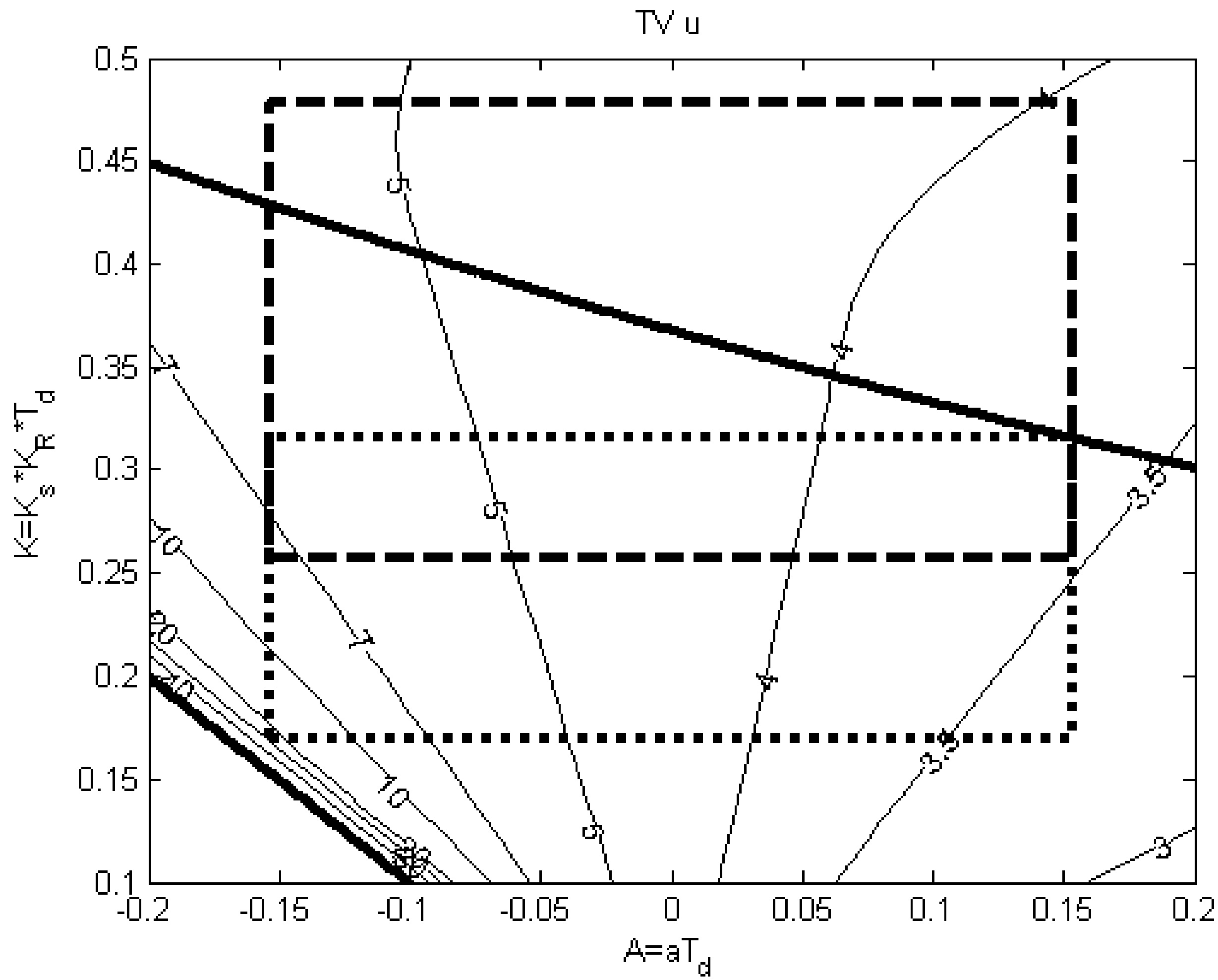

# Conclusions

- Robust tuning enables to guarantee some properties for all possible working points
- Analytical robust tuning is possible, but not flexible enough to cover practical requirements – much more flexible are experimental methods of generating PP
- Generation of the PP for normed variables enables its re-use for any numerical values of real parameters