INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Robustné PID-regulátory s obmedzeniami
Robust Constrained PID Control
DC2: Constrained PD_2 Controller

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Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.

Objectives

To explain:

- principle of relay minimum time control and its limitations,
- geometrical interpretation of the pole assignment control of 2nd order systems in the phase plane in the case of
  - real poles
  - complex poles
- influence of nonmodeled dynamics on choice of (equivalent) poles guaranteeing specified shape related requirements (output monotonicity, input 2P)

Objectives

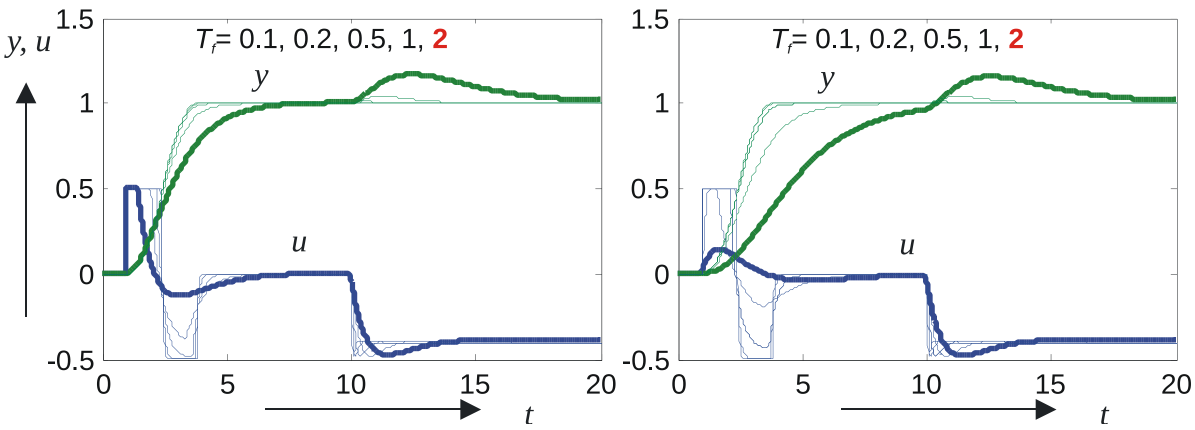
To explain:

- how the minimum time and pole assignment control principles may be combined to achieve monotonic responses at the plant output and 2P control at its input
- degrees of freedom in designing constrained pole assignment and hot to use them
- Use of the performance portrait method for different tasks
- how to apply PD_2 controller to a specific problem

Fundamental Solutions DC2

Dynamic class	I-action	K	Ke^{-T_d}	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}$	$\left[\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}\right]e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
		Dominant dynamics							
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	PD	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

Fundamental Solutions DC2



Fundamental Solutions DC2

Dynamic class	I-action	Dominant dynamics							
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0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
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	Y	-	-	-	-	-	-	PID	PrPID

Fundamental Solutions DC2

		Dominant dynamics							
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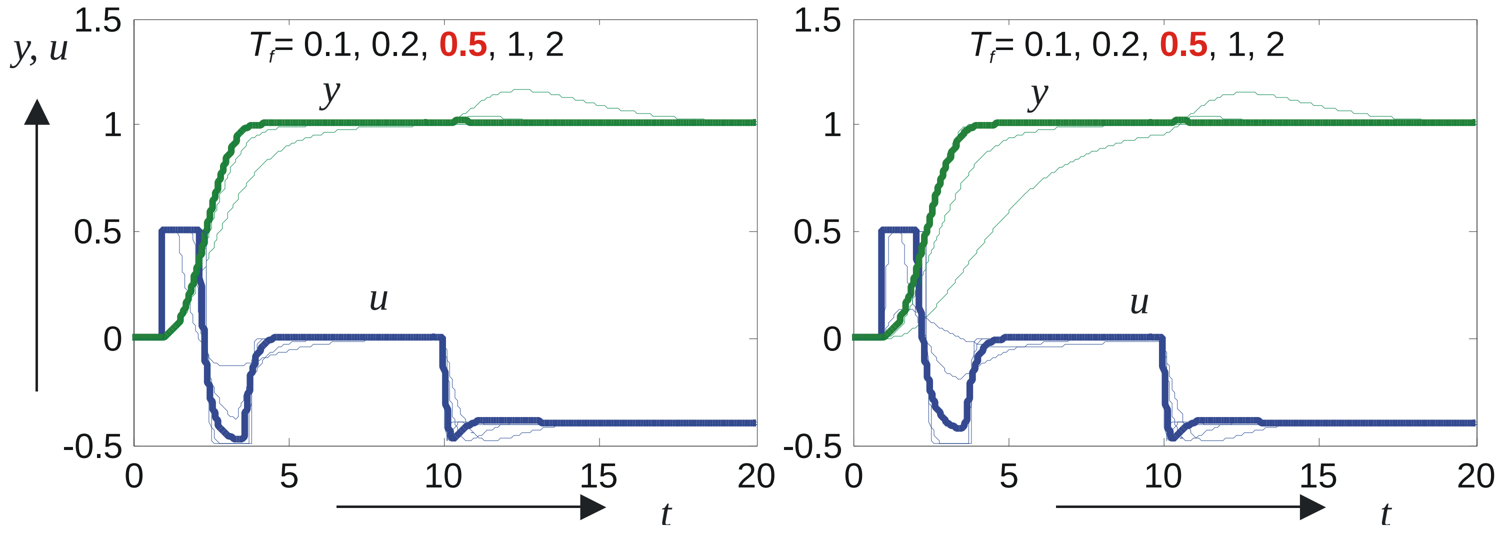
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	PD	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

Fundamental Solutions DC2



Fundamental Solutions DC2

Dynamic class	I-action	K	Ke^{-T_d}	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}$	$\left[\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}\right]e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
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0	N	FF	FF	FF	FF	FF	FF	FF	FF
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1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
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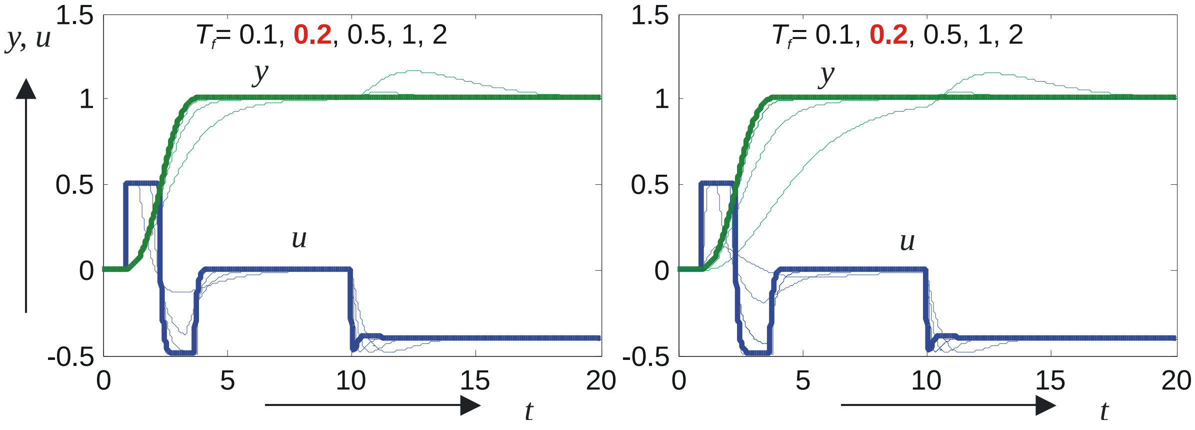
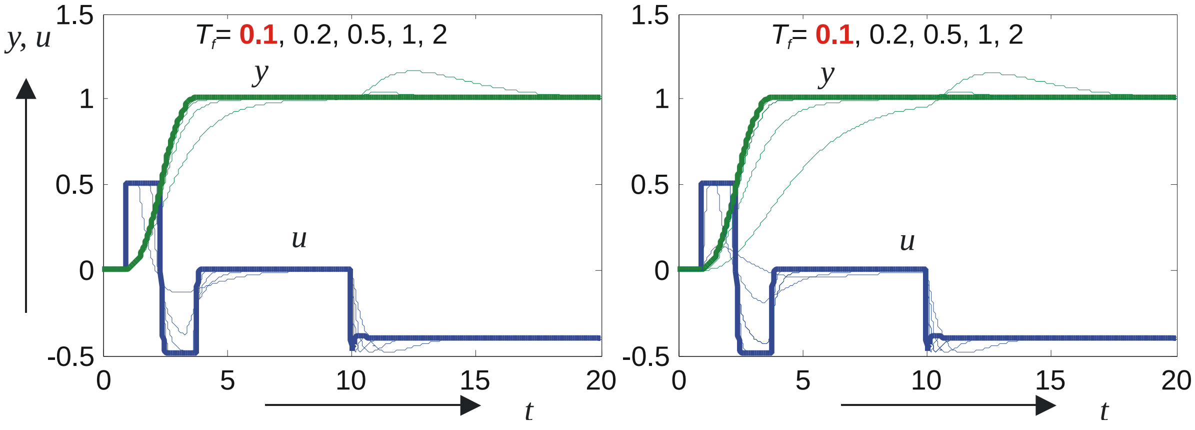
Fundamental Solutions DC2



Fundamental Solutions DC2

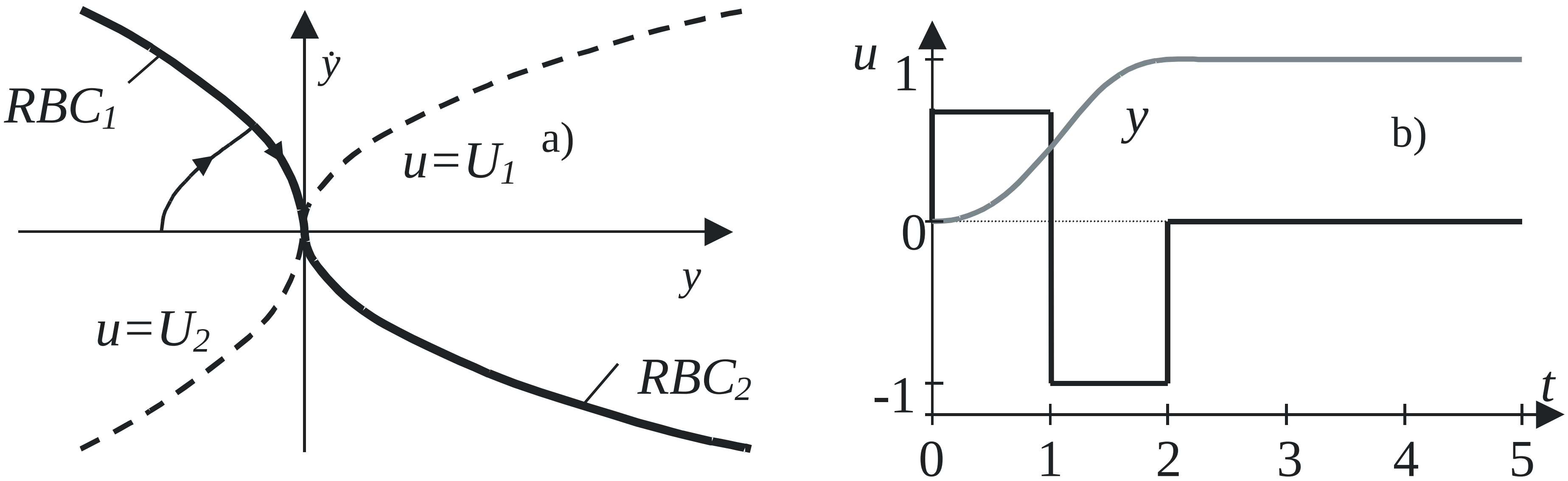
Dynamic class	I-action	Dominant dynamics: K	Ke^{-T_d}	$\frac{K_s}{s+a}$	$\frac{K_s e^{-T_d s}}{s+a}$	$\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}$	$\left[\frac{K_{s1}}{s+a_1}+\frac{K_{s2}}{s+a_2}\right]e^{-T_d s}$	$\frac{K_s}{s^2+a_1 s+a_0}$	$\frac{K_s e^{-T_d s}}{s^2+a_1 s+a_0}$
0	N	FF	FF	FF	FF	FF	FF	FF	FF
	Y	I	PrI	PI	PrPI	PID	PrPID	PID	PrPID
1	N	-	-	P	PrP	P-P	PrP-P	PD	PrPD
	Y	-	-	PI	PrPI	P-PI	PrP-PI	PID	PrPID
2	N	-	-	-	-	-	-	$\boxed{\text{PD}}$	PrPD
	Y	-	-	-	-	-	-	PID	PrPID

Fundamental Solutions DC2



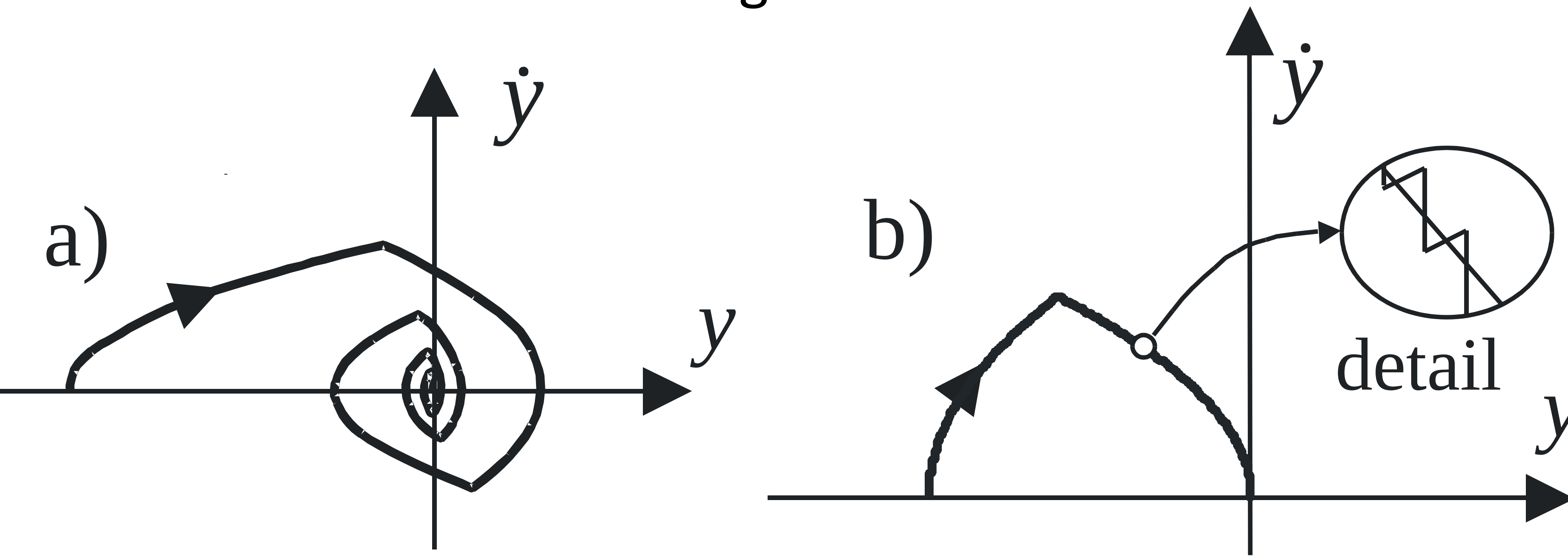
Dynamical Class “2”

- Minimum Time Control (MTC, late 1940s)
- Athans & Falb, Flüge-Lotz, Feldbaum, Pontrjagin et al., Ryan, Smith, Zeitz, etc.



Dynamical Class "2"

- High sensitivity to noise, parameter variations (relay chattering, overshoot), time delays
- Oscillations around origin



Overestimated dynamics

Underestimated dynamics

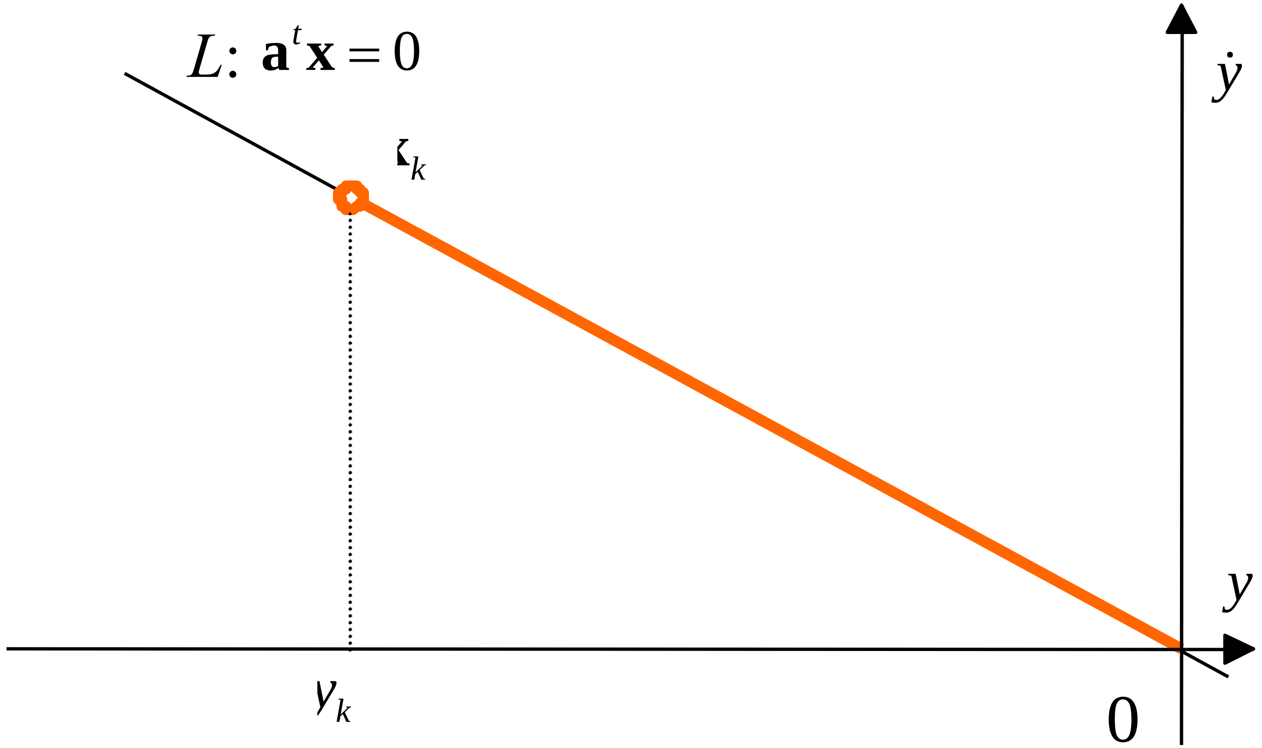
Dynamical Class “2”

- What does it mean
  Constrained Pole Assignment Control (CPAC), or
  Minimum Time Pole Assignment Control (MTPAC)?

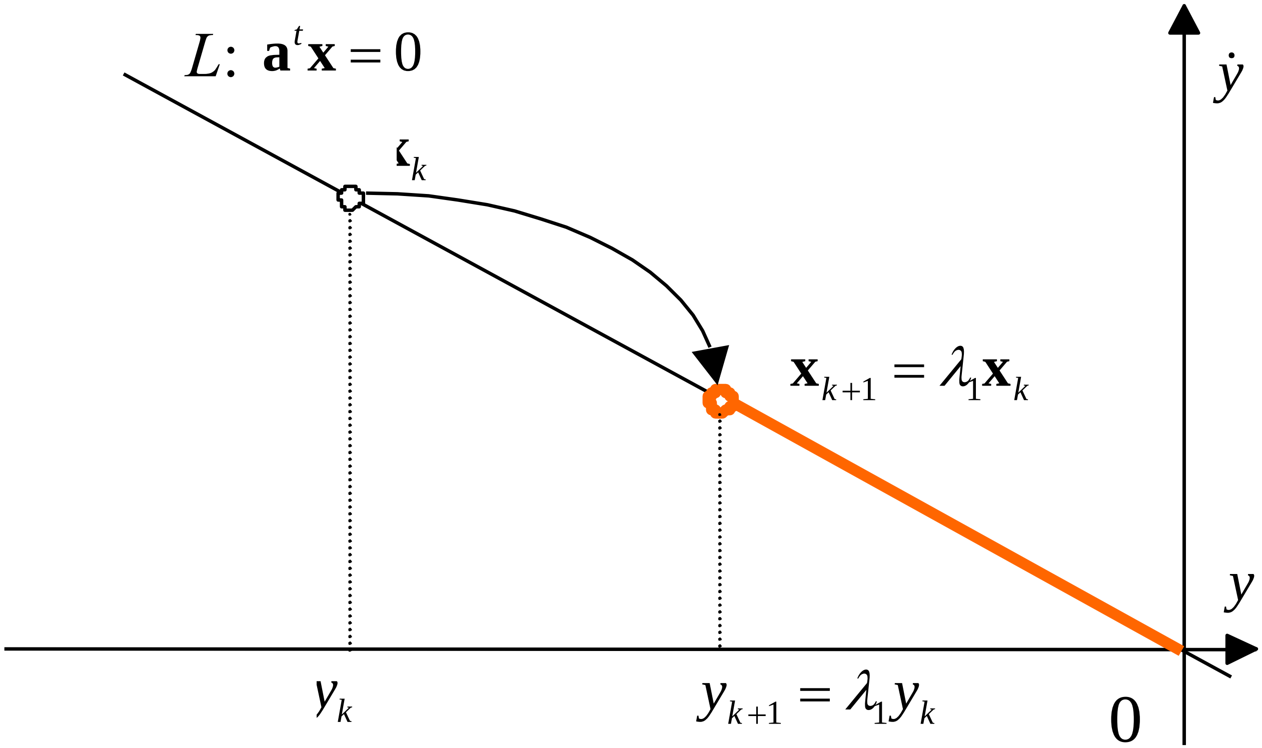
- What does it mean
  Linear Pole Assignment Control (LPAC)?

LPAC – Linear 2nd Order Systems

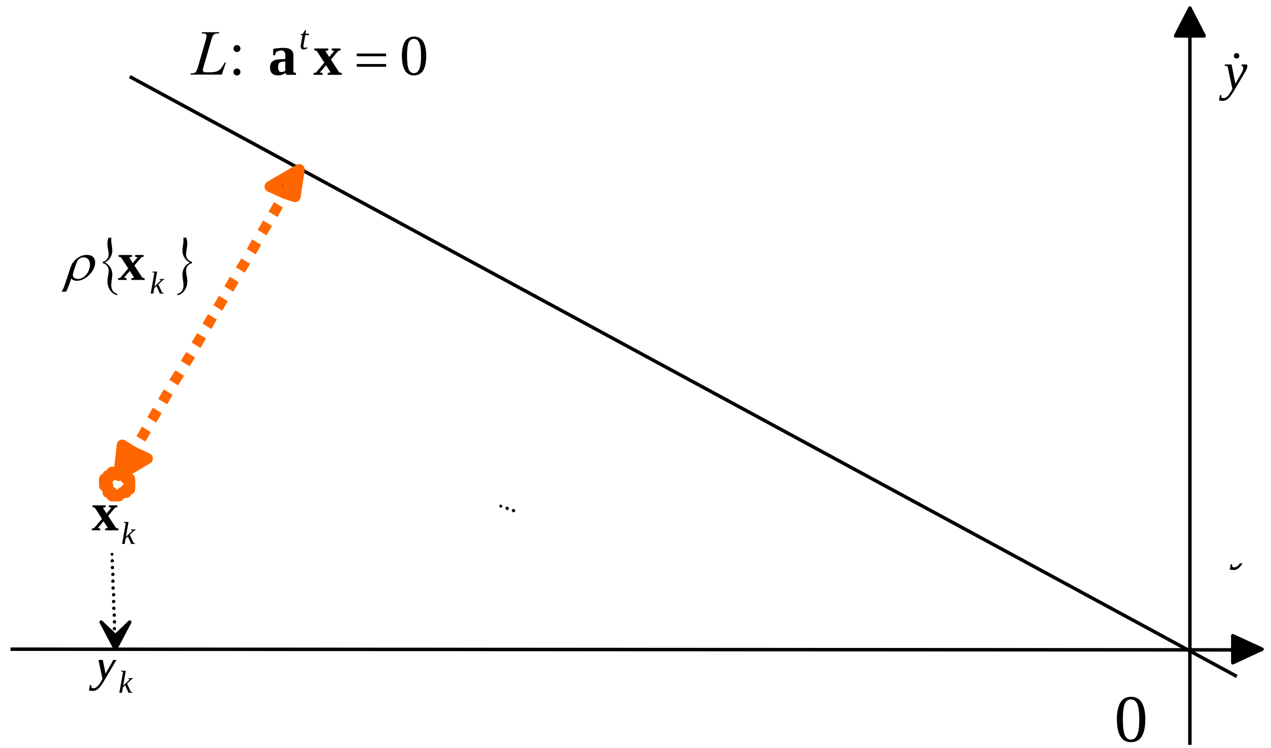
- Regular Decrease of the Distance from Origin Along Line *L*, Pole l1
- Eigenvectors



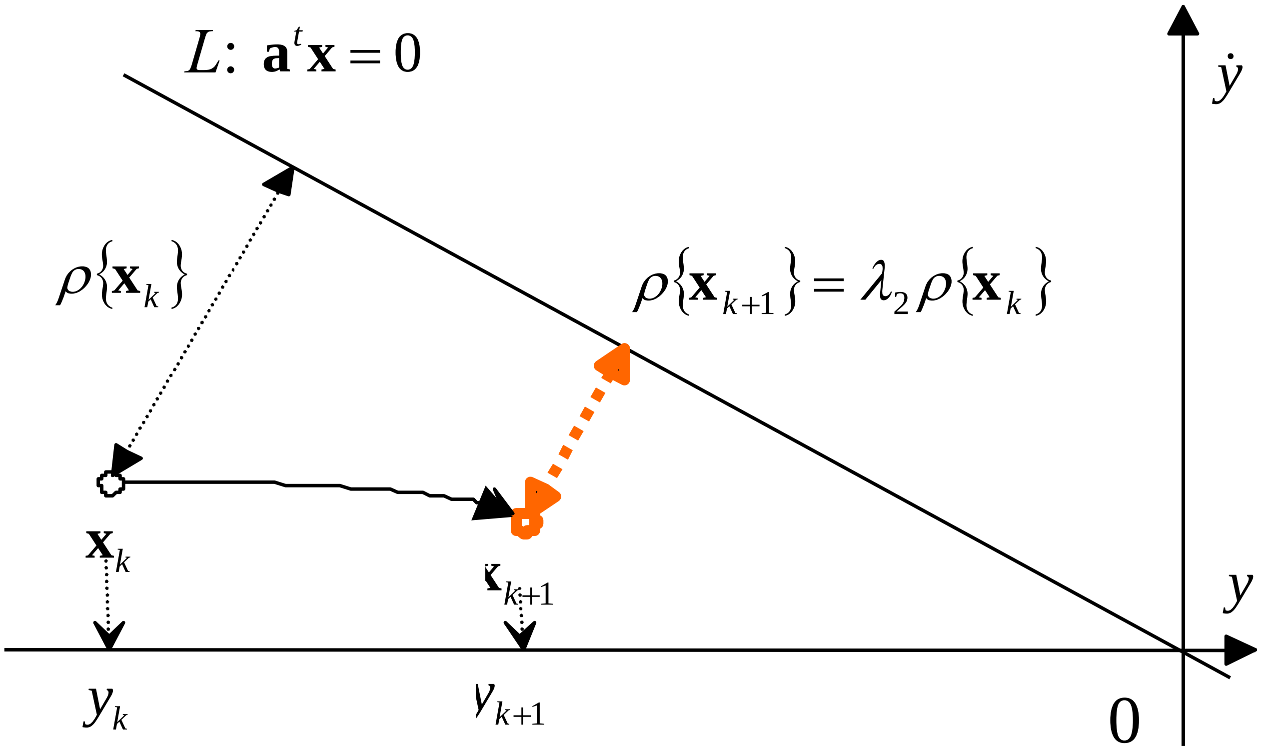
- Regular Decrease of the Distance from Origin Along Line *L*, Pole l1
- Eigenvectors



- Regular Decrease of the Distance from *L*, Pole l_2



- Regular Decrease of the Distance from *L*, Pole λ_2



Invariant Sets (La Salle)

- A generalization of the concept of equilibrium point
- Every system trajectory starting from a point in IS remains in IS for all future times
- An equilibrium point is an IS
- The domain of attraction of an equilibrium point, limit cycles, etc.

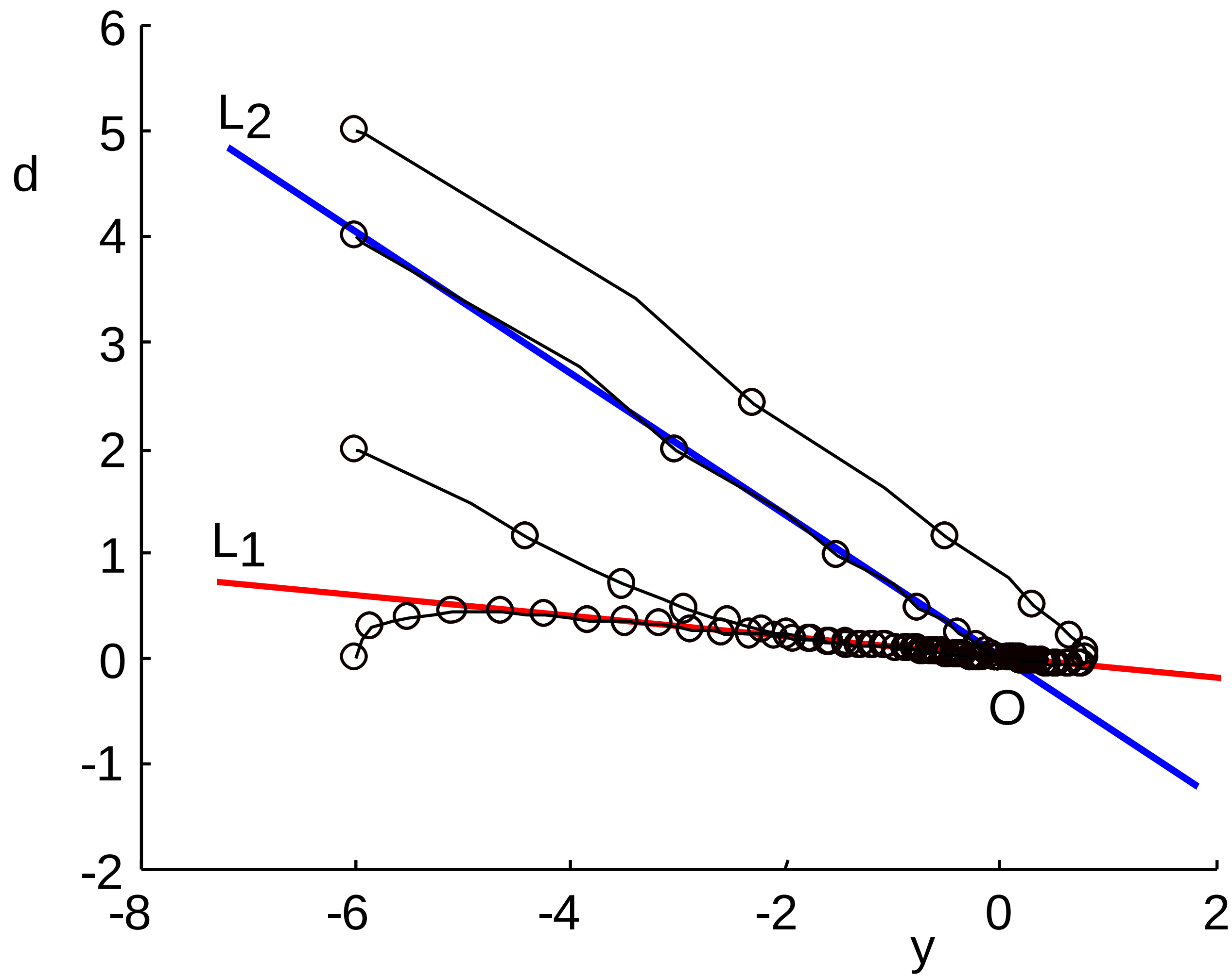
Pole Assignment Control

- A regular decrease
  of the representative point from the next invariant set of a lower dimension

$$\boxed{-\frac{\dfrac{d\rho_i}{dt}}{\rho} = \alpha_i}$$

$$\boxed{\frac{\rho_i(n+1)}{\rho_i(n)} = \lambda_i \; ; \;\; \lambda_i > 0}$$

LPAC: Phase-Plane Interpretation



Influence of Nonmodelled Dynamics

Restriction on closed loop poles admissible for output-monotonic and input-2P transients

Analytical derivation – Tripple Real Dominant Pole TRDP

Controller tuning

$$\boxed{r_0 = -0.079 / K_s T_d{}^2 \; ; \; r_1 = -0.461 / K_s T_d}$$

Equivalent poles

$$\boxed{\alpha_{1,2} = -(0.231 \pm j0.161)/T_d}$$

Approximations by real poles

Performance Portrait Based Tuning

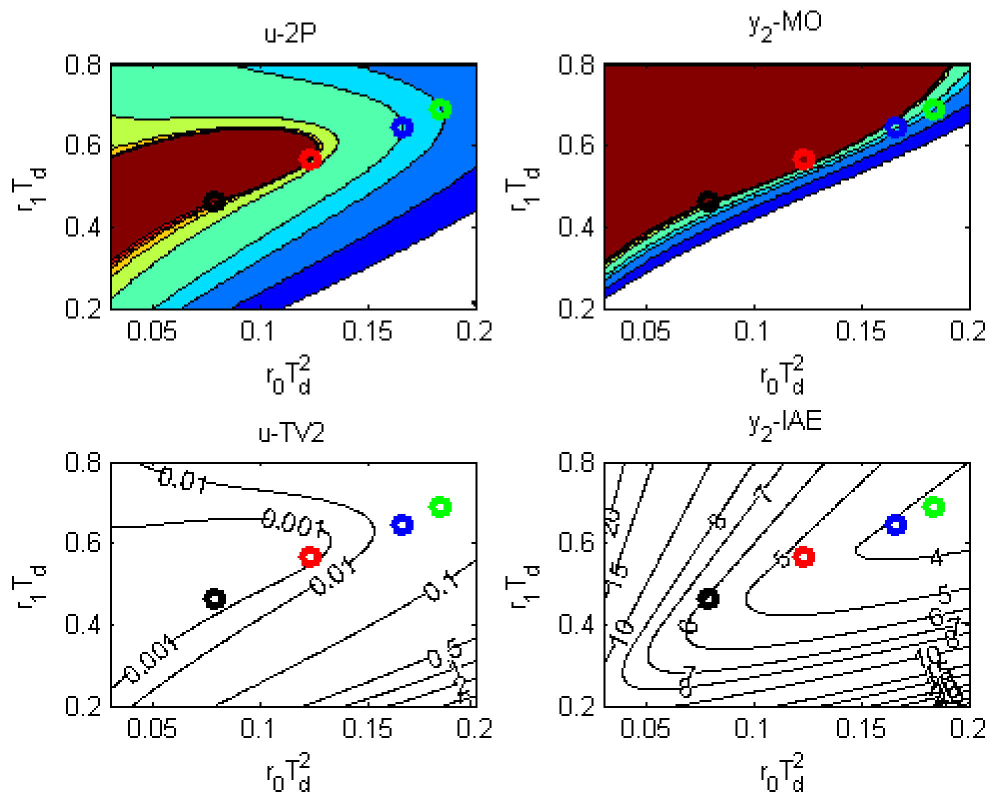
Performace Portrait with tolerated deviations from ideal shapes at the plant input and output & IAE

Black: TRDP

Red: ε=10^{-5}

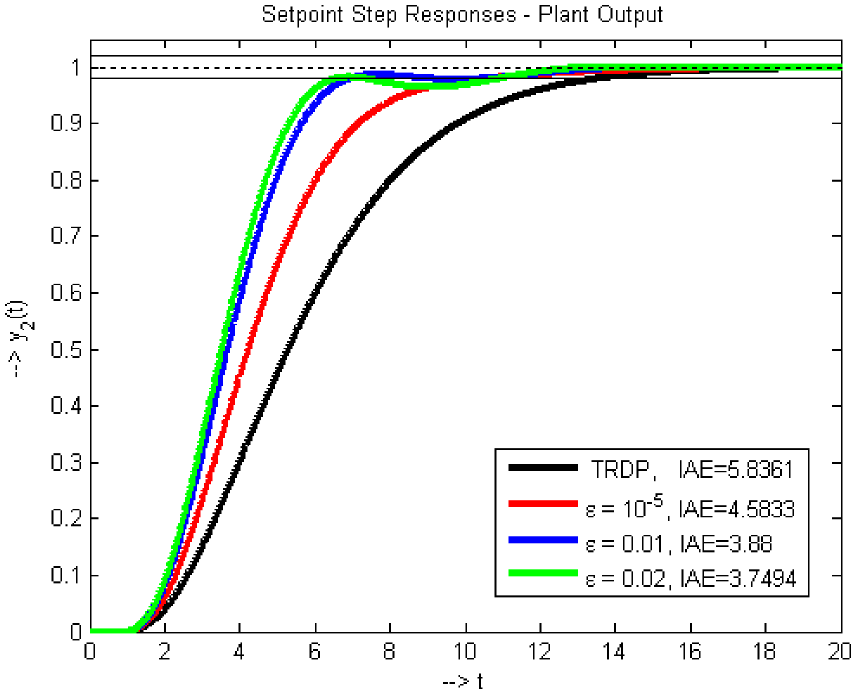
Blue: ε=0.01

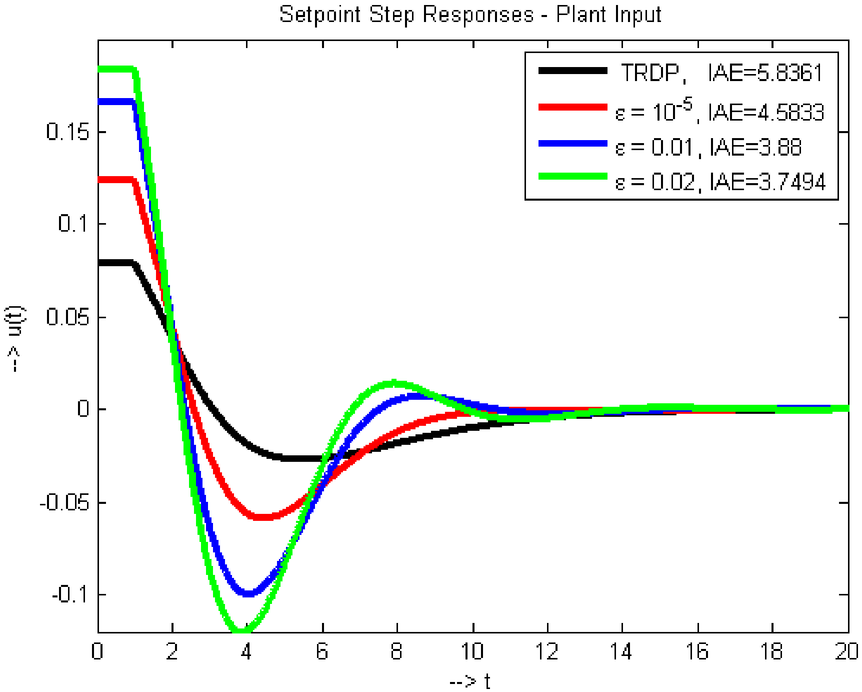
Green: ε=0.02



Corresponding setpoint steps

- Equivalent poles for tolerated deviations from ideal shapes at the plant input and output





Equivalent Poles

- Equivalent poles for tolerated deviations from ideal shapes at the plant input and output

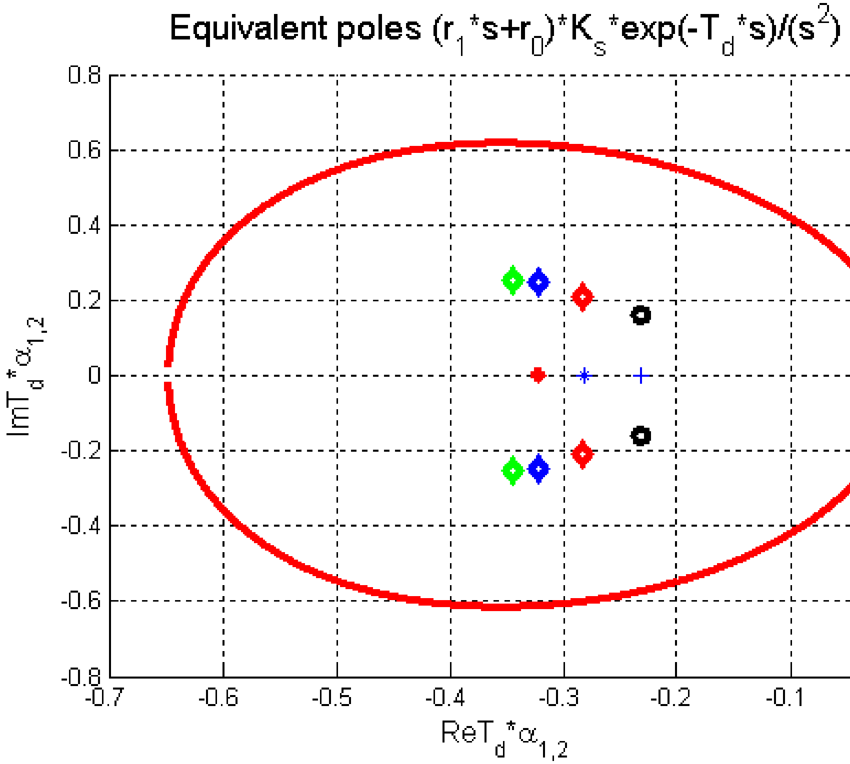
Black: TRDP

Red: ε=10^{-5}

Blue: ε=0.01

Green: ε=0.02

Red curve stability border

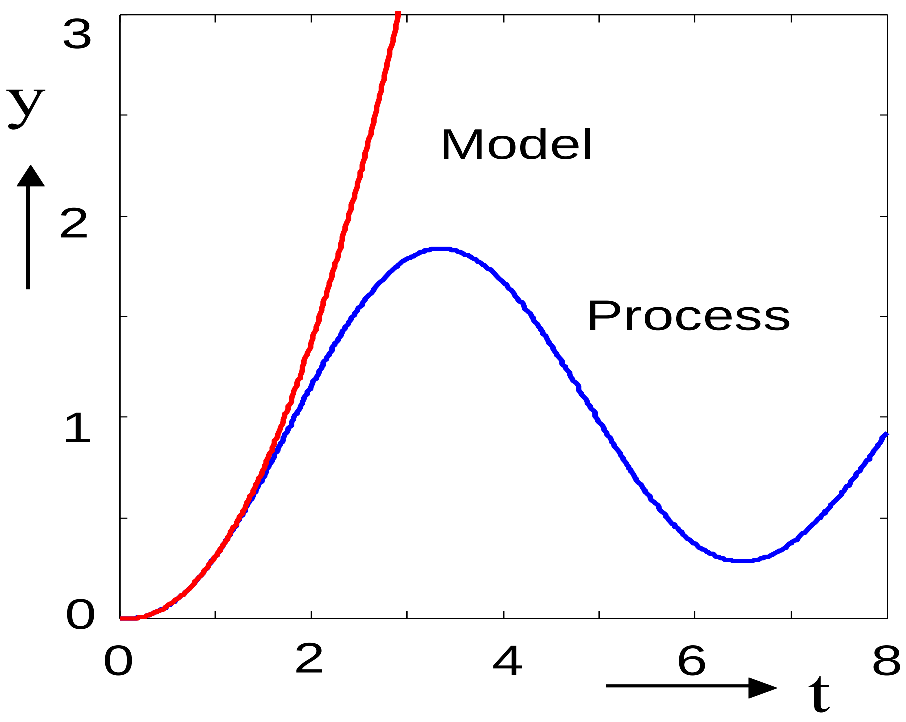


Example: I_2T_d - Approximation

Generalization of the approximation by Zielger and Nichols (1942) to double integrator + dead time

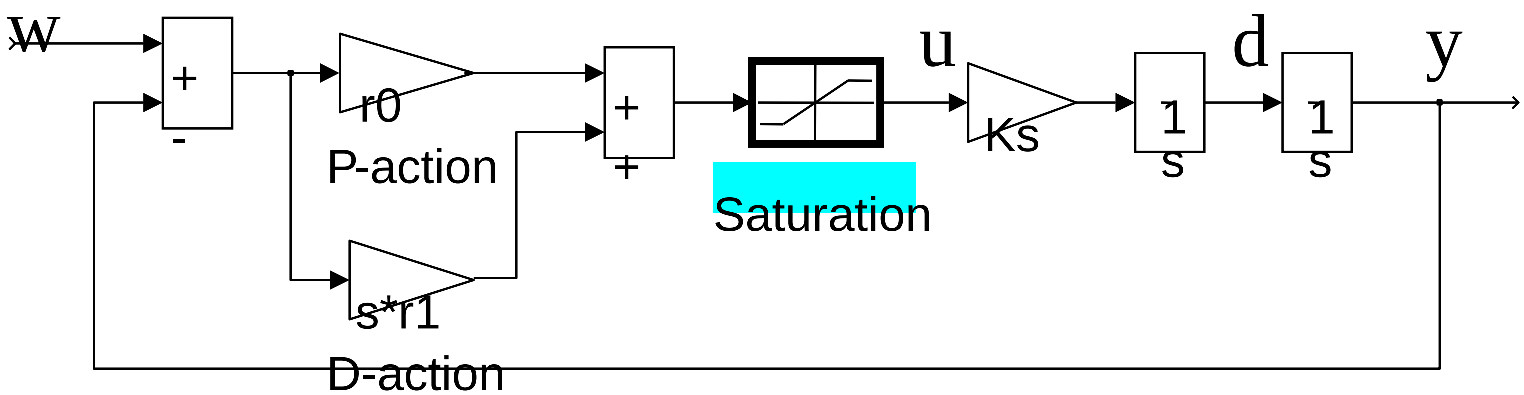
$$S(s) = \frac{1}{(s^2 + 0.1s + 1)(0.2s + 1)}$$

$$\hat{S}(s) = 0.76\frac{1}{s^2}e^{-0.1s}$$

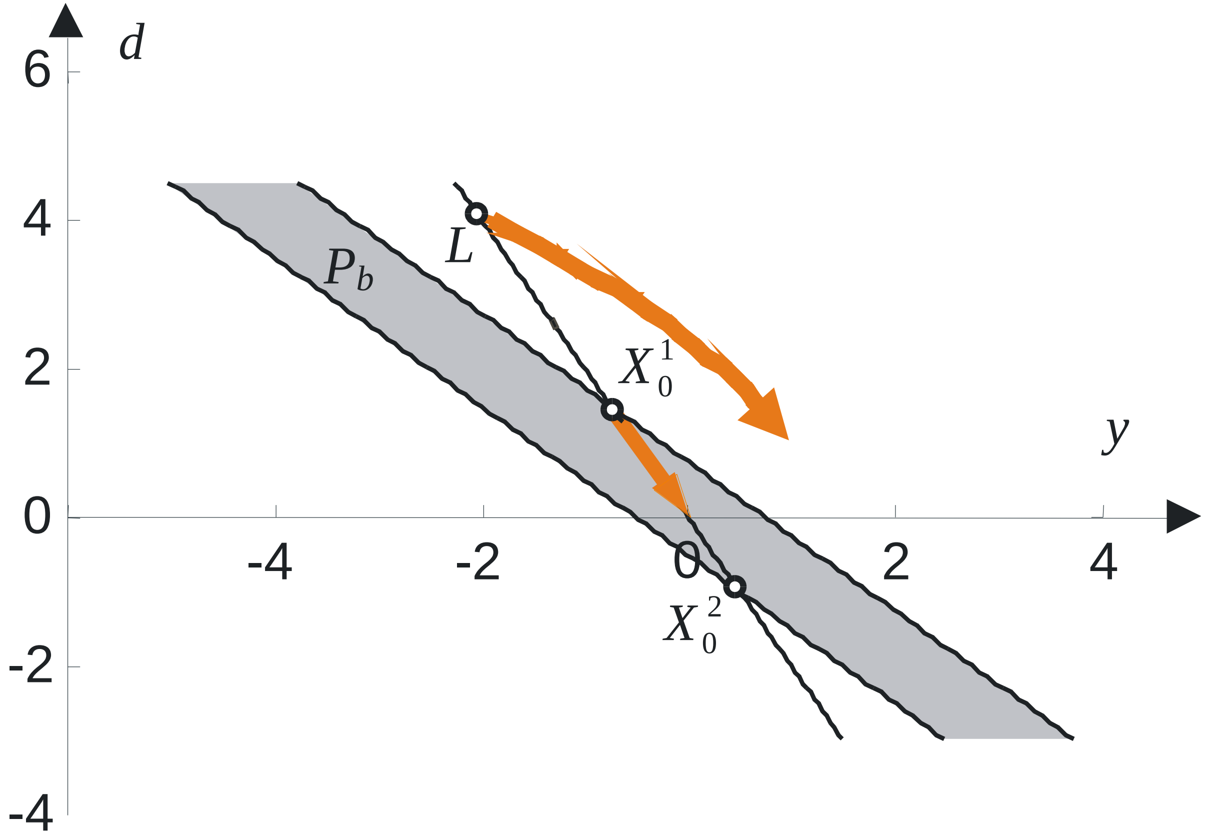


Saturating PD Controller

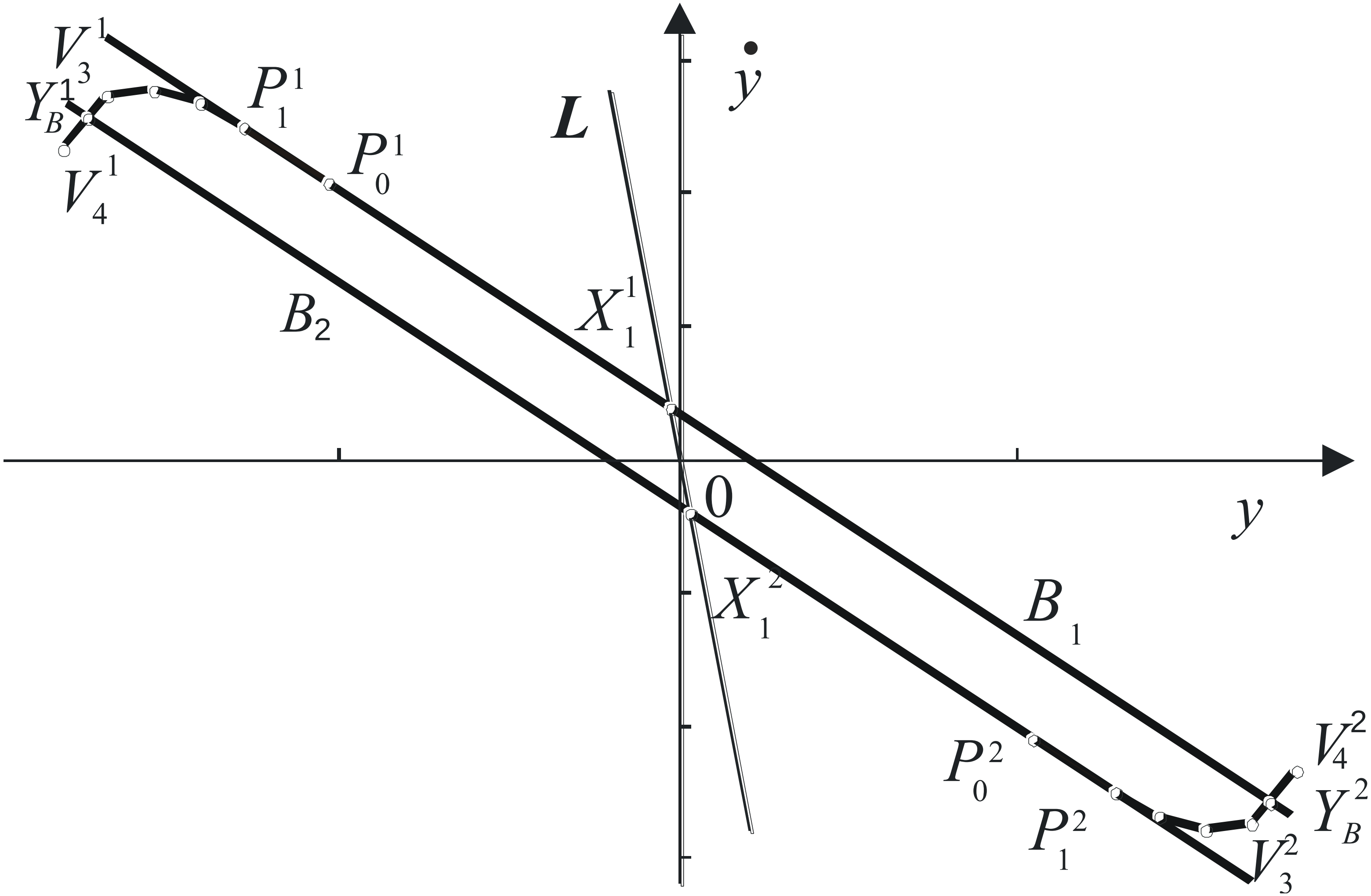
- Overshooting, instability
- **No optimal tuning of linear PD controller with output saturation!**
- New definition of invariant sets to consider constraints



Overshooting, instability due to the constraints



Invariant Set of Linear Control



CPAC - General Definition

- The fastest possible decrease of the distance between the representative point and the invariant set with a next lower dimension satisfying inequations:

$$\frac{\rho_i(n+1)}{\rho_i(n)} \le \lambda_i \; ; \;\; \lambda_i > 0$$

$$-\frac{\dfrac{d\rho_i}{dt}}{\rho} \le \left|\alpha_i\right|$$

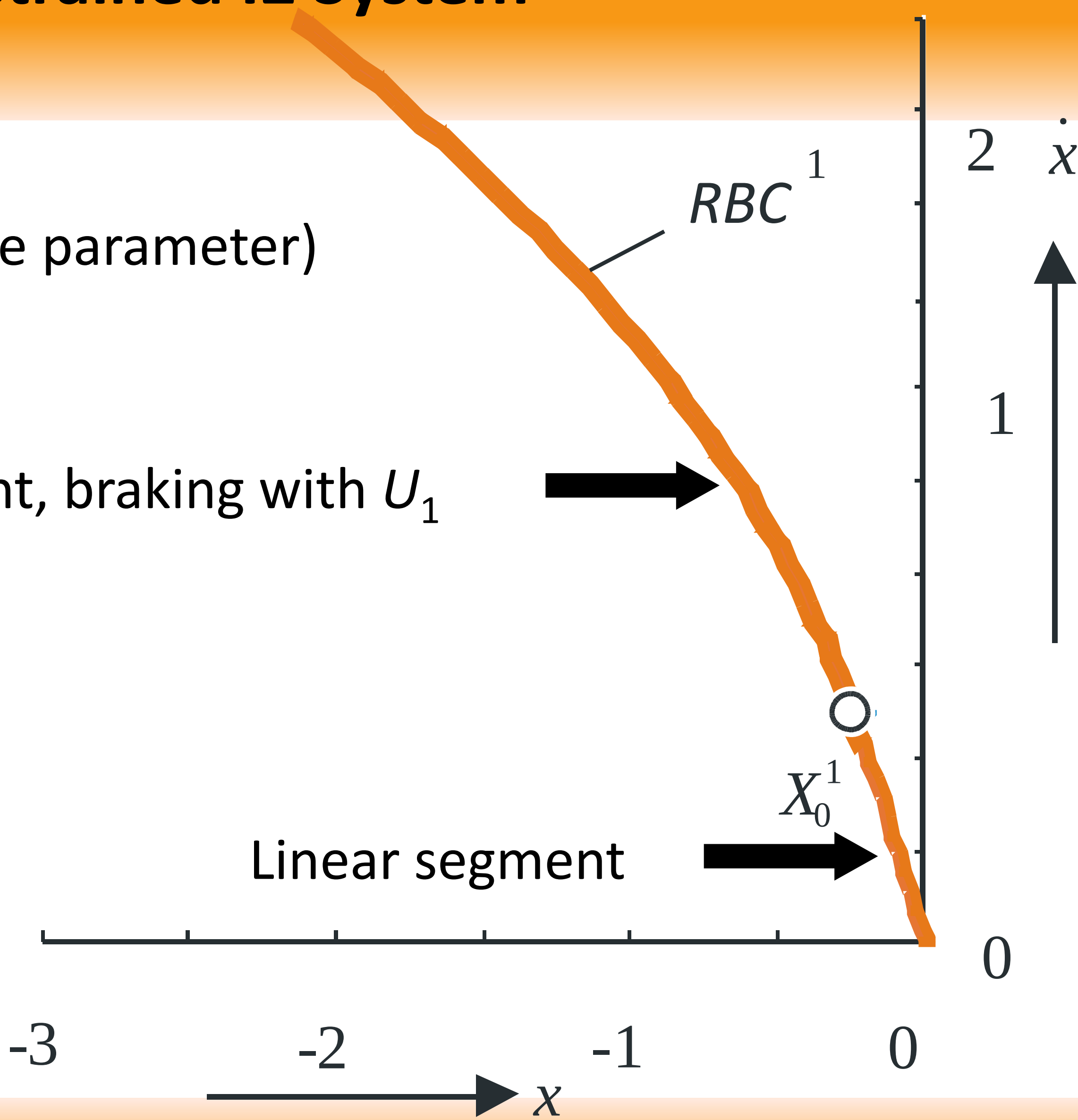
$$-\frac{\rho_i(n+1) - \rho_i(n)}{\rho_i(n)} \le 1 - \lambda_i$$

CPAC: Constrained I2 System

1D invariant set (1 free parameter)

Nonlinear segment, braking with U_1

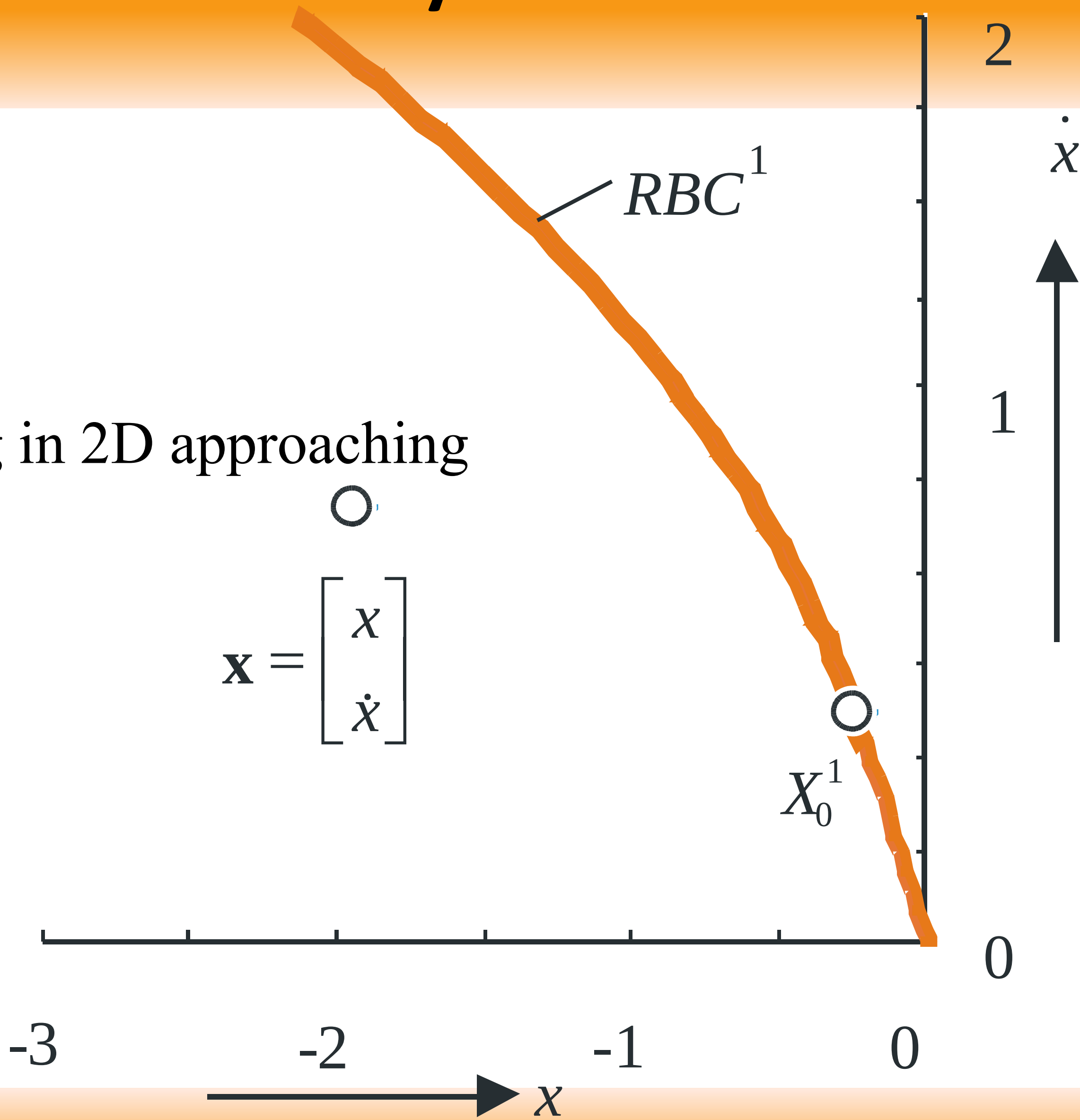
Linear segment



CPAC: Constrained I2 System

State moving in 2D approaching *RBC*

$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

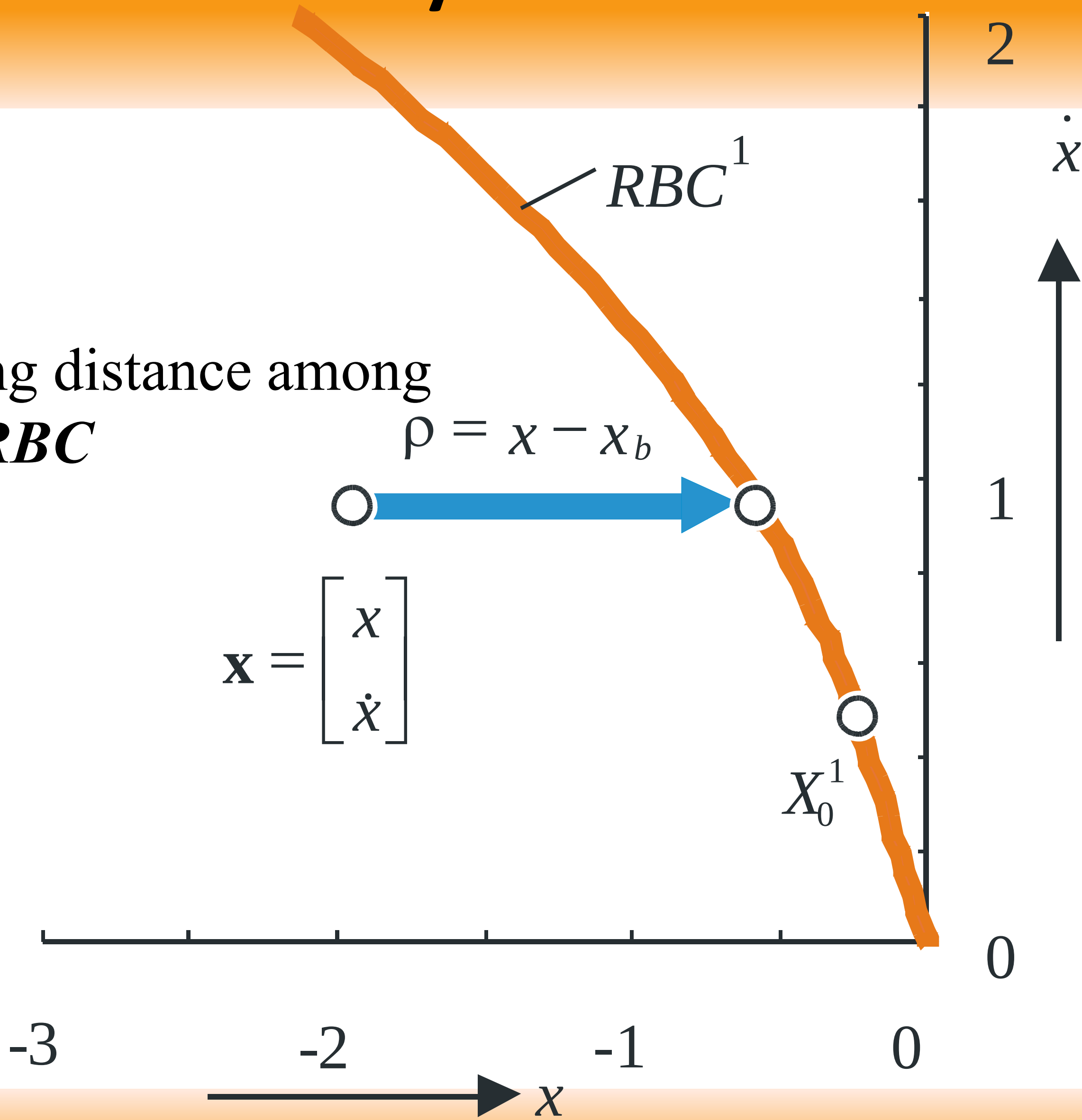


CPAC: Constrained I2 System

Defining distance among
x and ***RBC***

$\rho = x - x_b$

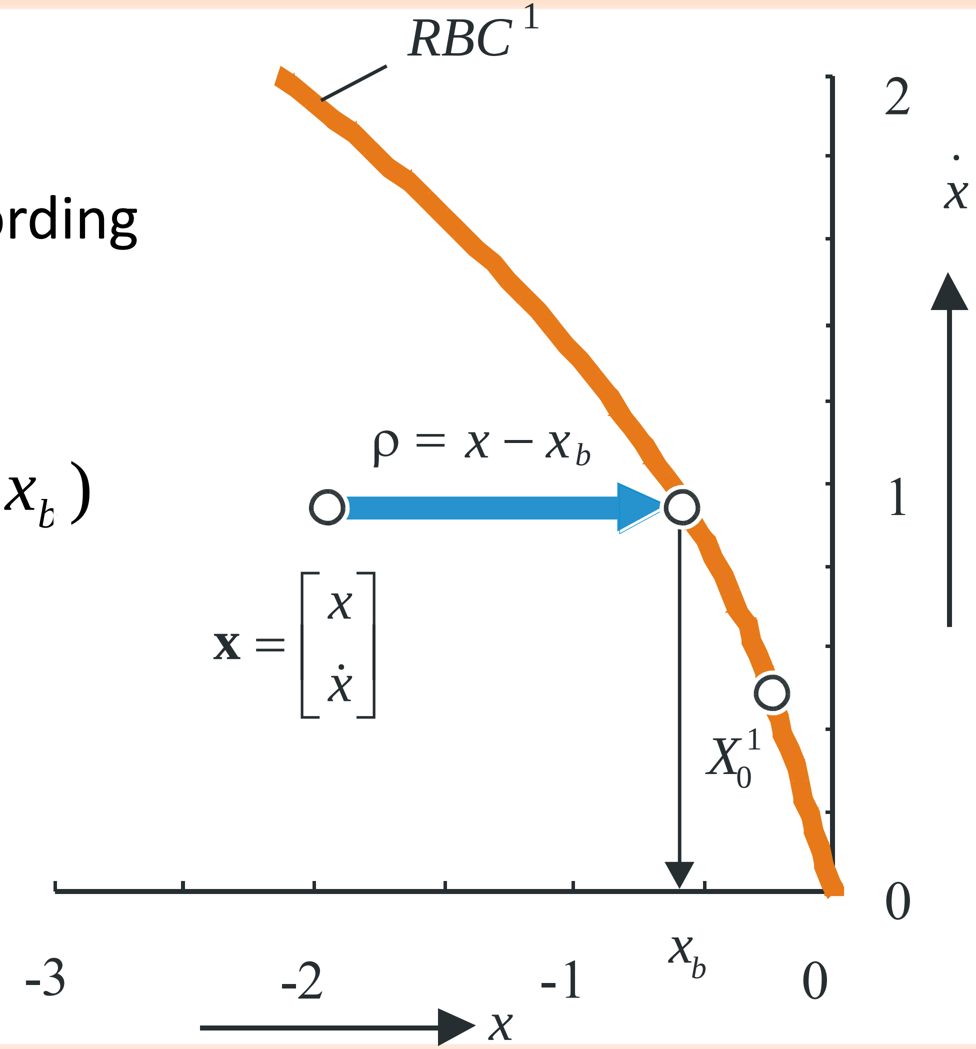
$$\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$



CPAC: Constrained I2 System

Decreasing distance acording to

$$\frac{d\rho}{dt} = \alpha_2 \rho \quad = \alpha_2 (x - x_b)$$

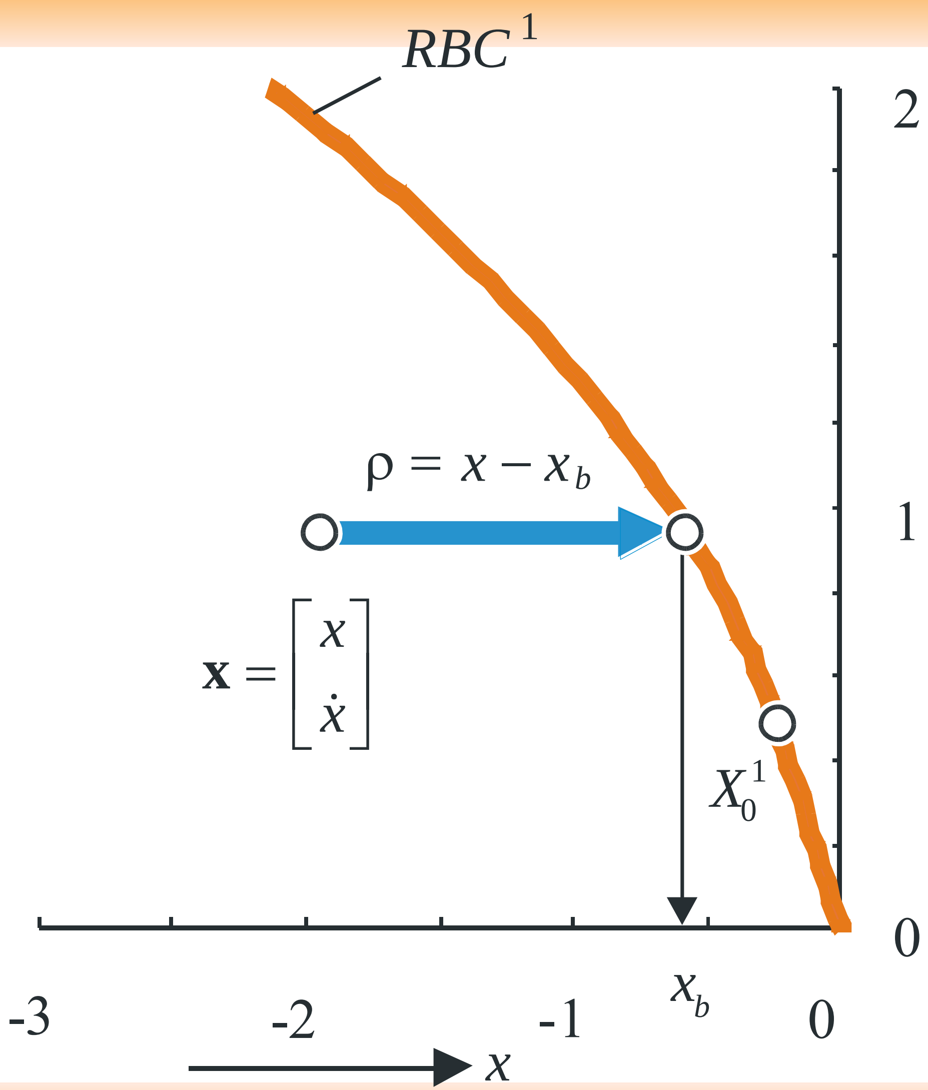


CPAC: Constrained I2 System

$$\rho = x - x_b$$

$$\frac{d\rho}{dt} = \alpha_2 \rho = \alpha_2 (x - x_b)$$

$$x_b = \frac{\dot{x}^2 + \left(\frac{U_j}{\alpha_1}\right)^2}{2U_j}$$



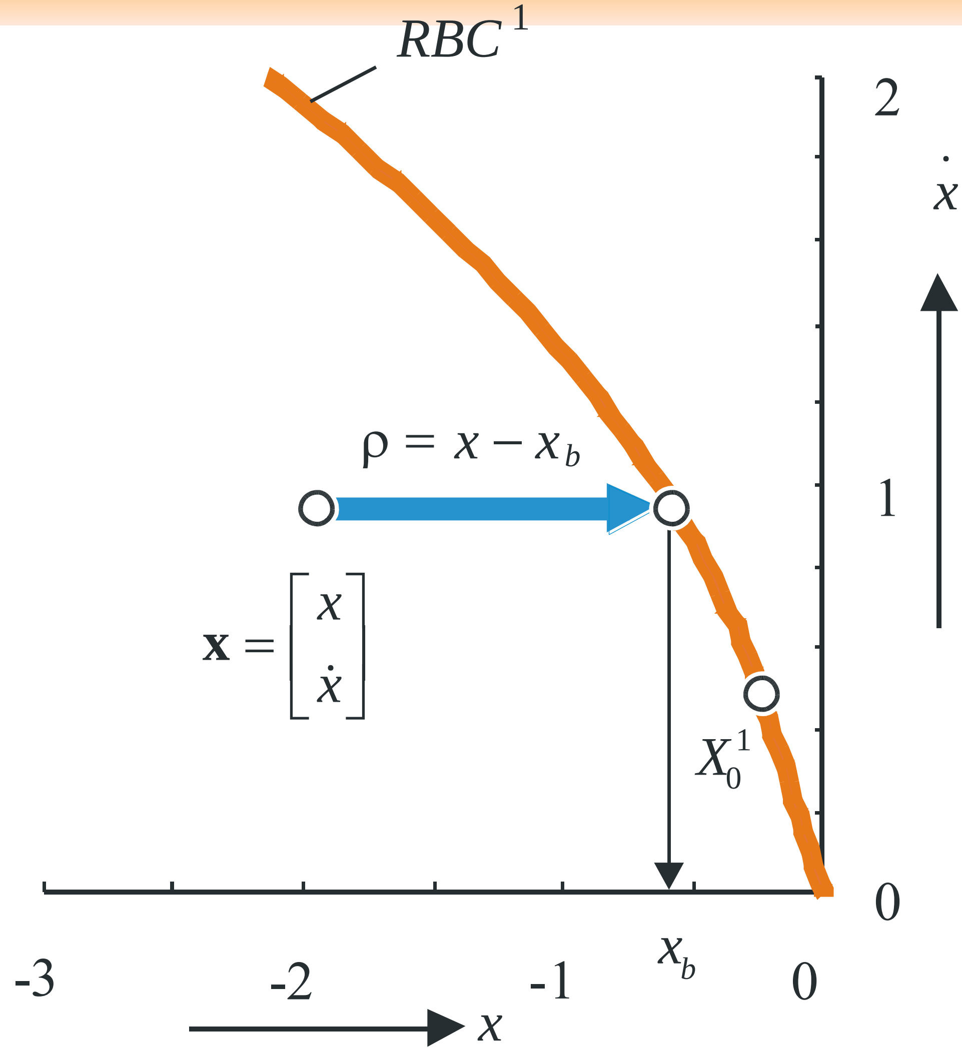
CPAC: Constrained I2 System

$$\rho = x - x_b$$

$$\frac{d\rho}{dt} = \alpha_2 \rho = \alpha_2 (x - x_b)$$

$$x_b = \frac{\dot{x}^2 + \left(\frac{U_j}{\alpha_1}\right)^2}{2U_j}$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial x}\dot{x} + \frac{\partial \rho}{\partial \dot{x}}\ddot{x}$$



CPAC: Constrained I2 System

- Saturation Limit for Braking

$$if \;\; x<0 \;\; then \;\; U_j = U_1 \;\; else \;\; U_j = U_2$$

- Choice Between the Linear and Nonlinear Algorithm

$$if \left(y<0 \;\; AND \; \dot{x} > \frac{K_s U_j}{\gamma} \right) OR \left(x>0 \;\; AND \;\; \dot{x} < \frac{K_s U_j}{\gamma} \right) then$$

$$u = \left[1-\alpha_2 \frac{x - \frac{1}{2}\left(\frac{\dot{x}^2}{K_s U_j} + \frac{K_s U_j}{\alpha_1^2} \right)}{\dot{x}} \right] U_j \; else \; u = r_0 x + r_1 \dot{x}$$

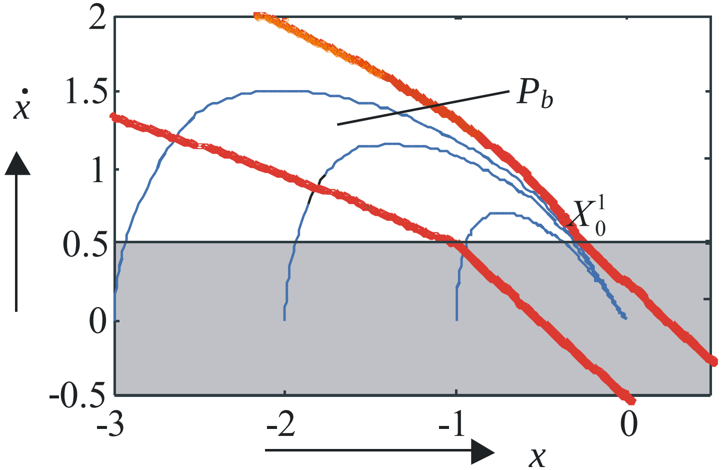
Final Control Saturation

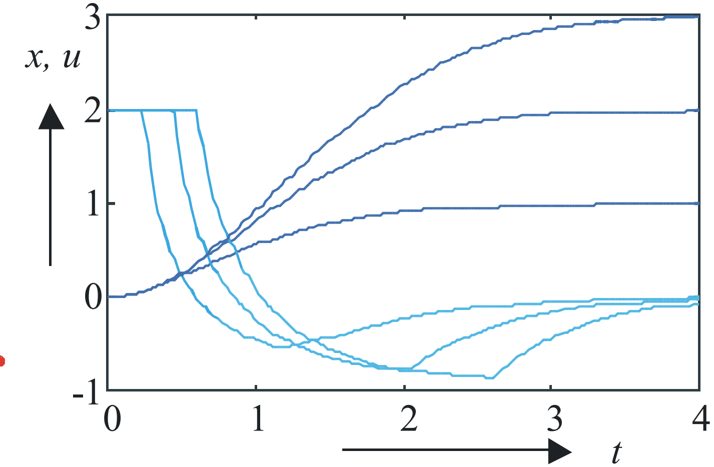
$$if \;\; u<U_1 \;\; then \;\; u=U_1 \qquad\qquad if \;\; u>U_2 \;\; then \;\; u=U_2$$

CPAC: Constrained I2 System

'Slow' and "fast" poles

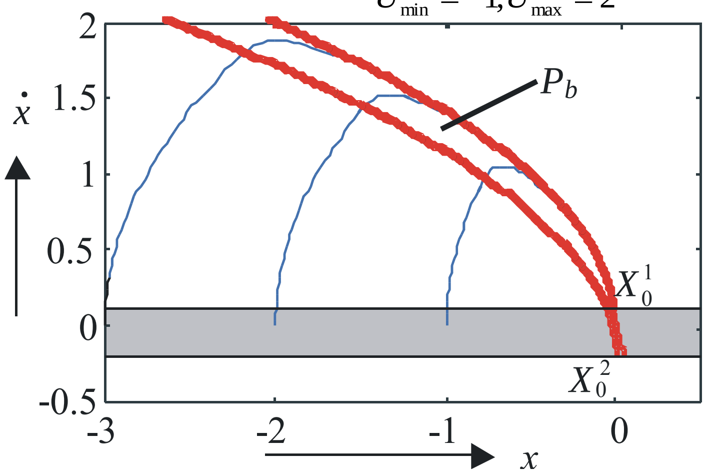
$$\alpha_{1,2} = -2$$

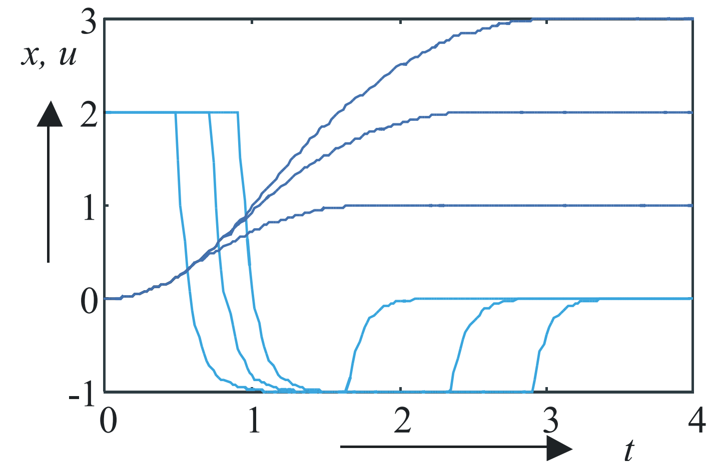




$U_{\min} = -1; U_{\max} = 2$

$$\alpha_{1,2} = -10$$



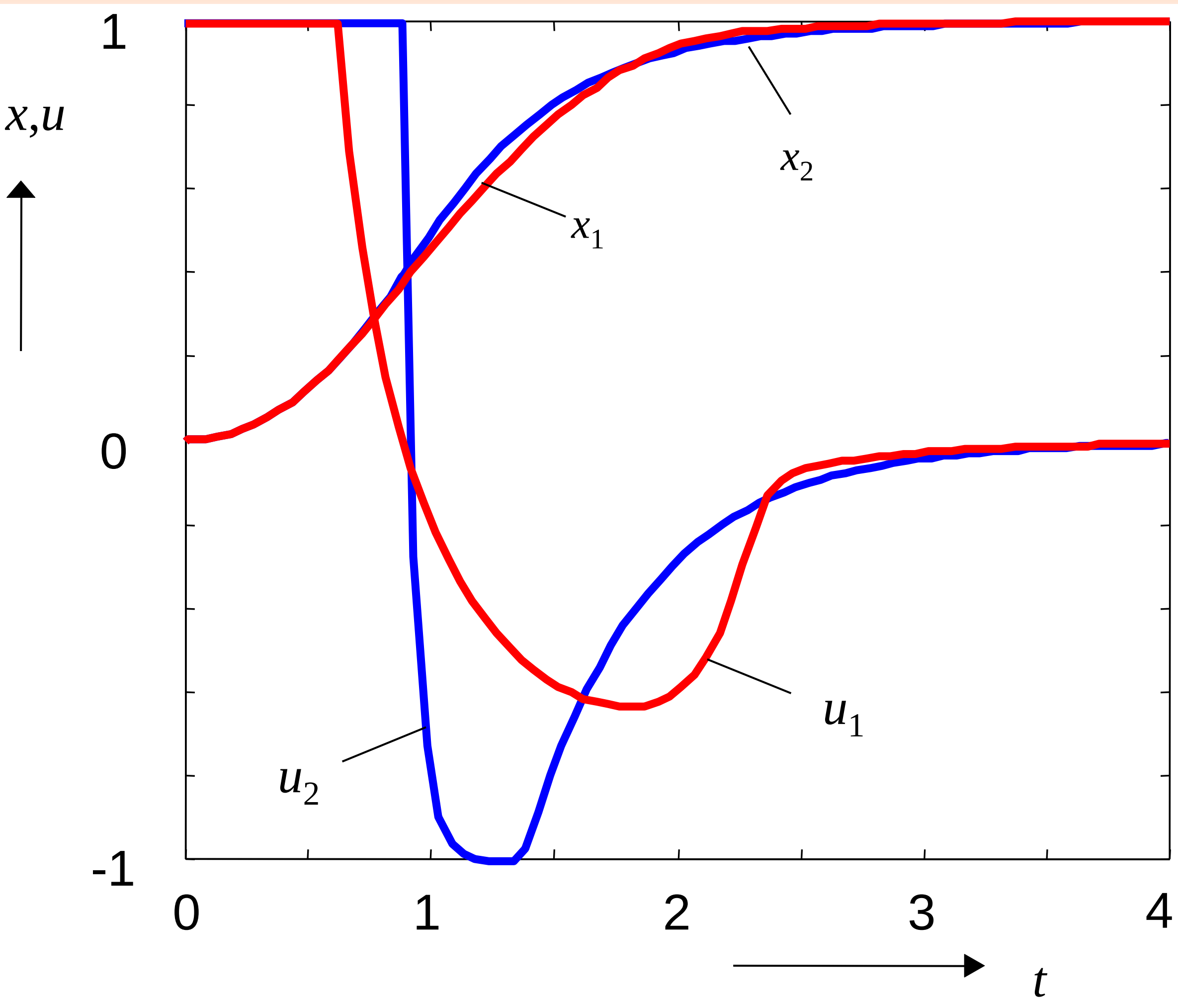


CPAC: Constrained I2 System

Oriented Vector
of Poles

- 1) (-2, -20)
- 2) (-20, -2)

Changed dynamics
of braking
and acceleration



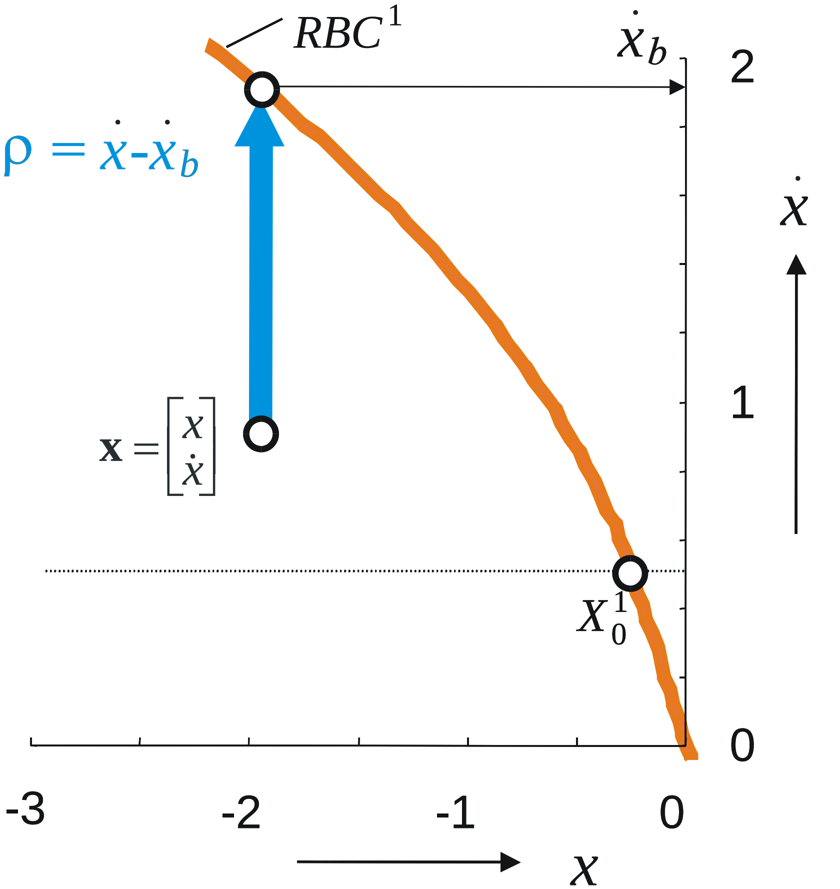
CPAC: Constrained I2 System

- Different distance definition

$$\rho = \dot{x} - \dot{x}_b$$

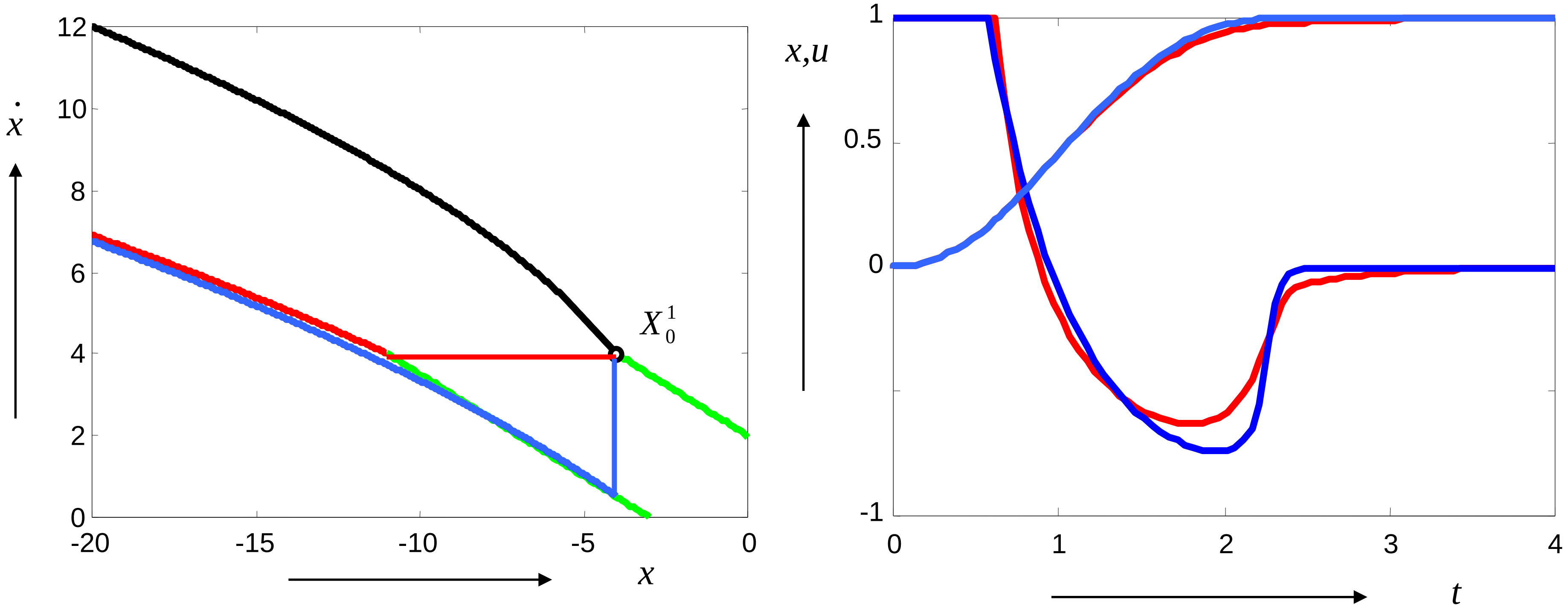
$$u = sign(x)\left[\alpha_2(p - \dot{x}) - \frac{U_j \dot{x}}{p}\right]$$

$$p = \sqrt{U_1\left(2x - \frac{U_j}{\alpha_1^2}\right)}$$



- Different Proportional Band
- Different transients

- Proportional Band and Transients for Different Distance definitions



CPAC: Constrained I2 System

- Modified shape of invariant sets
- Non-unique distance definition = many solutions

$$\rho = \inf_{\mathbf{x}_b \in RBC} \left\| \mathbf{x} - \mathbf{x}_b \right\|$$

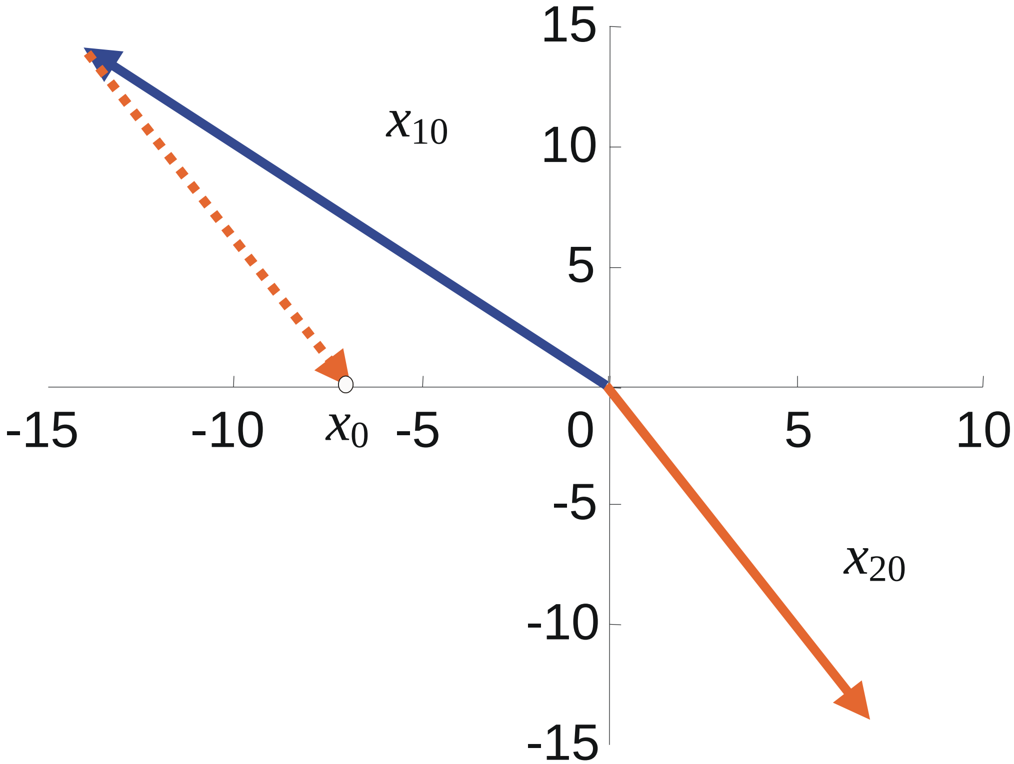
- Distance measured along a given vector

2nd Basic Question

- How to define distance of the representative point to the next Invariant Set - Reference Braking Curve?
- Several possible solutions

- **Simplicity of control algorithm!!!**

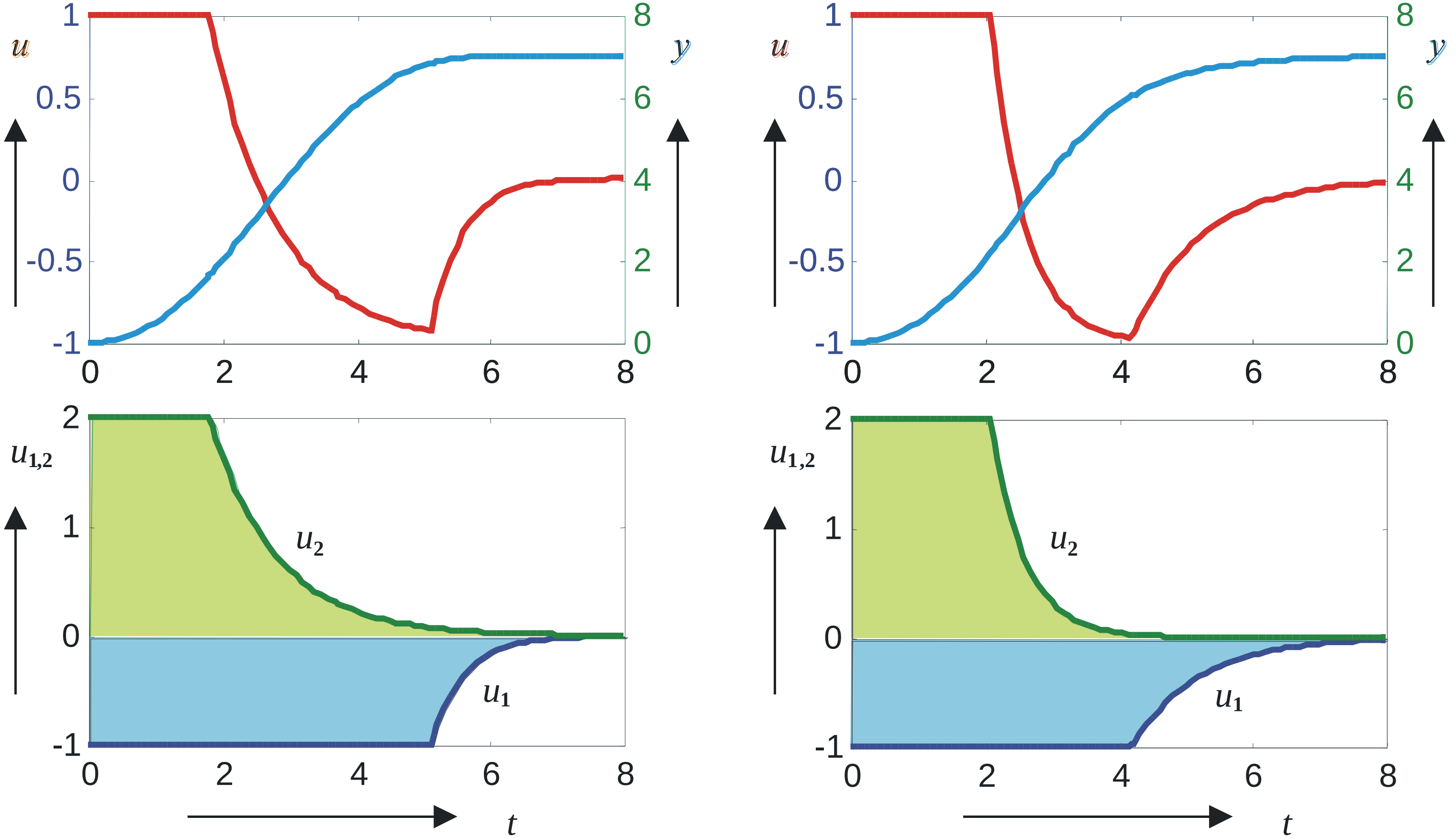
Linear Decomposition into Modes (Parallel Model)



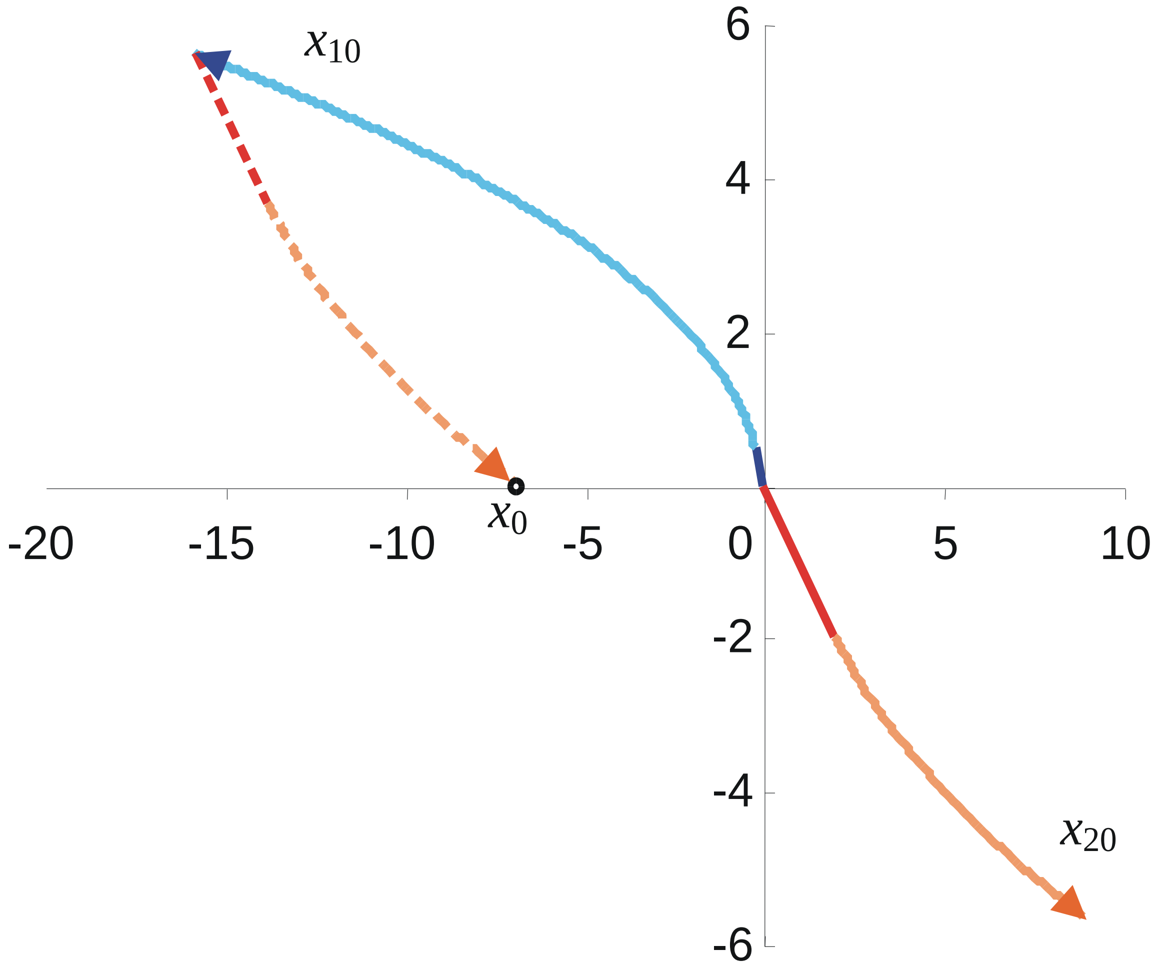
CPAC: Decomposition into Modes

2^{nd} order system - 2 ordered pole couples!

$\alpha_1 = -2; \alpha_2 = -1$ $\alpha_1 = -1; \alpha_2 = -2$



CPAC: Decomposition into Modes



CPAC: Decomposition into Modes

$$\mathbf{x}_0 = \mathbf{x}_{01} + \mathbf{x}_{02}$$

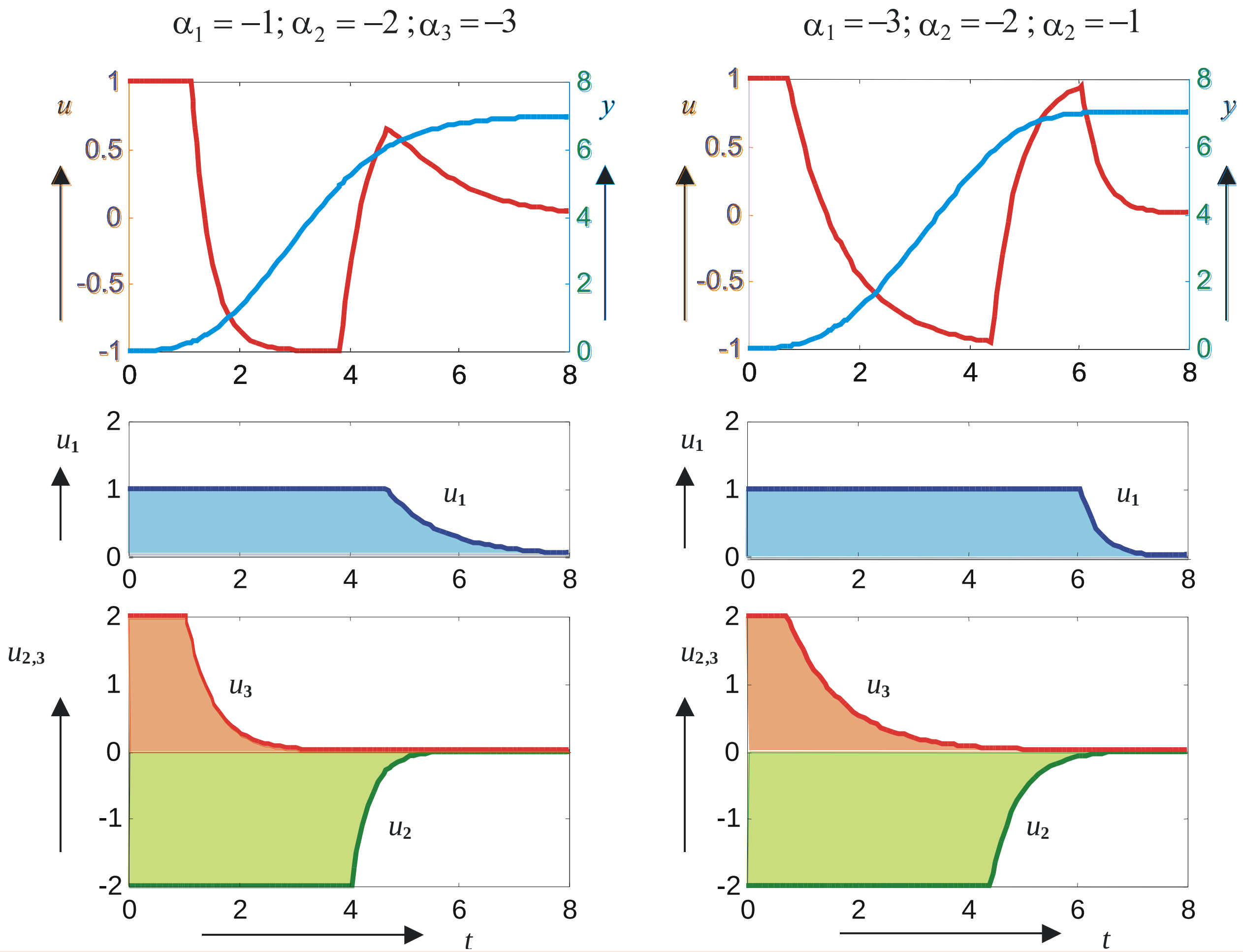
$$\mathbf{x}_{01} = \left[\Phi(-t_1)\mathbf{v}_1 + \Gamma(-t_1)\right]u_1 \;;\; u_1 = \left\langle \begin{matrix} \langle 0, U_j \rangle, t_1 = 0 \\ U_j,\; t_1 > 0 \end{matrix} \right.$$

$$\mathbf{x}_{02} = \left[\Phi(-t_2)\mathbf{v}_2 + \Gamma(-t_2)\right]u_2 ; u_2 = \left\langle \begin{matrix} \langle U_{3-j} - U_j \rangle, t_2 = 0 \\ U_{3-j} - U_j, t_2 > 0 \end{matrix} \right.$$

$$t_1 \geq t_2 \geq 0 \;;\; j = 1,\; or\; 2 \;;\; u = u_1 + u_2 \in \langle U_1, U_2 \rangle$$

$$\Phi(t) = e^{\mathbf{A}t} \;;\; \Gamma(t) = \int_0^t \Phi(\vartheta)\mathbf{b}\,d\vartheta$$

3rd order system - 6 ordered pole tripples!



Conclusions – Real Poles

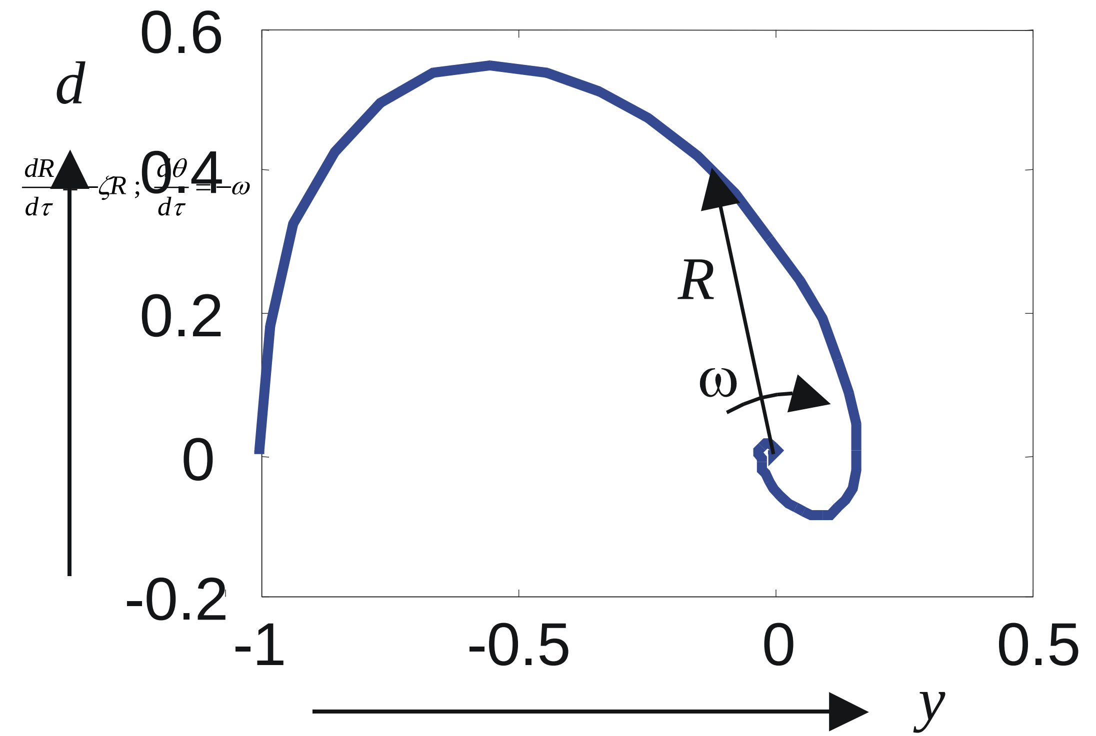
- New solutions for simple and advanced problems
- Many solutions of the constrained pole assignment problem - distance definition – ordered n-tuples of poles
- Simplicity of the control algorithms
  CPU Time $\approx$ 0.1ms - new opportunities for practice
- Nonsymetrical amplitude and rate constraints

Complex Closed Loop Poles

- Dynamics decomposition into saturating 1st order ones – limited to real closed loop poles
- What about the extension of the pole assignment control to constrained systems with complex closed loop poles?

Rotation, radius decreas

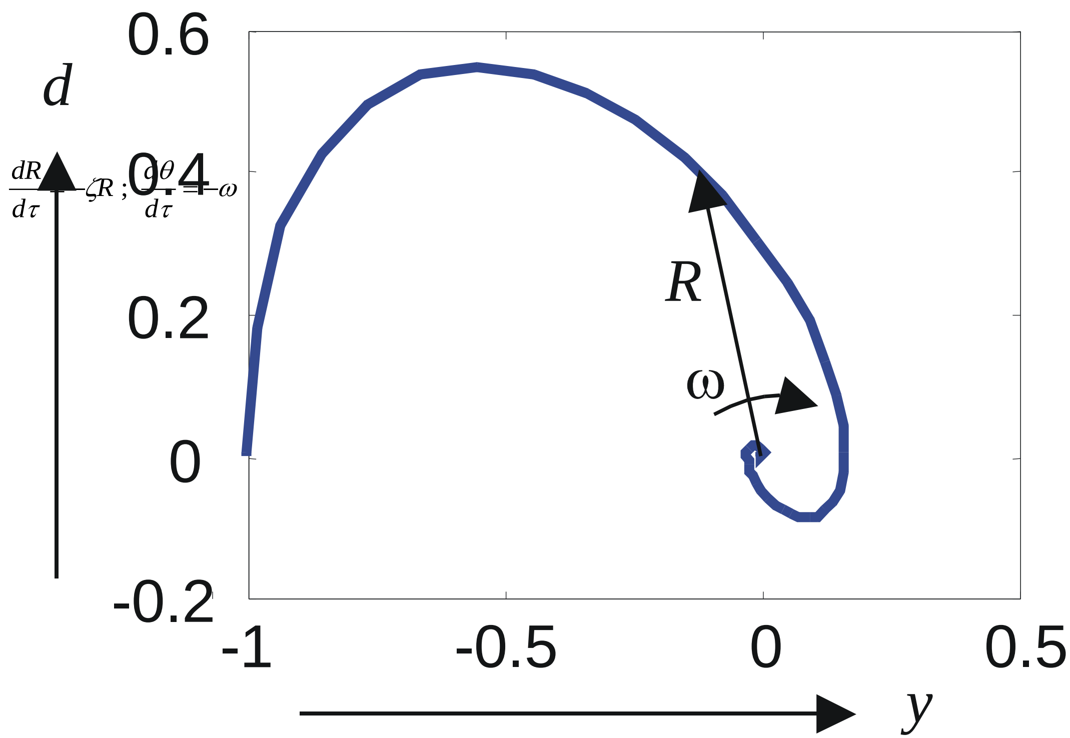
Single decrease parameter!!!



$$\frac{d\left(\mathbf{x}^t\mathbf{x}\right)}{d\tau} = -2\zeta\left(\mathbf{x}^t\mathbf{x}\right)$$

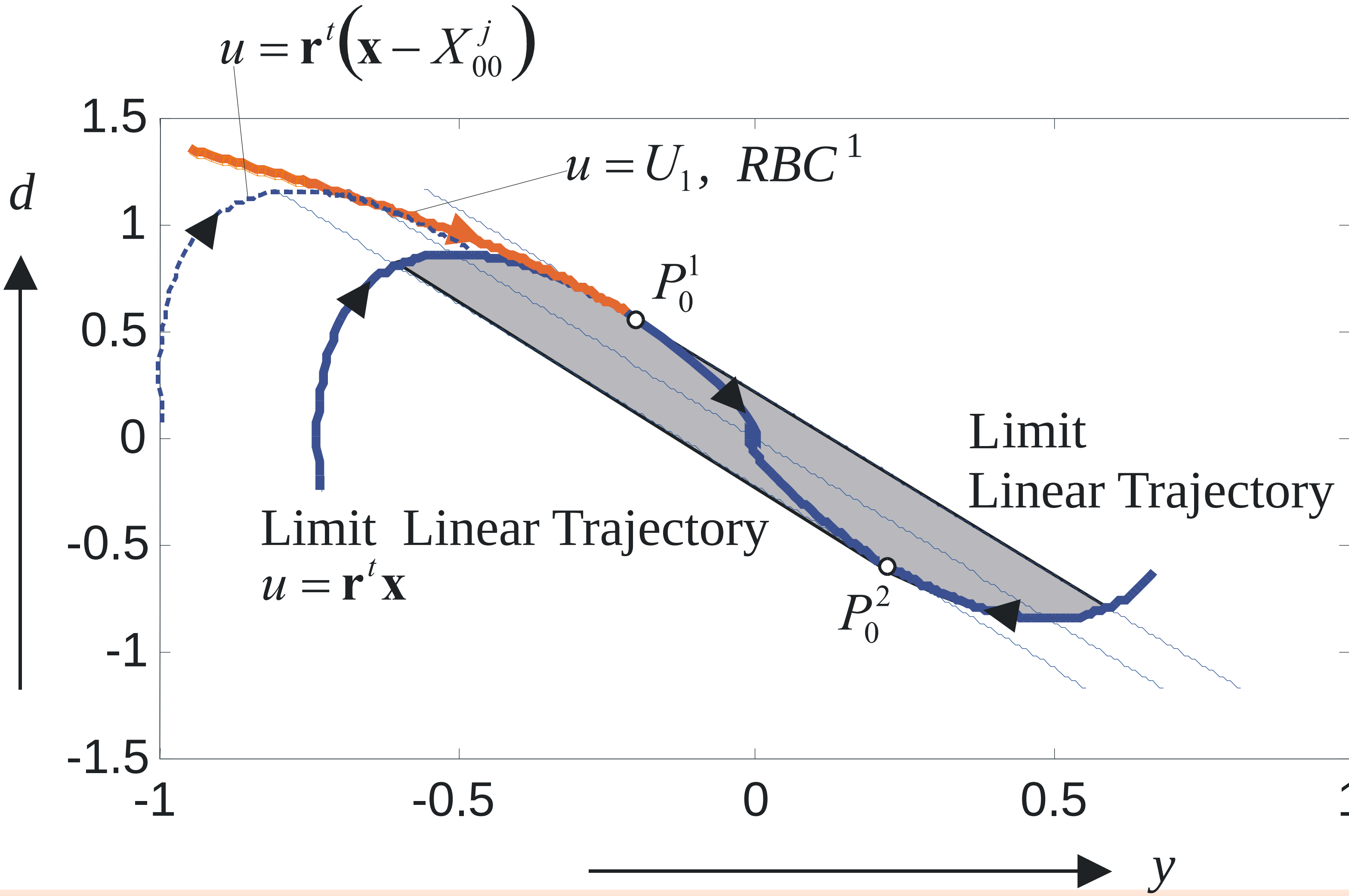
Complex Closed Loop Poles

What should be like the constrained PAC with complex poles?



$$\frac{d\left(\mathbf{x}^t\mathbf{x}\right)}{d\tau} = -2\zeta\left(\mathbf{x}^t\mathbf{x}\right)$$

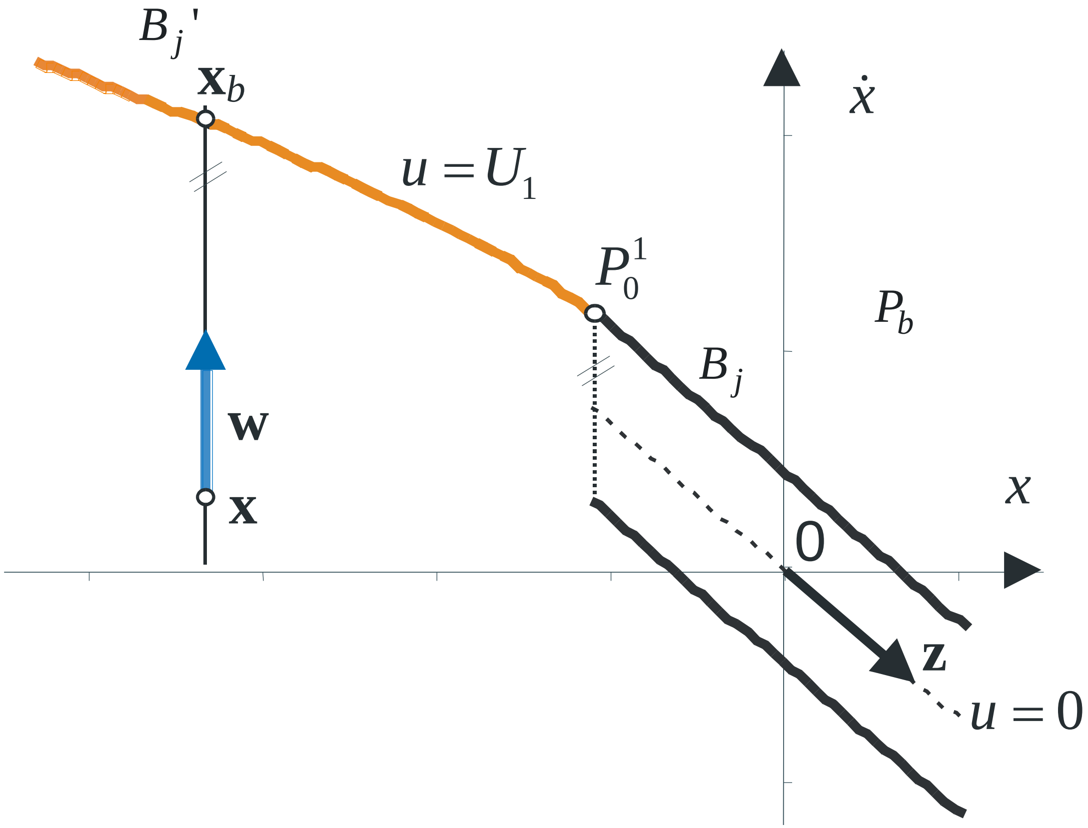
Complex Closed Loop Poles



Complex Closed Loop Poles

Position based reference shaping

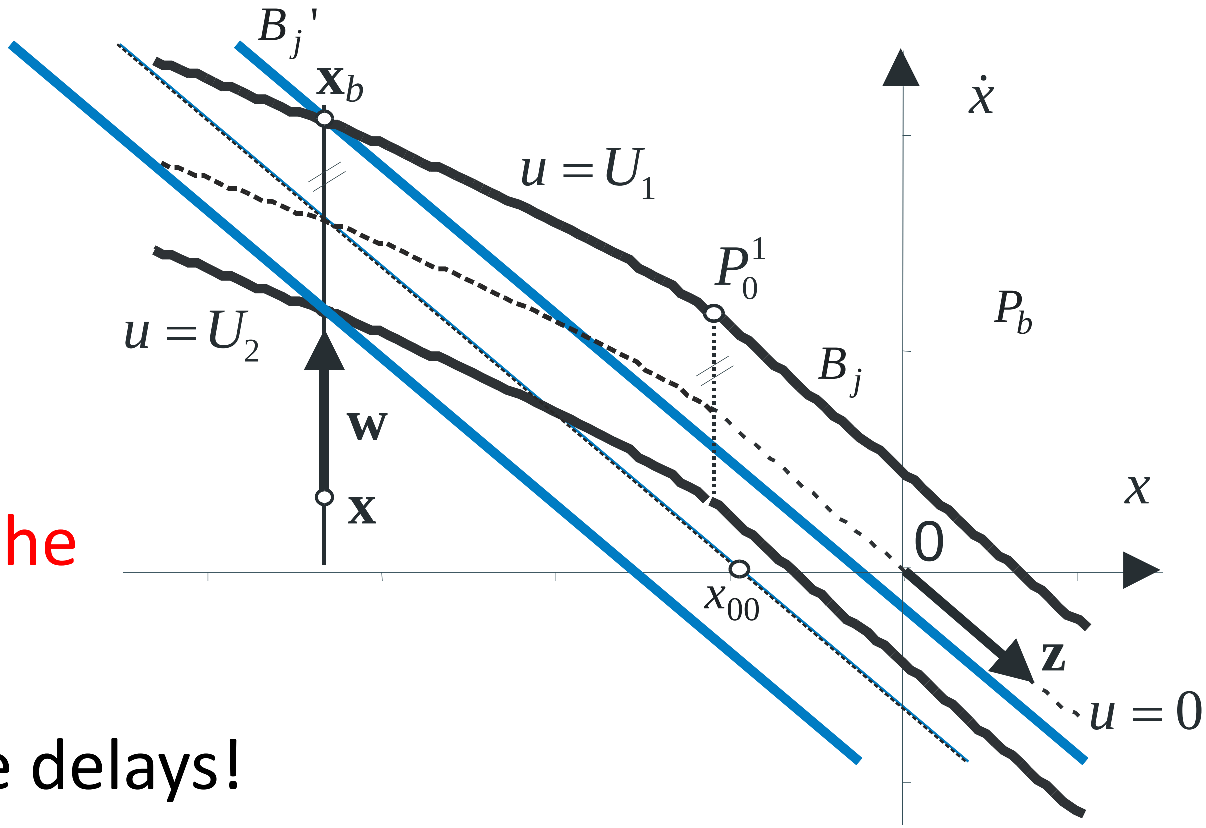
Limit linear trajectory – Points P_0^j - RBC



Complex Closed Loop Poles

Position based reference shaping

Limit linear trajectory – Points P_0^j - RBC

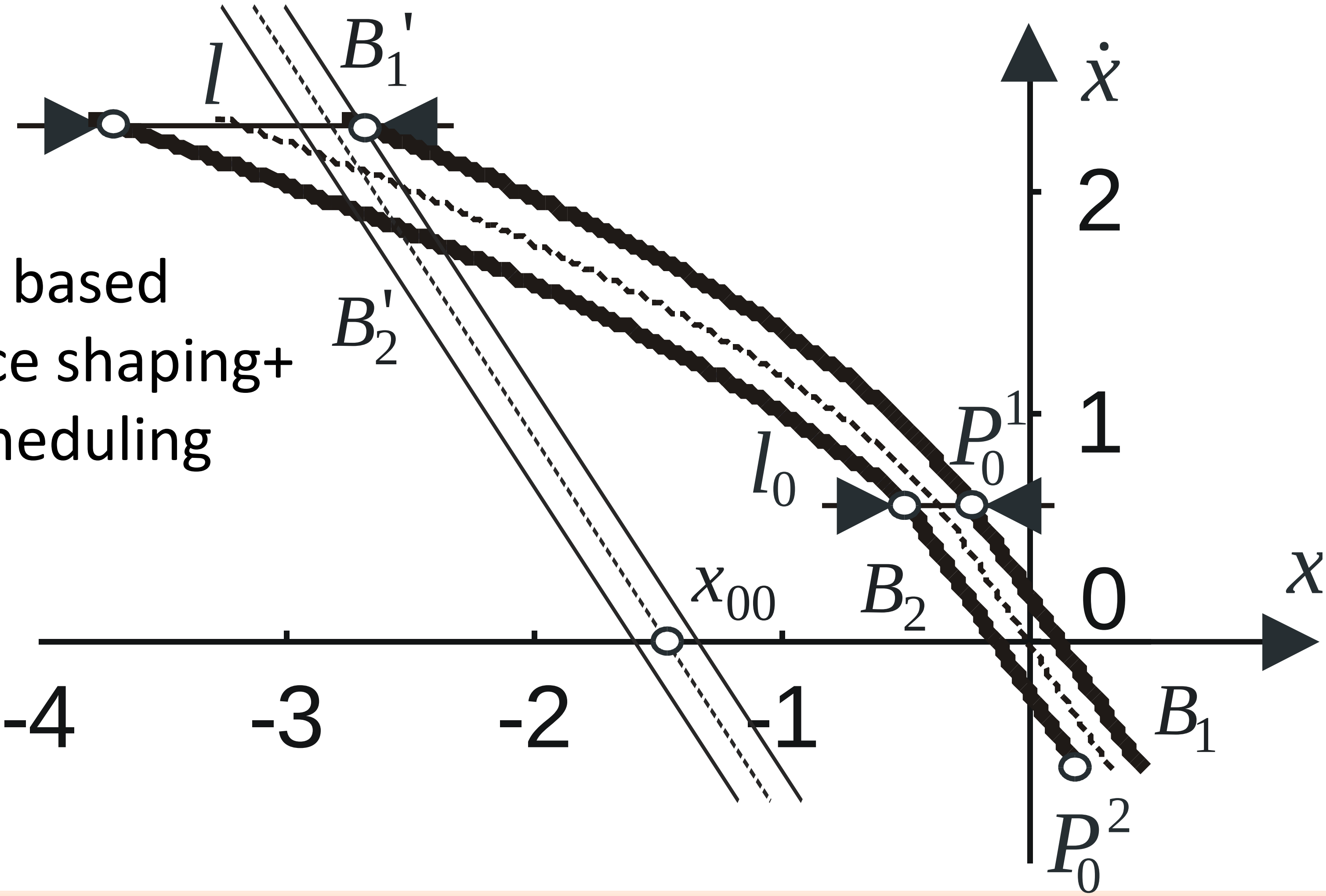


Constant width of the proportional band!

Problems with time delays!

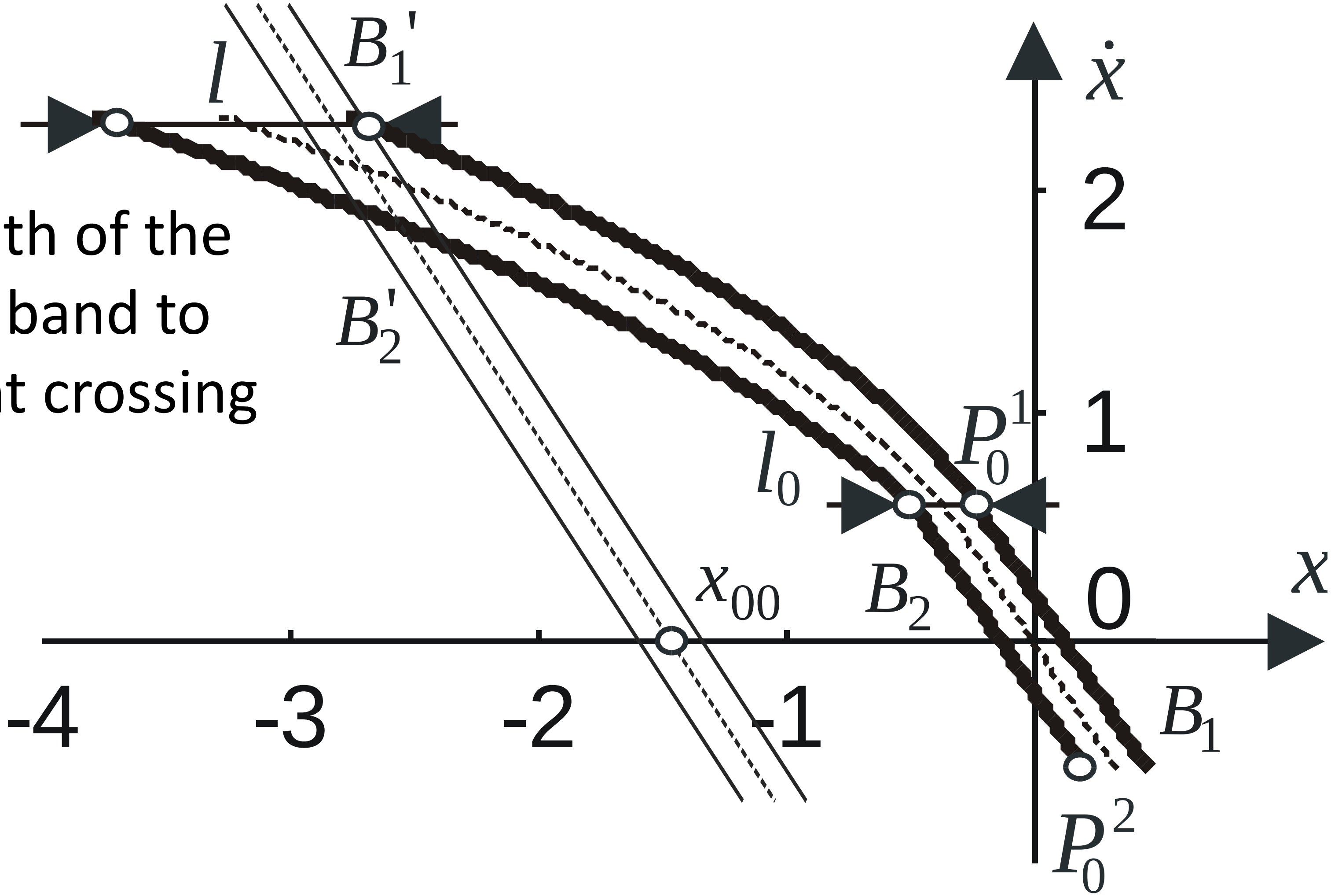
Complex Closed Loop Poles

Velocity based reference shaping+ Gain Scheduling



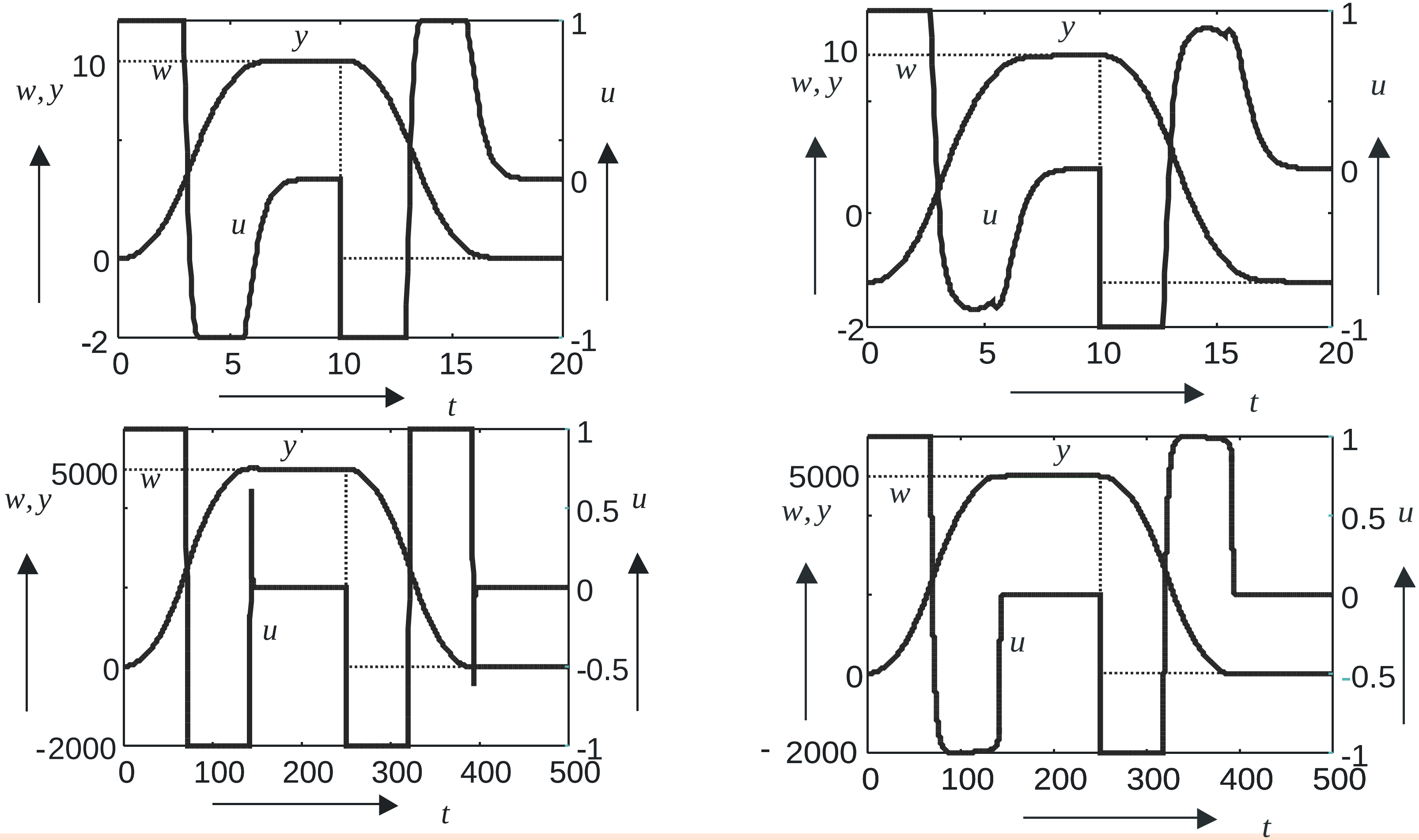
Complex Closed Loop Poles

Increasing with of the proportional band to keep constant crossing time



Complex Closed Loop Poles

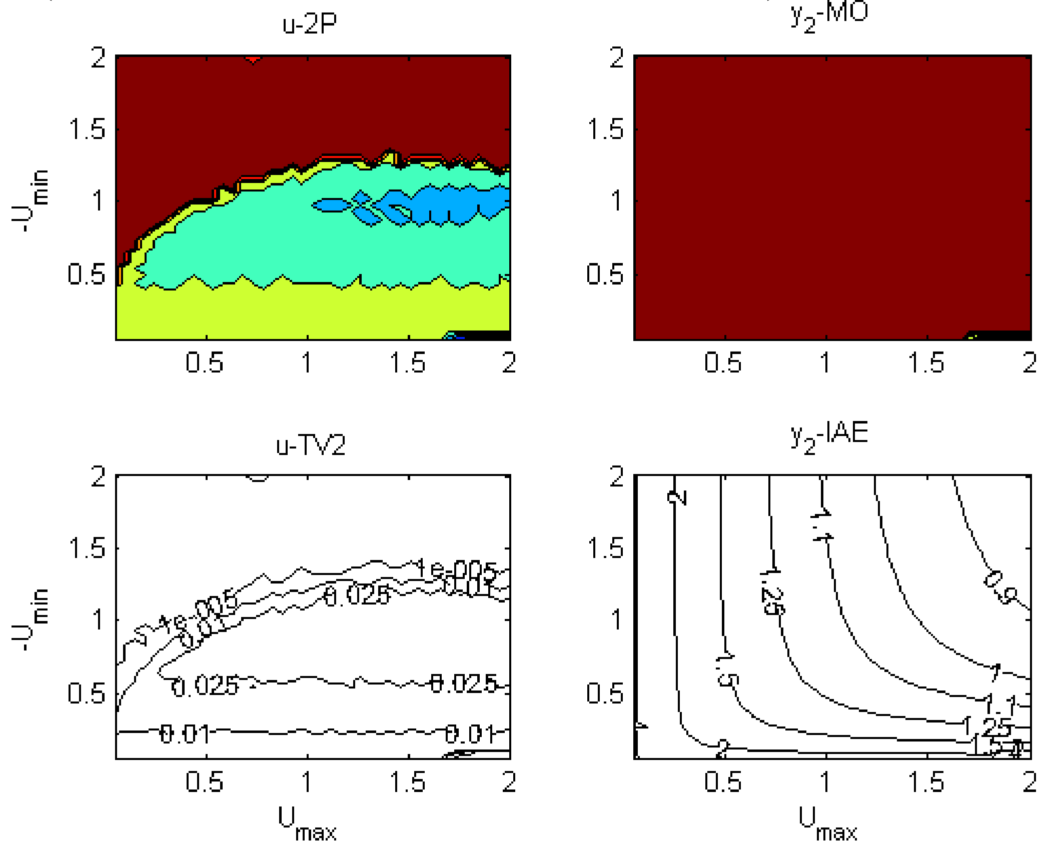
- I_2 system with time delay (T_d=0.2s)



Complex Poles – Perf. Portrait

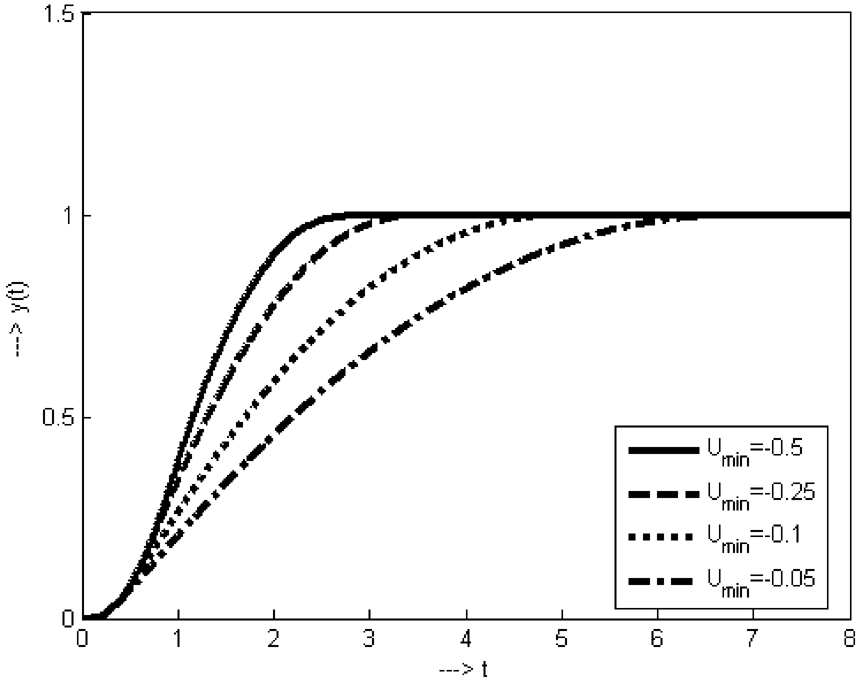
$T_d=0.1$; $\varepsilon=10^{-5}$ (red pole-pair)

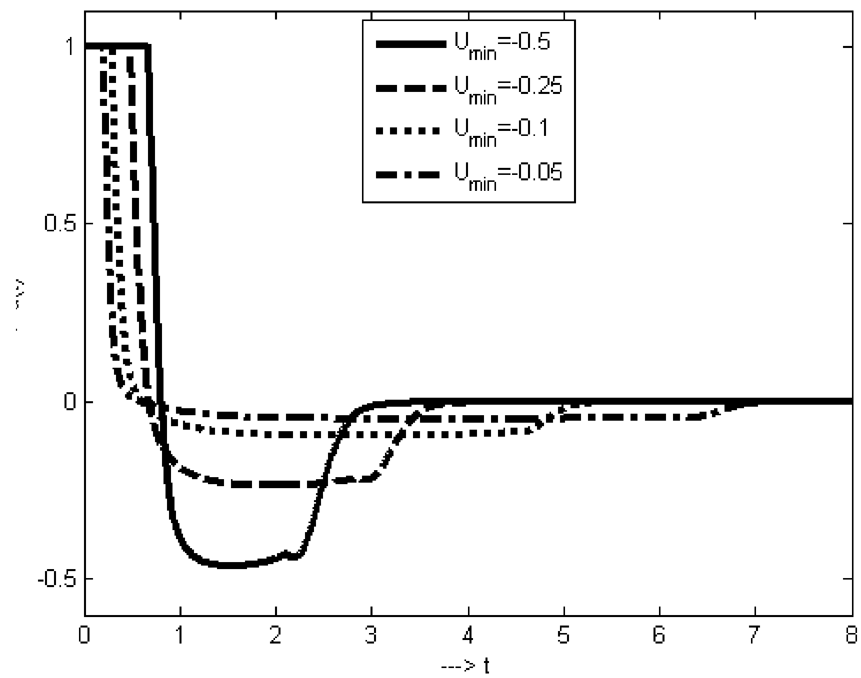
$$\varepsilon = \varepsilon_u = \varepsilon_y = \left\{0.1, 0.05, 0.02, 0.01, 10^{-3}, 10^{-4}, 10^{-5}\right\}$$



Complex Poles – Setpoint Steps

Td=0.1; ε=10^{-5} (red pole-pair)

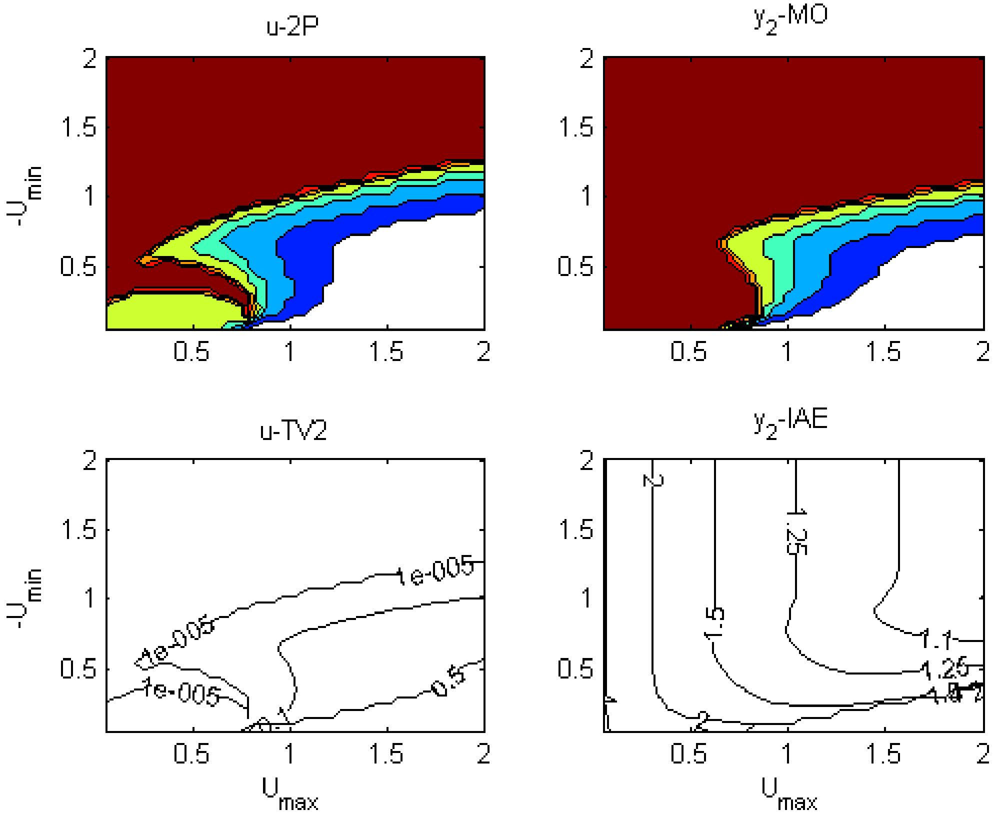




Complex Poles – Perf. Portrait

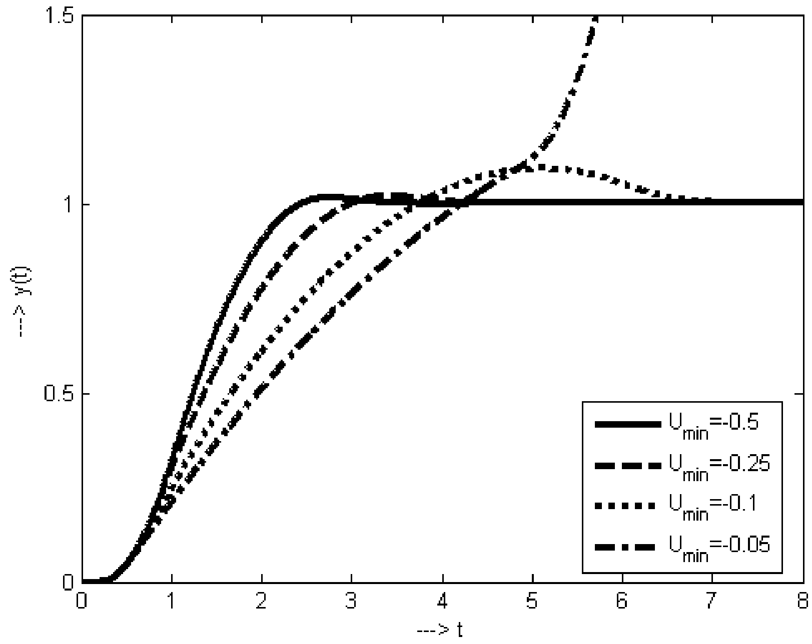
$$\varepsilon = \varepsilon_u = \varepsilon_y = \left\{0.1, 0.05, 0.02, 0.01, 10^{-3}, 10^{-4}, 10^{-5}\right\}$$

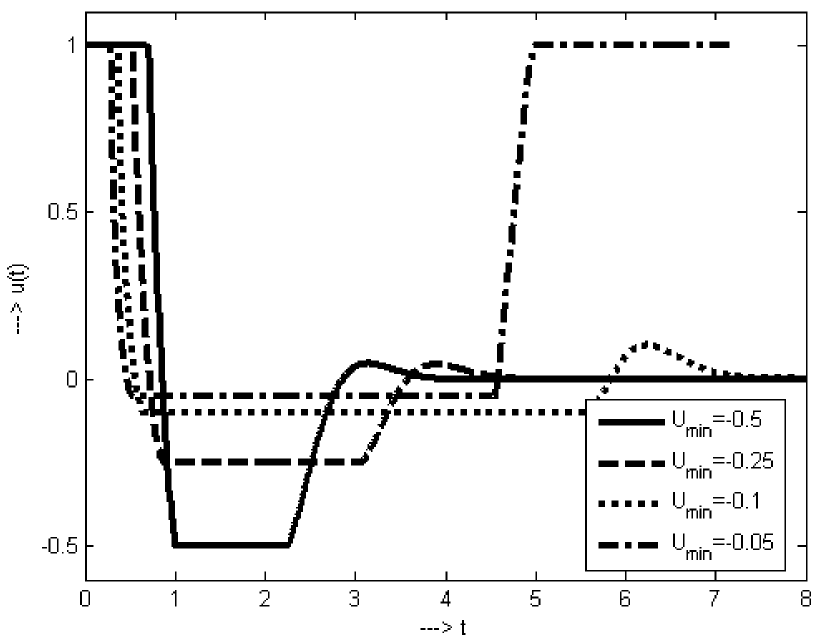
- Td=0.2; ε=10^{-5} (red pole-pair)



Complex Poles – Setpoint Steps

- Td=0.2; ε=10^{-5} (red pole-pair)
- Diverging for abs(U_{max}/U_{min})>20

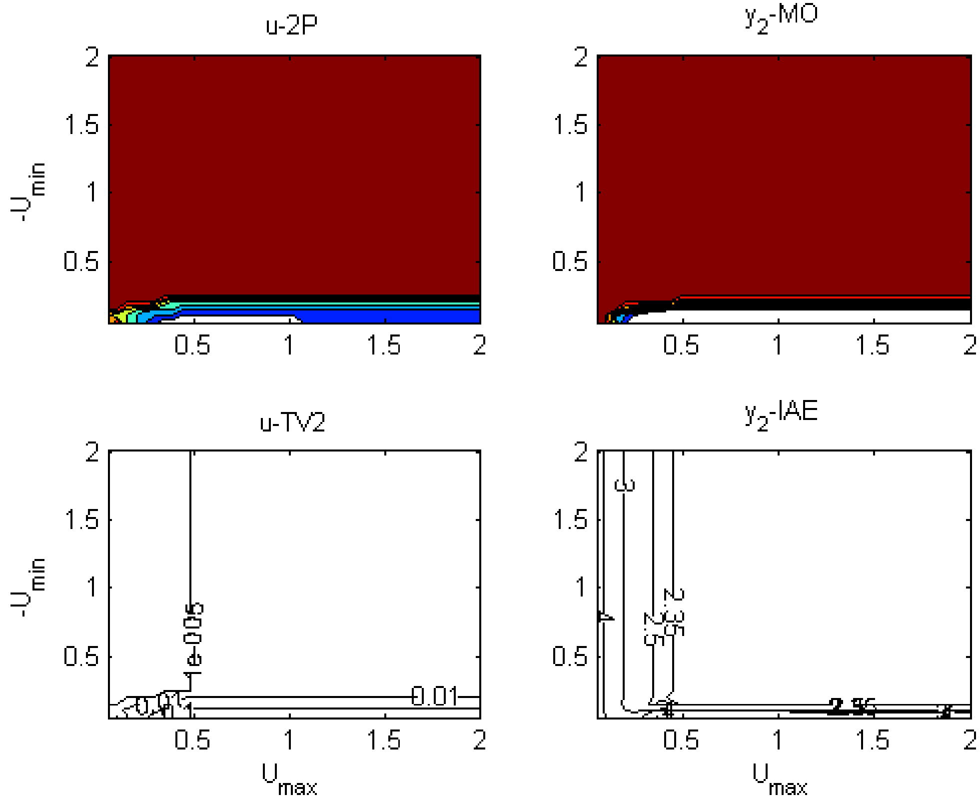




Complex Poles – P. Portrait

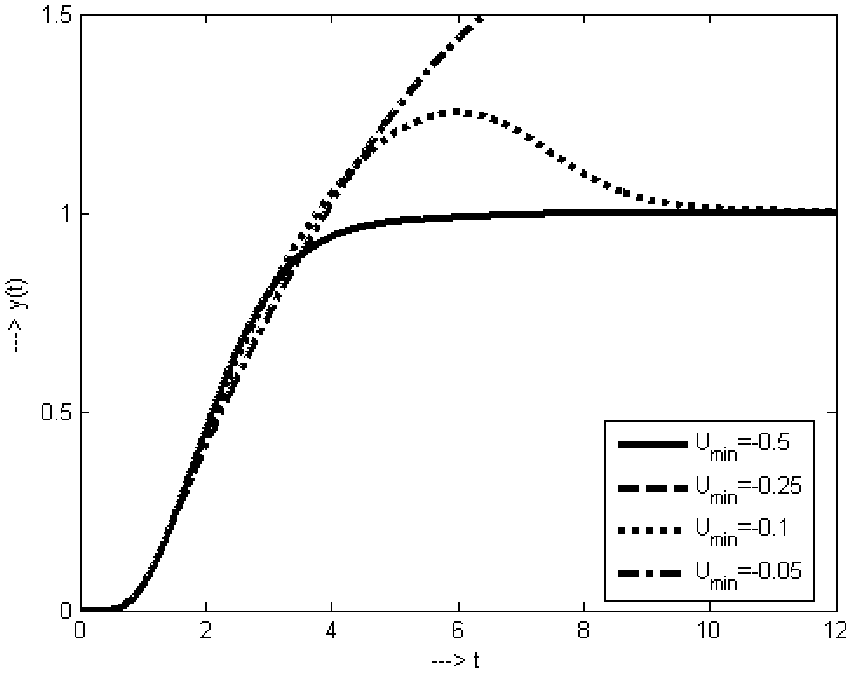
$$\varepsilon = \varepsilon_u = \varepsilon_y = \{0.1, 0.05, 0.02, 0.01, 10^{-3}, 10^{-4}, 10^{-5}\}$$

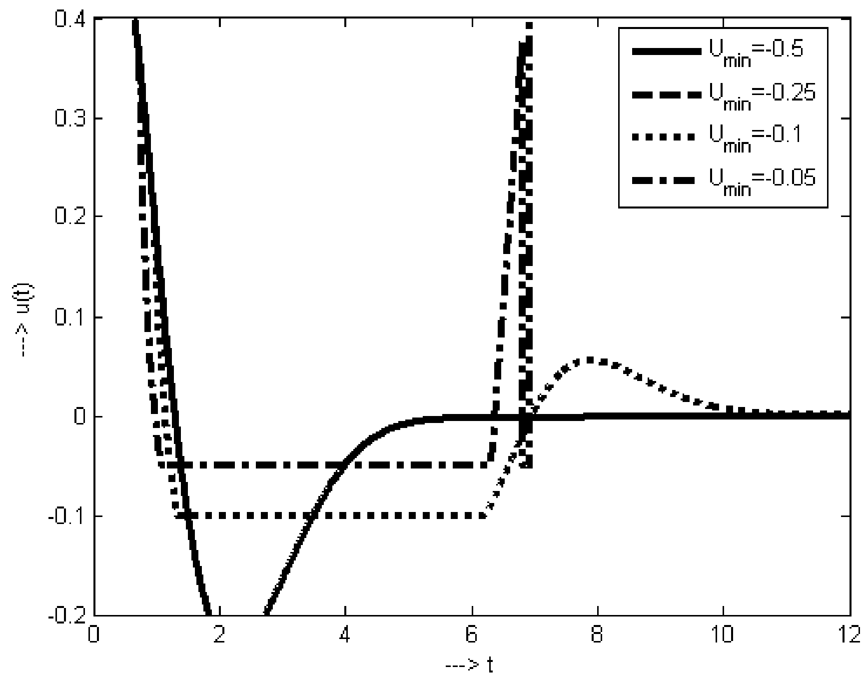
- Td=0.5; ε=10^{-5} (red pole-pair)



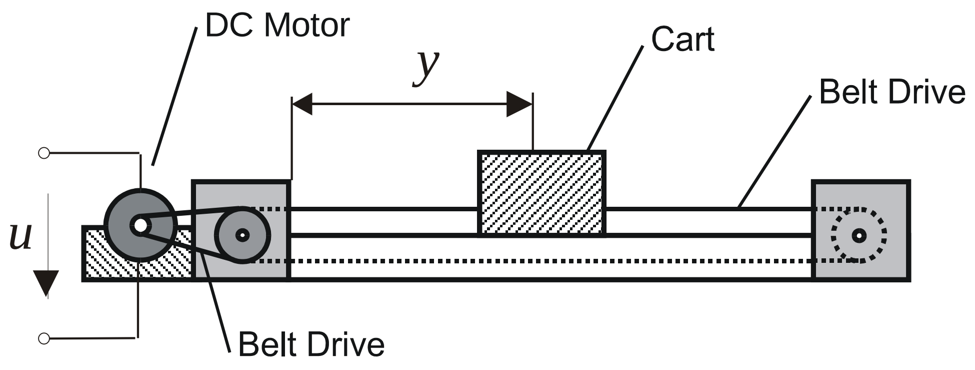
Complex Poles – Step responses

- Td=0.5; ε=10-5 (red pole-pair)
- Diverging for abs(Umax/Umin)>10

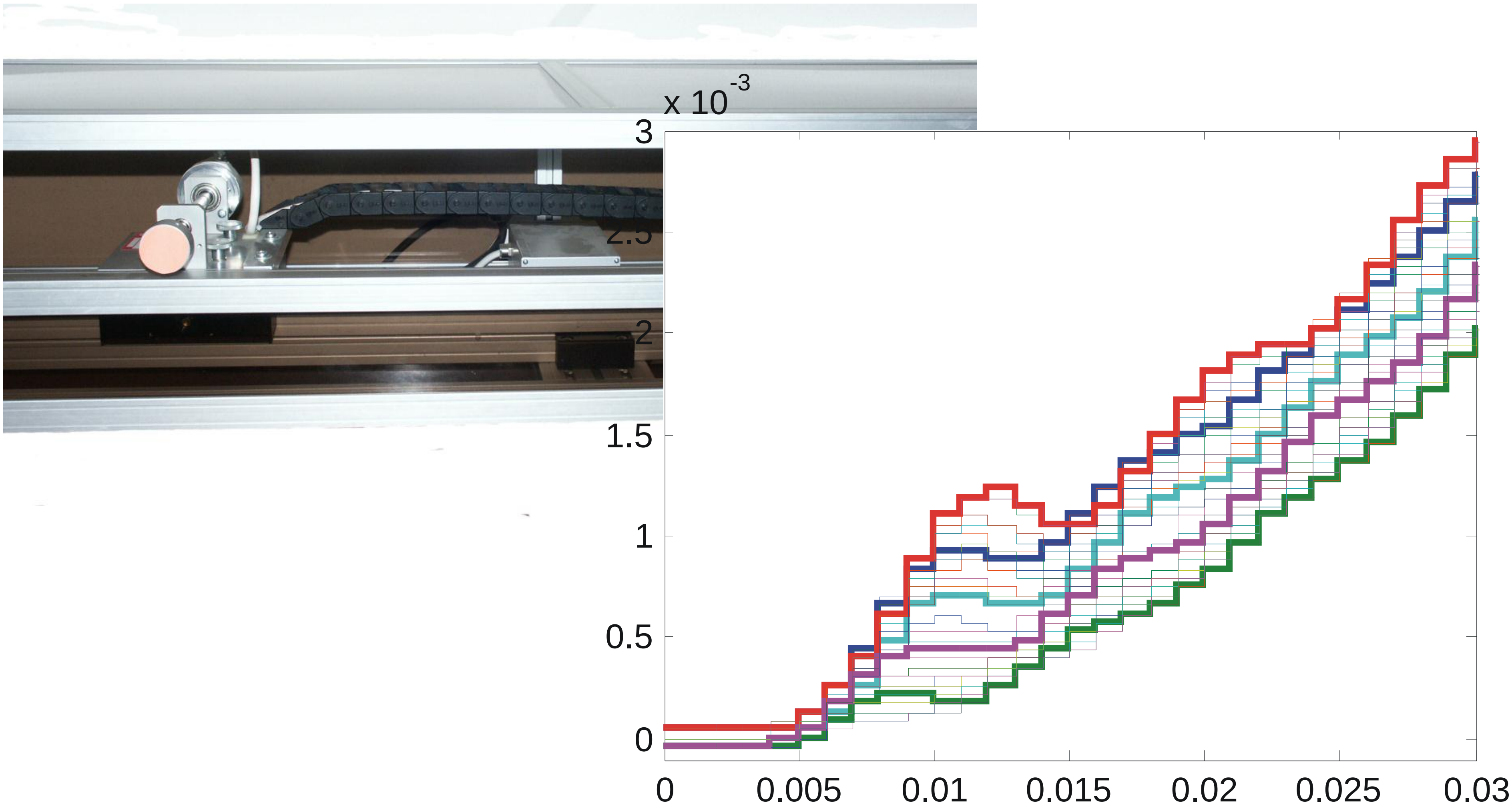




Example: Controller Tuning

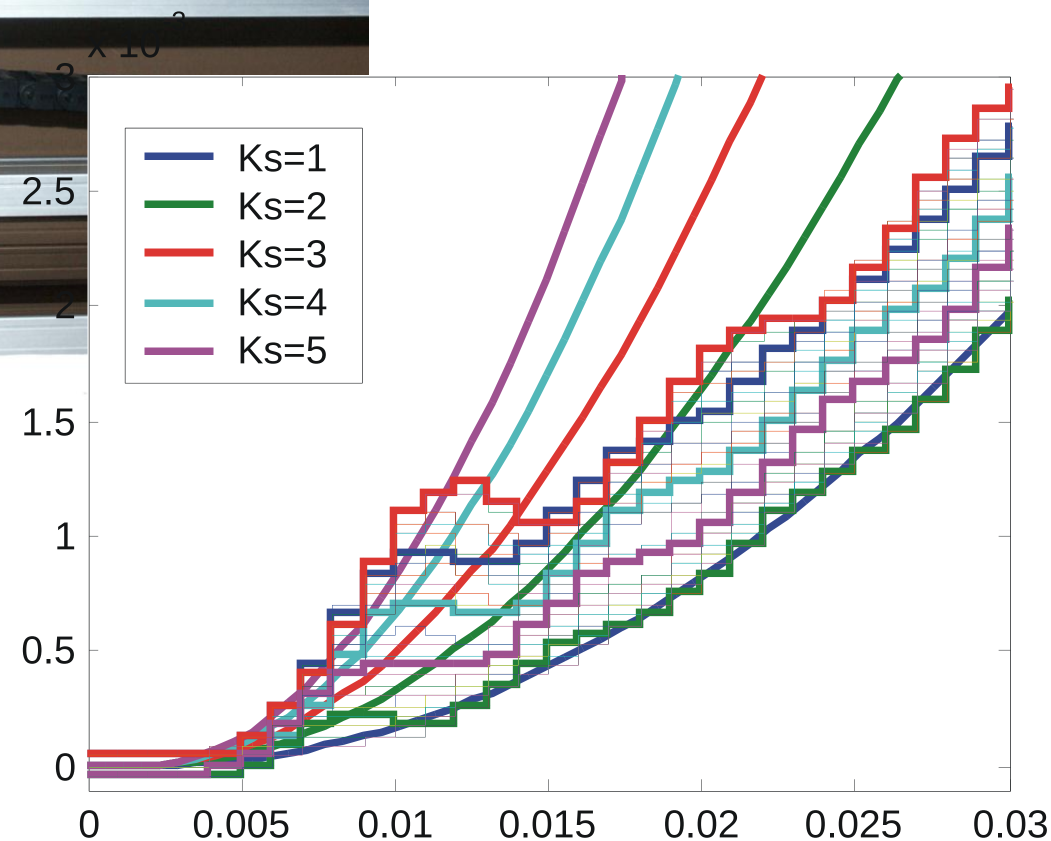


Example: Controller Tuning

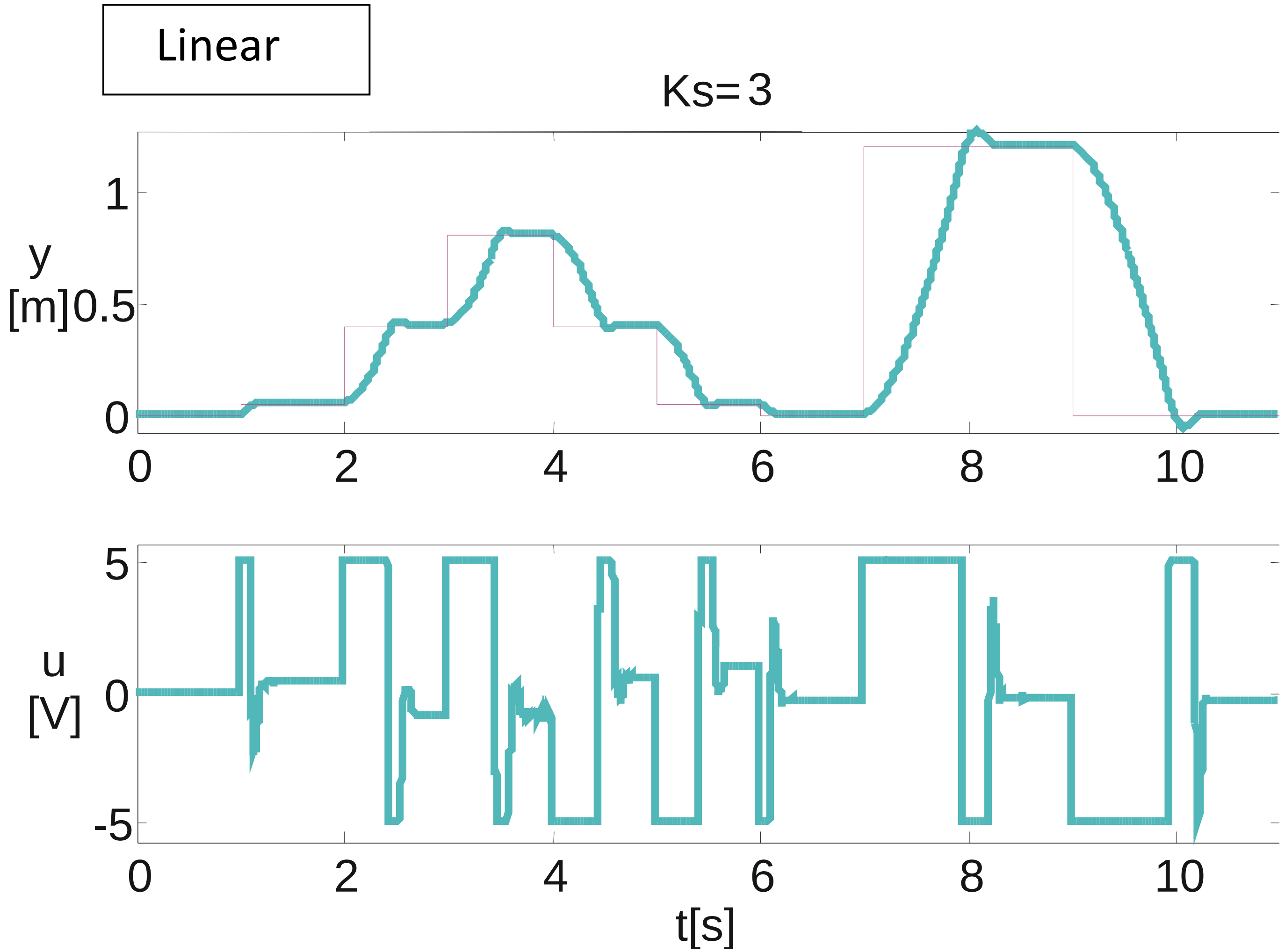


Example: Controller Tuning

$$S(s) = \frac{K_s}{s^2} e^{-0.003s}$$

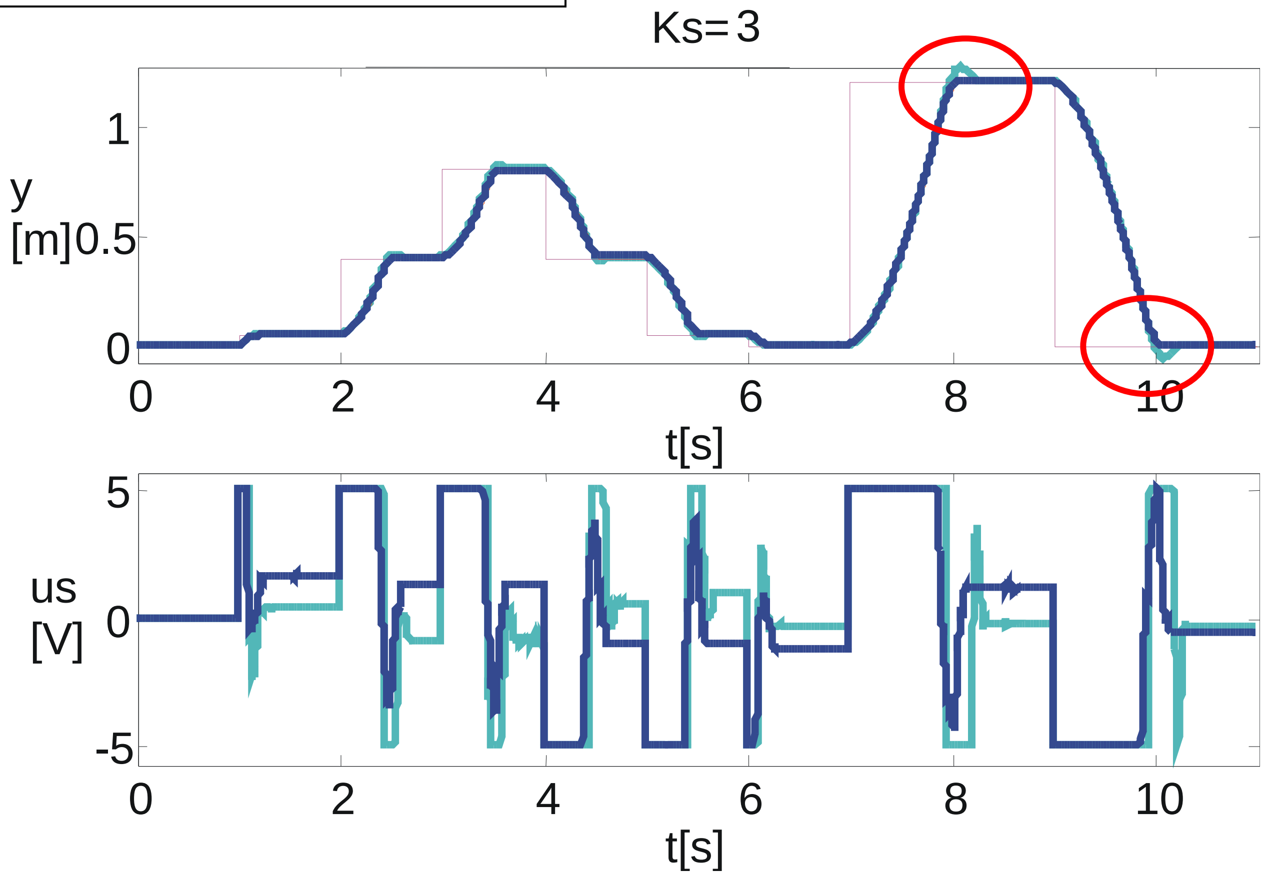


Constrained Linear Control

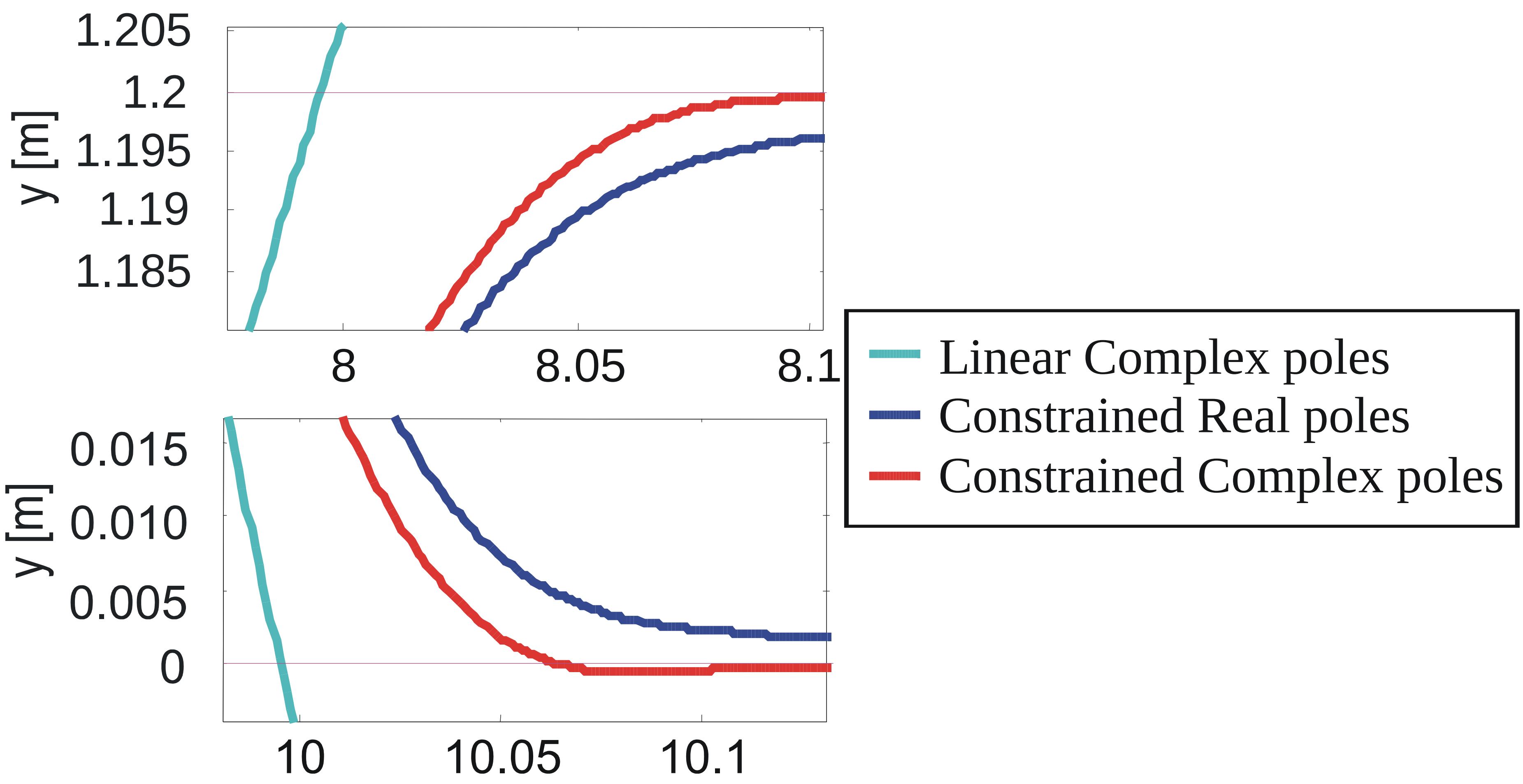


Real versus Complex Poles

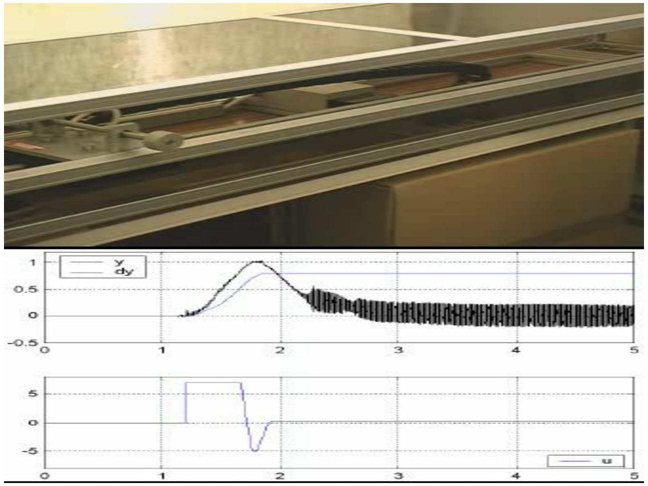
Linear +
Constrained Real poles

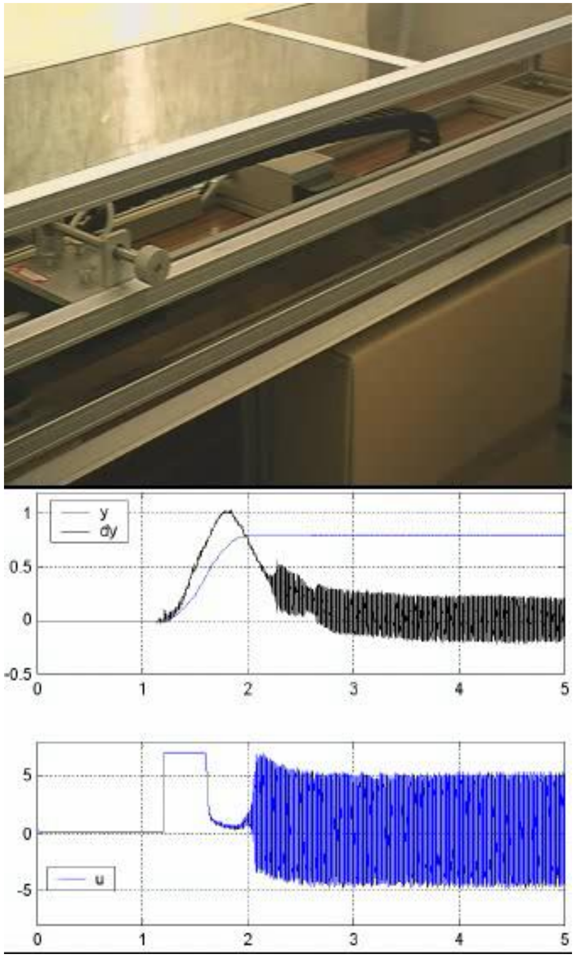


Real versus Complex Poles



Cart identification & control

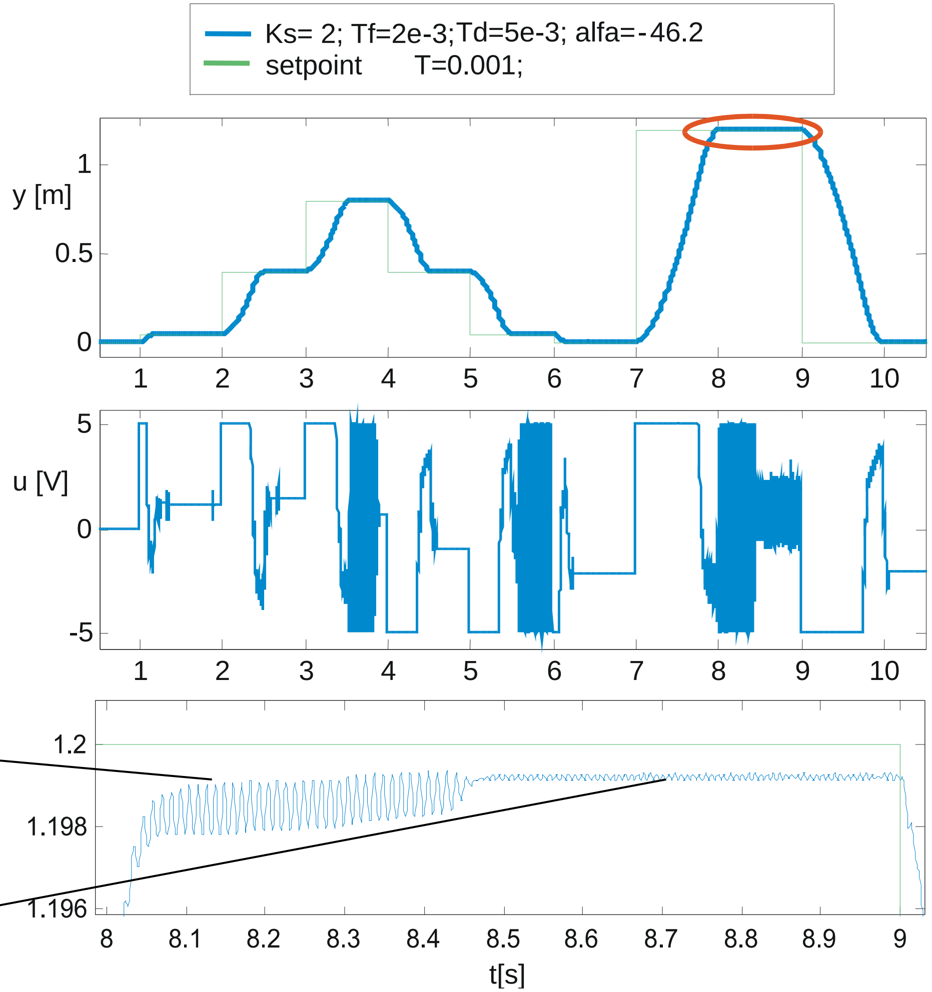




Identification – stability border

$$\hat{T}_d = \frac{r_1}{r_0}\frac{arctgx}{x}\ ;\ \ x = \frac{2\pi}{T_0}\frac{r_1}{r_0};$$

$$\hat{K}_s = \frac{\omega^2}{\sqrt{r_0^2 + \omega^2 r_1^2}};\quad \omega = \frac{2\pi}{T_0}$$



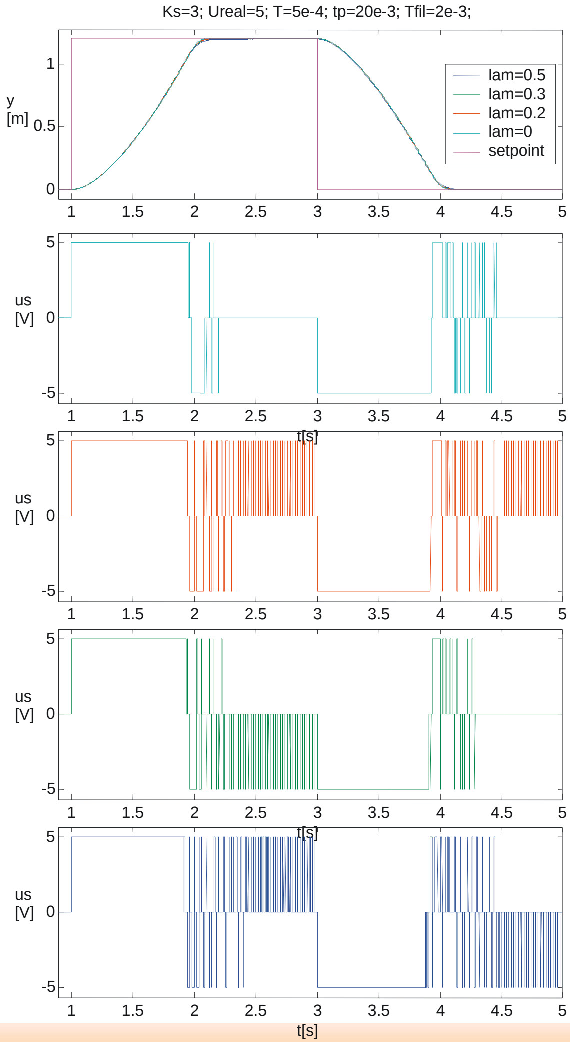
$$\hat{T}_d = 2.7ms\ ;\ \hat{K}_s = 12.1$$

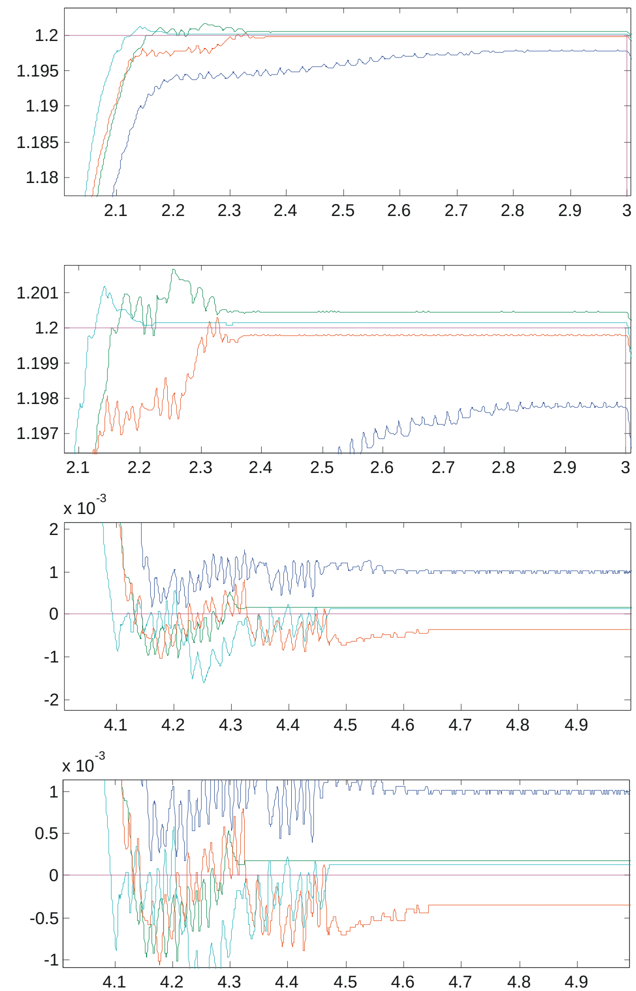
$$\hat{T}_d = 2.0ms\ ;\ \hat{K}_s = 17$$

PWM Pole Assignment Controller

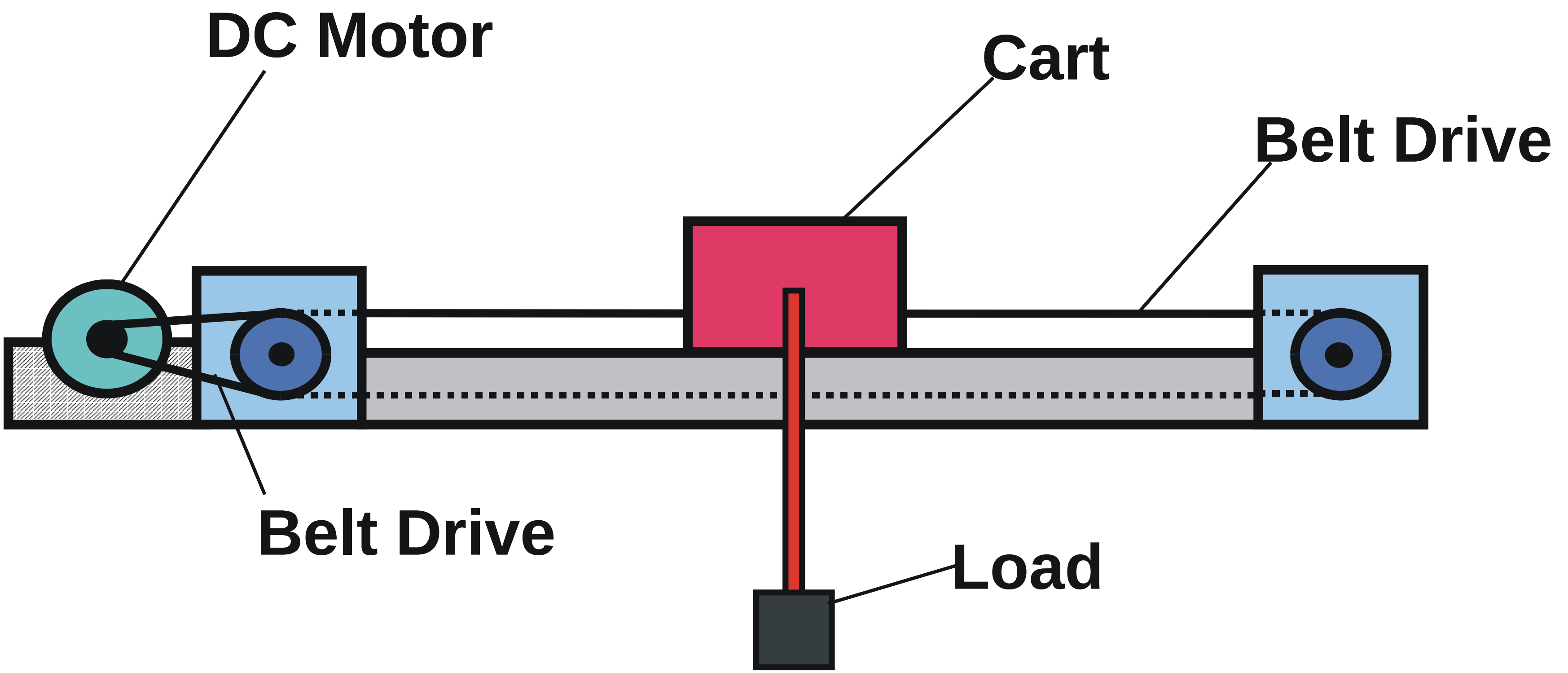
- Analogous approach to the PAM case
- Simpler actuator construction
- Higher steady state precision

PWM Pole Assignment Controller

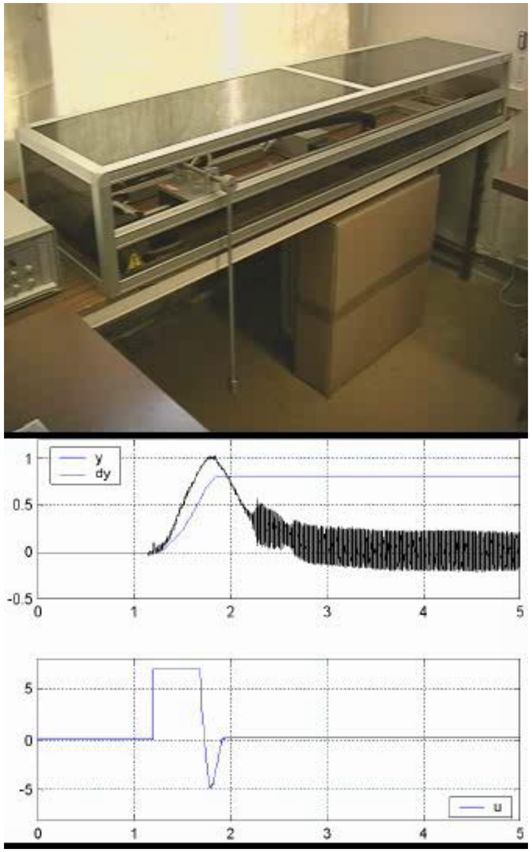


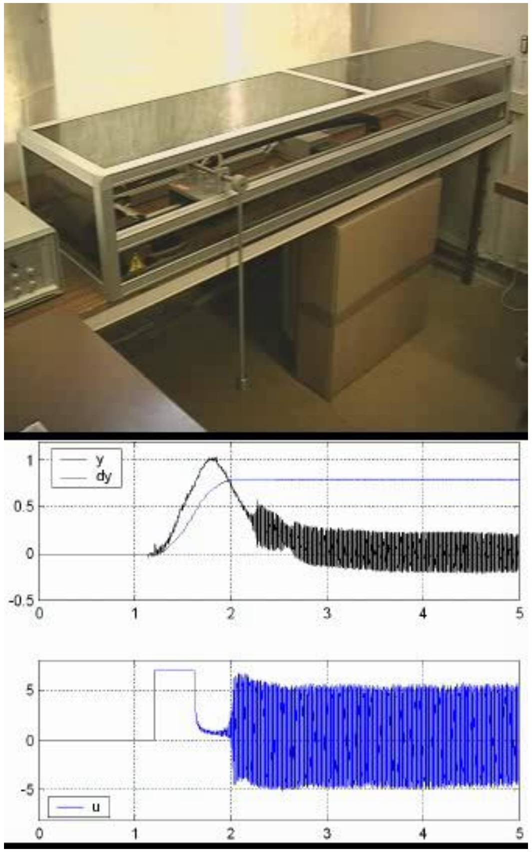


Pendulum identification & control

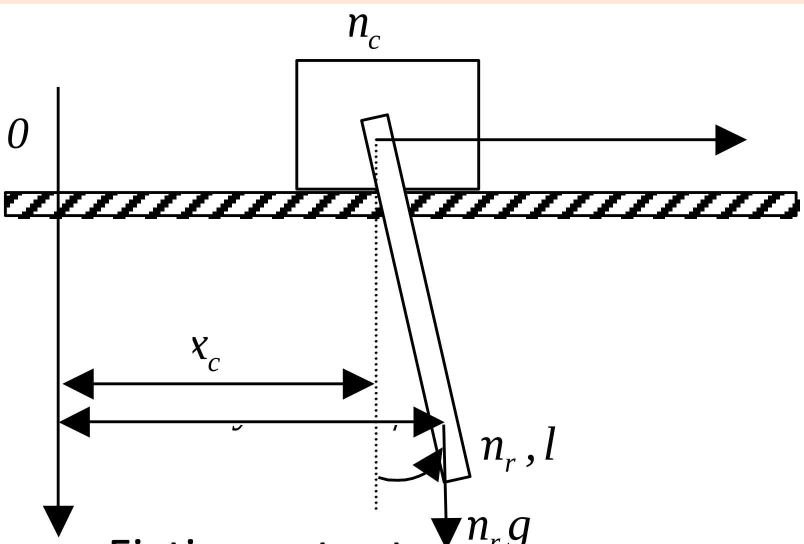


Pendulum identification & control





Reduction of the relative degree



Fictive output

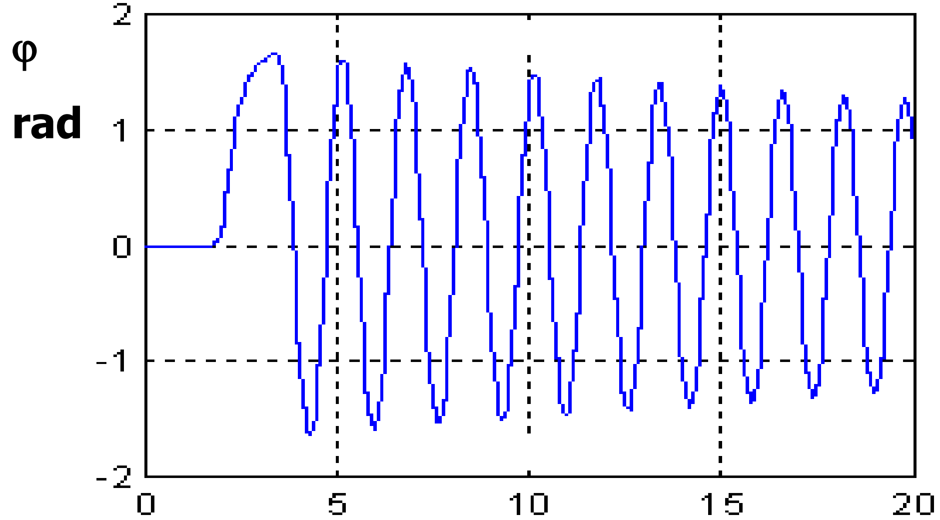
$$y_f = y + (T_1 + T_2)\dot{y} + T_1 T_2 \ddot{y}$$

Analytical tuning

$T_1 = T_2 = 3T_0$

Experimental tuning

$T_1 = T_2 = T_0/2$

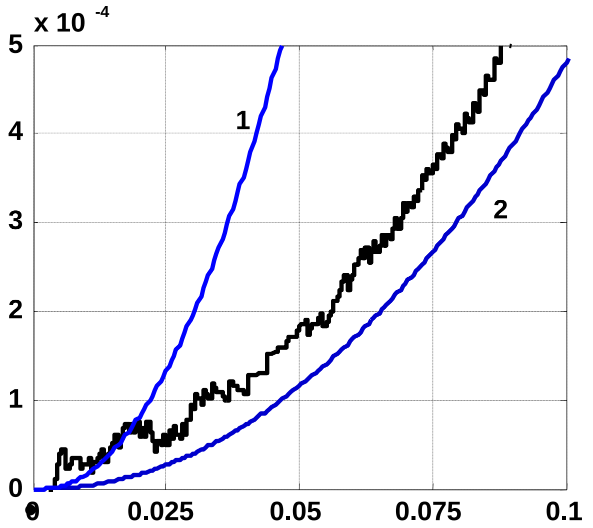


Autooscillation of pAendulum -

Measurement of time constant T_0

Aproximation by transfer function

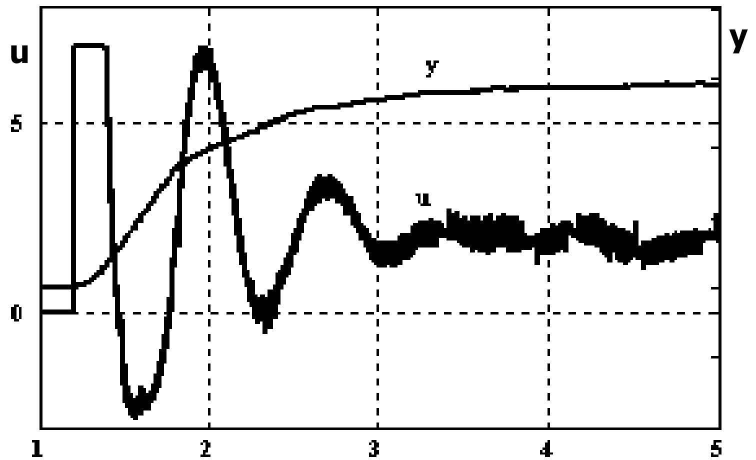
$$F(s) = \frac{K_s}{T_0^{\,2}.s^2 + 2.b_t.T_0.s + 1}$$



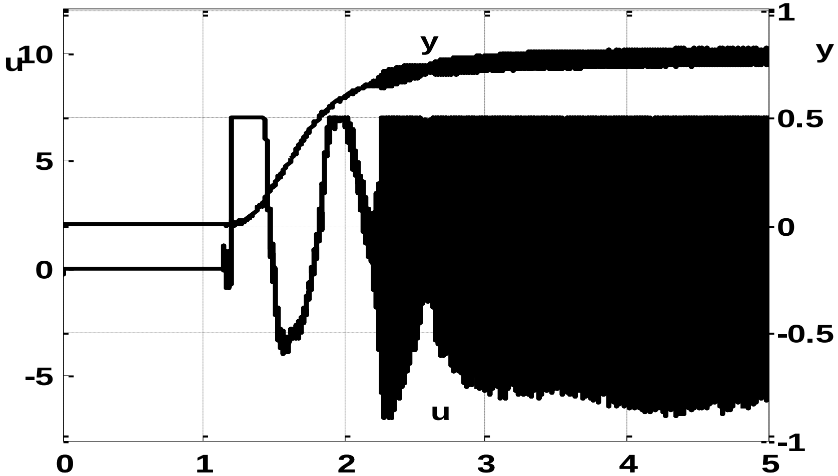
Approximations of the step response , Sampling period T=0.5 ms

1. Ks=0.5 , Td=1 ms
2. Ks=0.1 , Td=1 ms

Pole used for control : α=-3

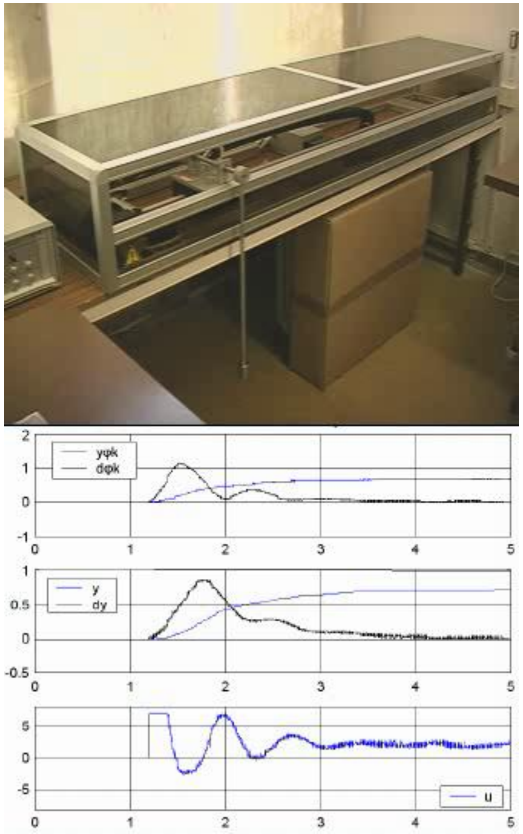


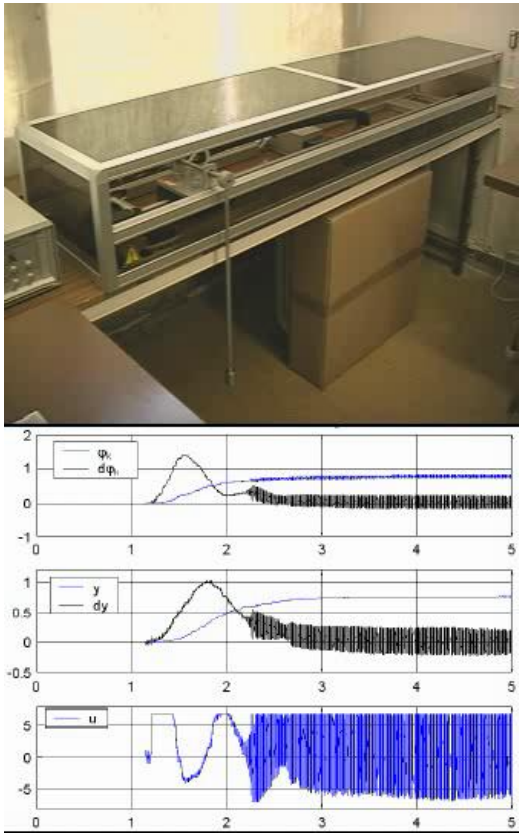
Real control using parameters no.1



Real control using parameters no.2

Pendulum identification & control





Conclusions I.

- Generalization of the method by Ziegler and Nichols (single integrator + dead time)
- Simple process approximation (double integrator + dead time) gives excellent results
- Simple and reliable controller tuning – analysis of system nonlinearity
- Suboptimal gain scheduling solutions for the case of complex poles / time delayed systems

Conclusions

- New solutions for simple and advanced problems
- Many solutions of the constrained pole assignment problem - distance definition, ordered n-tuples of poles
- Simplicity of the control algorithms
  CPU Time $\approx$ 0.1ms
- Simple and Reliable controller tuning
- New opportunities for practice
- Nonsymetrical amplitude and rate constraints
- Extension for nonlinear systems
- Windupless I-action