INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Lineární systémy: optimální a prediktivní řízení

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Tato prezentace je spolufinancována Evropským sociálním fondem a státním rozpočtem České republiky.

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Optimal and Suboptimal Decoupling Controllers

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Motivation

Decoupling is a way to decompose a complex system into non-interacting subsystems.

In fact, certain applications necessitate controlling independently different parts of the system.

Even if this is not required, the absence of interaction can significantly simplify the synthesis of the desired control laws.

Diagonal Decoupling

The basic form of decoupling
into single-input single-output subsystems
is often referred to as the *diagonal decoupling*.

Posed by Voznesenskij (1936),
studied by Kavanagh (1957), Strejc (1960), Mejerov (1965).

Related to the inversion problem of rational matrices.
Attention paid to the existence of proper transfer matrices.
The issue of stability, however, was not properly addressed.

State Space Approach

A deeper insight was provided by the state-space approach.
Pioneering work by Morgan (1964), who posed
the problem of decoupling by static state feedback.
Falb and Wolovich (1967) established a solvability condition.

The use of restricted static state feedback,
namely the static output feedback, was studied by
Howze and Pearson (1970), Descusse, Lafay, Kučera (1984).
This is a very restricted problem,
whose solution is hard to obtain, but useful in applications.

Block Decoupling

A more general form of decoupling
into multi-input multi-output subsystems
is referred to as the *block decoupling*.

Introduced by Wonham and Morse (1970)
and Basile and Marro (1970).
Using a geometric approach,
they determined the solvability of the problem
by static state feedback in several special cases.

Dynamic State Feedback

The decoupling by dynamic state feedback was studied via the geometric approach by Morse and Wonham (1970), who obtained a deep insight into the internal structure of the decoupled system.

By that time, decoupling by dynamic state feedback was solved, including stability or pole distributions that may be achieved while preserving a decoupled structure.

The status of noninteracting control was reviewed by Morse and Wonhan (1971).

Transfer Function Methods

A comeback of transfer function methods is witnessed in the works of Hautus and Heymann (1983) and Kučera (1983).

A dynamic state feedback was shown to be equivalent with combined dynamic output feedback and reference feedforward compensation.
Called the *two-degree-of-freedom controller* structure, it is ideally suited to decoupling since stability and non-interaction can be treated as two independent constraints.

Outline

This paper adopts the most general setting:
a system in which the measurement output
may be different from the output to be decoupled
and a two-degree-of-freedom dynamic controller .

The class of all such controllers
that decouple and stabilize the system
is determined in parametric form
and the parameter is used to obtain the H_2-optimal controller
that achieves asymptotic reference tracking.

Contribution

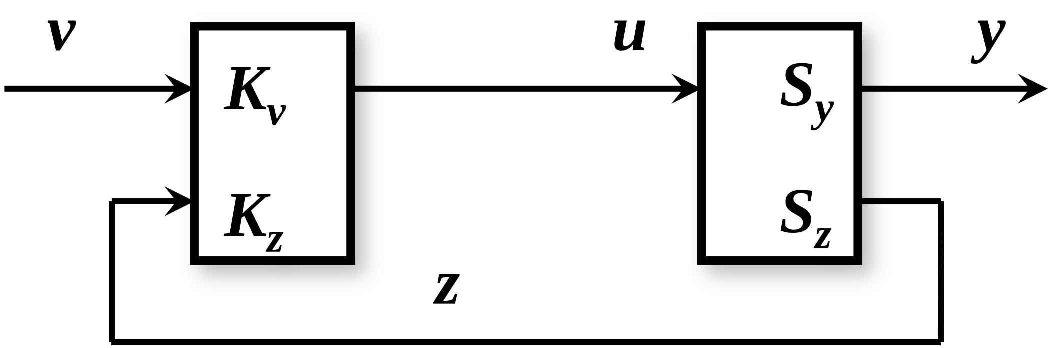
The main contribution of the present paper
is a streamlined and transparent exposition
and a simple and direct solution.

The paper draws inspiration from the works of
Hautus and Heymann (1983), Kučera (1983),
Desoer and Gündeş (1986), and Lee and Bongiorno (1993).

The problem is solved using an algebraic approach.
The parameterization of the decoupling controllers
is achieved via the Youla-Kučera parameterization.

Control System

Consider the control system configuration



where

$\boldsymbol{y}$ is the $\boldsymbol{p}$- vector output to be controlled,

$\boldsymbol{z}$ is the $\boldsymbol{m}$- vector measurement output,

$\boldsymbol{u}$ is the $\boldsymbol{q}$- vector control input,

$\boldsymbol{v}$ is the $\boldsymbol{r}$- vector external reference input.

Transfer Functions

We assume that the plant and the controller are linear, time-invariant, differential systems described by the input-output equations

$$\boldsymbol{y} = \boldsymbol{S}_y \boldsymbol{u}, \ \ \boldsymbol{z} = \boldsymbol{S}_z \boldsymbol{u}$$

and

$$\boldsymbol{u} = \boldsymbol{K}_v \boldsymbol{v} + \boldsymbol{K}_z \boldsymbol{z}$$

where the transfer functions $\boldsymbol{S}_y$, $\boldsymbol{S}_z$ and $\boldsymbol{K}_v$,$\boldsymbol{K}_z$ are real rational and proper matrices of appropriate sizes.

Decoupling Problem

Given $\boldsymbol{S_y}$ and $\boldsymbol{S_z}$.

Let ($\boldsymbol{p_1, ..., p_k}$) be a partition of $\boldsymbol{p}$ into $\boldsymbol{k}$ positive integers.

The *decoupling problem* is then to find matrices $\boldsymbol{K_v}$ and $\boldsymbol{K_z}$ such that the transfer function

$$\boldsymbol{T} = \boldsymbol{S_y}(\boldsymbol{I} - \boldsymbol{K_z}\boldsymbol{S_z})^{-1}\boldsymbol{K_v}$$

from $\boldsymbol{v}$ to $\boldsymbol{y}$ has the block diagonal form

$$\boldsymbol{T} := \begin{bmatrix} \boldsymbol{T_1} & & \\ & \ddots & \\ & & \boldsymbol{T_k} \end{bmatrix}$$

where $\boldsymbol{T_i}$ is $\boldsymbol{p_i} \times \boldsymbol{r_i}$ for some partition ($\boldsymbol{r_1, ..., r_k}$) of $\boldsymbol{r}$.

Admissibility Condition

Obviously, unless additional provisions are made,
the decoupling problem is trivial as it could be solved by $\boldsymbol{K}_v = \mathbf{0}$.
Thus it is necessary to impose a certain ***admissibility condition***
on the decoupling controller to make the problem meaningful,
for example

$$\mathbf{rank}\, T = \mathbf{rank}\, S_y$$

This condition is equivalent to the preservation
of the class of controlled output trajectories.
We thus require that no essential loss of control occurs
through the decoupling process.

Proper and Stable Fractions

In order to study stability of the decoupled system
it is convenient to express
the transfer functions of the plant and those of the controller
in the form of *proper and stable rational fractions*

$$\begin{bmatrix} S_z \\ S_y \end{bmatrix} := \begin{bmatrix} B \\ C \end{bmatrix} A^{-1} \qquad \begin{bmatrix} K_z & K_v \end{bmatrix} := P^{-1} \begin{bmatrix} -Q & R \end{bmatrix}$$

where the plant representation is right coprime
and the controller representation is left coprime.

The overall system transfer function then reads

$$T = C(PA + QB)^{-1} R$$

Stability

It is assumed that

the plant is such that

its part that is not controllable from $\boldsymbol{u}$ is stable

and its part that is not jointly observable from $\boldsymbol{y}, \boldsymbol{z}$ is stable,

and

the controller is realized in such a manner that

its part that is not jointly controllable from $\boldsymbol{v}, \boldsymbol{z}$ is stable

and its part that is not observable from $\boldsymbol{u}$ is stable.

Then the overall system is stable

if and only if the matrix $\boldsymbol{PA} + \boldsymbol{QB}$ is unimodular.

Problem Solvability

A simple necessary and sufficient condition

is established for a system to be decoupled and stable.

Based on the partition $(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_k)$, write

$$\boldsymbol{C} := \begin{bmatrix} \boldsymbol{C}_1 \\ \vdots \\ \boldsymbol{C}_k \end{bmatrix}$$

There exists an admissible controller

such that the overall system is

- ❖ stable if and only if $\boldsymbol{A}$ and $\boldsymbol{B}$ are right coprime,
- ❖ decoupled if and only if

$$\sum_{i=1}^{k} \textbf{rank}\, \boldsymbol{C}_i = \textbf{rank}\, \boldsymbol{C}$$

Interpretation

The stability condition requires
the stability of the part of the plant
that is not observable at the measured output $\boldsymbol{z}$.

The decoupling condition calls for
the linear independence of any two outputs of the plant
that belong to different blocks.

The solvability of the decoupling problem
thus strongly depends on the partition $(\boldsymbol{p}_1, \dots, \boldsymbol{p}_k)$, that is to say,
upon the allocation of the outputs into the blocks.

Controller Construction

Matrices $\boldsymbol{P}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$ that define a particular controller are obtained as follows.

Determine any proper and stable rational matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ such that

$$\boldsymbol{PA}+\boldsymbol{QB}=\boldsymbol{I}$$

with *P* invertible and the inverse of *P* proper.

Let $\boldsymbol{r_i} := \textbf{rank}\, \boldsymbol{C_i},\ \boldsymbol{i}=\mathbf{1}, \ldots, \boldsymbol{k}$ and
let $\boldsymbol{U_i}$ be a unimodular proper and stable rational matrix such that

$$\boldsymbol{C_i}=\boldsymbol{U_i}\begin{bmatrix}\boldsymbol{C_i'}\\ \mathbf{0}\end{bmatrix}$$

where the rows of $\boldsymbol{C_i'}$ are linearly independent.

Controller Construction

Denote

$$\boldsymbol{C}' := \begin{bmatrix} \boldsymbol{C}'_1 \\ \vdots \\ \boldsymbol{C}'_k \end{bmatrix}$$

and let $\boldsymbol{U}'$ be a unimodular proper and stable rational matrix such that

$$\boldsymbol{C}'\boldsymbol{U}' := \begin{bmatrix} \boldsymbol{D}_1 & & & \mathbf{0} \\ & \ddots & & \vdots \\ & & \boldsymbol{D}_k & \mathbf{0} \end{bmatrix}$$

where $\boldsymbol{D}_i$ is an $\boldsymbol{r}_i \times \boldsymbol{r}_i$ proper and stable rational matrix.

Then $\boldsymbol{R}$ is formed by the first $\boldsymbol{r}$ columns of $\boldsymbol{U}'$, where

$$\boldsymbol{r} := \sum_{i=1}^{k} \boldsymbol{r}_i$$

Controller Parameterization

The set of admissible, stabilizing and decoupling controllers is given by

$$\boldsymbol{P} = \overline{\boldsymbol{P}} + \boldsymbol{W}\overline{\boldsymbol{B}},\ \boldsymbol{Q} = \overline{\boldsymbol{Q}} - \boldsymbol{W}\overline{\boldsymbol{A}}$$

where $\overline{\boldsymbol{P}}, \overline{\boldsymbol{Q}}$ is any solution pair of equation $\boldsymbol{PA} + \boldsymbol{QB} = \boldsymbol{I}$,

where $\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}$ are left coprime,
proper and stable rational matrices such that $\overline{\boldsymbol{A}}^{-1}\overline{\boldsymbol{B}} = \boldsymbol{BA}^{-1}$

and $\boldsymbol{W}$ is a free proper and stable rational matrix parameter constrained so that $\boldsymbol{P}(\infty)$ is nonsingular;

Controller Parameterization

and further by

$$\boldsymbol{R} = \boldsymbol{U}'_r \begin{bmatrix} \boldsymbol{V}_1 & & \\ & \ddots & \\ & & \boldsymbol{V}_k \end{bmatrix}$$

where $\boldsymbol{U}'_r$ consists of the first *r* columns of $\boldsymbol{U}'$
and $\boldsymbol{V}_i$ is a free proper and stable rational matrix parameter.

The parameterization of decoupling stabilizing controllers reveals that decoupling and stabilization are two independent issues.

Achievable Transfer Functions

The transfer function of the decoupled system

is block diagonal

$$\boldsymbol{T} = \boldsymbol{C}(\boldsymbol{PA}+\boldsymbol{QB})^{-1}\boldsymbol{R} = \begin{bmatrix} \boldsymbol{U}_1 & & \\ & \ddots & \\ & & \boldsymbol{U}_k \end{bmatrix} \begin{bmatrix} \begin{smallmatrix}\boldsymbol{D}_1\\ \mathbf{0}\end{smallmatrix} & & \\ & \ddots & \\ & & \begin{smallmatrix}\boldsymbol{D}_k\\ \mathbf{0}\end{smallmatrix} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 & & \\ & \ddots & \\ & & \boldsymbol{V}_k \end{bmatrix}$$

The decoupling can deteriorate system's performance.

The bonus of having a parameterized solution set

is that the lost performance can easily be optimized.

Specific decoupling controllers can be obtained

by an appropriate choice of the parameters $V_1, \ldots, V_k$.

Asymptotic Tracking

Suppose that the control objective is
for each block of outputs y_i to track
the corresponding block of reference inputs v_i .

The tracking error for each block is given by

$$\boldsymbol{e}_i := \boldsymbol{v}_i - \boldsymbol{y}_i = \boldsymbol{H}_i \boldsymbol{v}_i$$

Suppose that the reference input is given by

$$\boldsymbol{v}_i = \boldsymbol{G}_i^{-1}, \boldsymbol{g}_i$$

where G_i is a fixed proper and stable rational matrix
and g_i is an unspecified proper and stable rational vector
that captures the effect of initial conditions.

Asymptotic Tracking

Asymptotic tracking means that

$$\boldsymbol{e}_i = \boldsymbol{H}_i \boldsymbol{G}_i^{-1} \boldsymbol{g}_i$$

is a proper and stable rational vector.

Thus G_i must be absorbed in H_i .

The generic form of H_i is

$$\boldsymbol{H}_i = \boldsymbol{I} - \boldsymbol{F}_i \boldsymbol{V}_i$$

where $F_i := U_i D_i$ and V_i are proper and stable rational with F_i fixed and V_i an arbitrary parameter to be specified.

Asymptotic Tracking

Therefore, asymptotic tracking is possible

if and only if there is a proper and stable rational matrix Z_i

such that
$$F_i V_i + Z_i G_i = I.$$

For a particular solution $\bar{V}_i, \bar{Z}_i$, the solution class is given by

$$V_i = \bar{V}_i + N_i G_i, \;\; Z_i = \bar{Z}_i - F_i N_i,$$

where N_i is a proper and stable rational matrix parameter.

Thus, the set of reference-to-error transfer

functions that achieve asymptotic reference tracking

in a decoupled system is
$$H_i = \bar{Z}_i G_i - F_i N_i G_i.$$

H_2 Optimal Controllers

Suppose that for each block,
the reference-to-error transfer function H_i
that is parameterized above
is to have least H_2 norm.

To achieve this task,
determine the inner-outer factorization of F_i,

$$F_i = F_{iI} F_{iO},$$

and note that G_i is outer for a typical refernce.

Then $$\|\boldsymbol{H}_i\| = \|\boldsymbol{F}_{iI}^{-1}\boldsymbol{H}_i\| = \|\boldsymbol{F}_{iI}^{-1}\overline{\boldsymbol{Z}}_i\boldsymbol{G}_i - \boldsymbol{F}_{iO}\boldsymbol{N}_i\boldsymbol{G}_i\|$$

.

H_2 Optimal Controllers

Write

$$\boldsymbol{F}_{iI}^{-1}\overline{\boldsymbol{Z}}_i\boldsymbol{G}_i = \boldsymbol{F}_{iI}^{-1}\boldsymbol{K}_i, + \boldsymbol{L}_i$$

where K_i, L_i are proper and stable rational matrices

with K_i strictly proper.

Note that $\boldsymbol{F}_{iI}^{-1}$ has poles only in $\mathrm{Re}s > 0$.

Then

$$\left\|\boldsymbol{H}_i\right\|^2 = \left\|\boldsymbol{F}_{iI}^{-1}\boldsymbol{K}_i + (\boldsymbol{L}_i - \boldsymbol{F}_{iO}\boldsymbol{N}_i\boldsymbol{G}_i)\right\|^2 = \left\|\boldsymbol{F}_{iI}^{-1}\boldsymbol{K}_i\right\|^2 + \left\|\boldsymbol{L}_i - \boldsymbol{F}_{iO}\boldsymbol{N}_i\boldsymbol{G}_i\right\|^2$$

because the cross terms contribute nothing to the norm.

This is a complete square

in which only the second term depends on N_i .

H_2 Optimal Controllers

Therefore, a unique N_i that attains the minimum of the norm for subsystem i is

$$\boldsymbol{N}_i = \boldsymbol{F}_{iO}^{-1} \boldsymbol{L}_i \boldsymbol{G}_i^{-1}$$

provided N_i is a proper and stable rational matrix.

Suboptimal Controllers

Unfortunately, matrix N_i is generically unstable
for typical references due to the presence of $j\omega$-zeros in G_i.

This impasse can be obviated
by sacrificing the optimality
and focusing on suboptimal controllers.

Suboptimal Controllers

Select proper and stable rational matrices M_i, N_i

so that

$$\boldsymbol{L}_i = \boldsymbol{M}_i + \boldsymbol{F}_{iO}\boldsymbol{N}_i\boldsymbol{G}_i$$

holds with M_i being strictly proper

and having a small H_2 norm; in fact, as small as desired.

Then

$$\|\boldsymbol{H}_i\|^2 = \|\boldsymbol{F}_{iI}^{-1}\boldsymbol{K}_i\|^2 + \|\boldsymbol{M}_i\|^2$$

and the parameter M_i defines a suboptimal controller,

for which the resulting H_2 norm of H_i

is only an incremental addition to the unattainable infimum.

Example

Consider a system with transfer matrices

$$S_y = \begin{bmatrix} 1 & \dfrac{s+2}{s-1} \\ \dfrac{s-1}{s+2} & 2 \end{bmatrix}, \quad S_z = \begin{bmatrix} \dfrac{2s+1}{s+2} & \dfrac{3s}{s-1} \\ \dfrac{s-1}{s+2} & 2 \end{bmatrix}.$$

Thus the measurement output z

is different from the output y to be decoupled

in that it involves a non-unity feedback sensor.

The task is to determine a two-degree-of-freedom controller that $(1, 1)$-decouples and stabilizes the system.

Example

The first step is to obtain a proper and stable fractional representation (6) for the system. Standard calculations yield

$$A=\begin{bmatrix} 1 & 0 \\ 0 & \dfrac{s-1}{s+2} \end{bmatrix},\ B=\begin{bmatrix} \dfrac{2s+1}{s+2} & \dfrac{3s}{s+2} \\ \dfrac{s-1}{s+2} & 2\dfrac{s-1}{s+2} \end{bmatrix},\ C=\begin{bmatrix} \dfrac{1}{s-1} & \dfrac{1}{s-1} \\ \dfrac{}{s+2} & 2\dfrac{}{s+2} \end{bmatrix}$$

Since A and B are right coprime,

a stabilizing controller exists.

Since $\mathrm{rank}C_1 + \mathrm{rank}C_2 = \mathrm{rank}\ C$,

an admissible decoupling controller exists as well.

Example

To parametrize the feedback part of the controller,

we consider the solution class of equation $\boldsymbol{PA}+\boldsymbol{QB}=\boldsymbol{I}$

$$\boldsymbol{P}=\begin{bmatrix} 1 & 0 \\ -\dfrac{2s+1}{s+2} & -2 \end{bmatrix}+\boldsymbol{W}\begin{bmatrix} \dfrac{s-1}{s+2} & 2 \\ -\dfrac{(s-1)(2s+1)}{(s+2)^2} & -\dfrac{3s}{s+2} \end{bmatrix}$$

$$\boldsymbol{Q}=\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}-\boldsymbol{W}\begin{bmatrix} 0 & 1 \\ -\dfrac{s-1}{s+2}. & 0 \end{bmatrix}$$

Example

To obtain the feedforward part of the controller,

note that $U_1 = U_2 = 1$ and

$$\boldsymbol{U'} = \begin{bmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}.$$

Thus

$$\boldsymbol{R} = \begin{bmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{V_2} \end{bmatrix}.$$

The matrices P, Q, and R define the class of all controllers that solve the decoupling problem.

The parameters V_1, V_2 are free non-zero proper and stable rational functions

and W ranges over proper and stable rational 2 $\times$ 2 matrices so that the inverse of P exists and is proper.

Example

The decoupled transfer matrices

that can be achieved in this example are given by

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{s-1}{s+2} \end{bmatrix} \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix}.$$

Suppose that the decoupled outputs

are to follow any step references

and the transients are to be optimized in terms of the H_2 norm.

Thus, put

$$G_1 = G_2 = \frac{s}{s+1}$$

and solve equation $F_i V_i + Z_i G_i = I$ channel by channel.

Example

Clearly $V_1 = 1$, $Z_1 = 0$ is an optimal solution that yields $\mathrm{H}_1 = 0$. On the other hand,

$$\boldsymbol{V}_2 = \frac{s-2}{s+1} + N_2 \frac{s}{s+1}, \quad Z_2 = \frac{6}{s+2} - \frac{s-1}{s+2} N_2$$

and the inner-outer factorization of

$$F_2 = \frac{s-1}{s+2}$$

is seen to be

$$F_{2I} = \frac{s-1}{s+1}, \quad F_{2O} = \frac{s+1}{s+2}.$$

Example

Then

$$F_{iI}^{-1}\overline{Z}_2 G_2 = \frac{s+1}{s-1}\frac{6}{s+2}\frac{s}{s+1} = \frac{s+1}{s-1}\frac{2}{s+1} + \frac{4}{s+2}$$

so that

$$K_2 = \frac{2}{s+1},\ \ L_2 = \frac{4}{s+2}.$$

Thus,

$$N_2 = \frac{6}{s}$$

and the infimum of $\|H\|$ cannot be attained.

Example

To obtain a suboptimal controller, choose

$$M_2 = \frac{2\varepsilon}{s+\varepsilon}, \; N_2 = \frac{4-2\varepsilon}{s+\varepsilon}$$

with $\varepsilon > 0$ arbitrarily small, in order to satisfy (22). Then

$$V_2 = \frac{s+2}{s+1}\frac{s-\varepsilon}{s+\varepsilon}, \; Z_2 = \frac{2+2\varepsilon}{s+\varepsilon}, \; H_2 = \frac{2+2\varepsilon}{s+\varepsilon}\frac{s}{s+1}$$

and

$$\left\| H_2 \right\|^2 = \left\| \frac{2}{s+1} \right\|^2 + \left\| \frac{2\varepsilon}{s+\varepsilon} \right\|^2 = 2+2\varepsilon$$

is arbitrarily close to the infimum value of 2.

Example

It follows that a suboptimal R is

$$R = \begin{bmatrix} 2 & -\dfrac{s+2}{s+1}\dfrac{s-\varepsilon}{s+\varepsilon} \\ -1 & \dfrac{s+2}{s+1}\dfrac{s-\varepsilon}{s+\varepsilon} \end{bmatrix}$$

and the overall system has the transfer function

$$T = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{s-1}{s+1}\dfrac{s-\varepsilon}{s+\varepsilon} \end{bmatrix}.$$

Conclusions

An optimal decoupling control problem
has been studied in the most general setting:
for systems in which the measurement output
may be different from the output to be decoupled
and for two-degree-of-freedom dynamic controllers.

The class of all such controllers
that decouple and stabilize the system has been determined
in parametric form and the parameter has been used
to obtain the H_2-optimal controller
that achieves asymptotic reference tracking.

Conclusions

The adopted controller configuration
is ideally suited to decoupling
since stability and non-interaction
can be treated as two independent constraints.

The parameterization of the decoupling controllers
is achieved via the Youla-Kučera parameterization
of all stabilizing controllers.

Optimal decoupling controllers are then obtained
by an appropriate choice of the parameters.

Conclusions

In case the H_2 optimal control problem has no solution,
suboptimal controllers have been determined
for which the H_2 norm
of the reference-to-error transfer function
is only an incremental addition to the unattainable infimum.

References

Basile, G. and G. Marro (1970). A state space approach to non interacting controls. *Ric. Autom.*, 1, 68-77.

Descusse, J., J.F. Lafay, and V. Kučera (1984). Decoupling by restricted static state feedback: The general case. *IEEE Trans. Automatic Control* ,29, 79-81.

Falb, P.L. and W.A. Wolovich (1967). Decoupling in the design and synthesis of multivariable control systems. *IEEE Trans. Automatic Control* ,12, 651-659.

Howze, J.W. and J.B. Pearson (1970). Decoupling and arbitrary pole placement in linear systems using output feedback. *IEEE Trans. Automatic Control* ,15, 660-663.

Kavanagh, R.J. (1957). Noninteracting controls in linear multivariable systems. *AIEE Trans. Applications and Industry*, 76, 95-100.

Mejerov, M.V. (1965). *Multivariable Control Systems* (in Russian). Nauka. Moscow.

References

Morgan, B.S. (1964). The synthesis of linear multi-variable systems by state feedback. In: *Proc. Joint Automatic Control Conference*, pp. 468-472.

Morse, A.S. and W.M. Wonham (1970). Decoupling and pole placement by dynamic compensation. *SIAM J. Control,* 8, 317-337.

Morse, A.S. and W.M. Wonham (1971). Status of noninteracting control. *IEEE Trans. Automatic Control*, 16, 568-581.

Strejc, V. (1960). The general theory of autonomity and invariance of linear systems of control. *Acta Technica*, 5, 235-258.

Voznesenskij, I.N. (1936). A control system with many outputs (in Russian). *Automat. i Telemech*., 4, 7-38.

Wonham, W.M. and A.S. Morse (1970). Decoupling and pole assignment in linear multivariable sys-tems: A geometric approach. *SIAM J. Control*, 8, 1-18.

References

Kučera, V. (1975). Stability of discrete linear feed-back systems. In: *Preprints 6th IFAC Congress*. Vol.1, paper 44.1.

Kučera, V. (1979). *Discrete Linear Control: The Polynomial Equation Approach,* Wiley, Chichester.

Youla, D.C., H. Jabr, and J.J. Bongiorno (1976). Modern Wiener-Hopf design of optimal controllers – Part II: The multivariable case. *IEEE Trans. Automatic Control*, 21, 319-338.

Desoer, C.A., R.W. Liu, J. Murray, and R. Saex (1980). Feedback system design: The fractional representation approach to analysis and synthesis. *IEEE Trans. Automatic Control*, 25, 399-412.

C.A. Desoer and A.N. Gündeş (1986). Decoupling linear multiinput-multioutput plant by dynamic output feedback: An algebraic theory. *IEEE Trans. Automatic Control*, 31, 744-750.

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References

Hautus, M.L.J. and M. Heymann (1983). Linear feedback decoupling – Transfer function analysis. *IEEE Trans. Automatic Control*, 28, 823-832.

Kučera, V. (1983). Block decoupling by dynamic compensation with internal properness and stability," *Probl. Control Info. Theory*, 12, 379-389.

Lee, H.P. and J.J. Bongiorno (1993). Wiener-Hopf design of optimal decoupling controllers for plants with non-square transfer matrices. *Int. J. Control*, 58, 1227-1246.

Park, K.H. (2008). H_2 design of one-degree-of-freedom decoupling controllers for square plants," *Int. J. Control*, 81, 1343-1351.

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Modelování a fyzika budov

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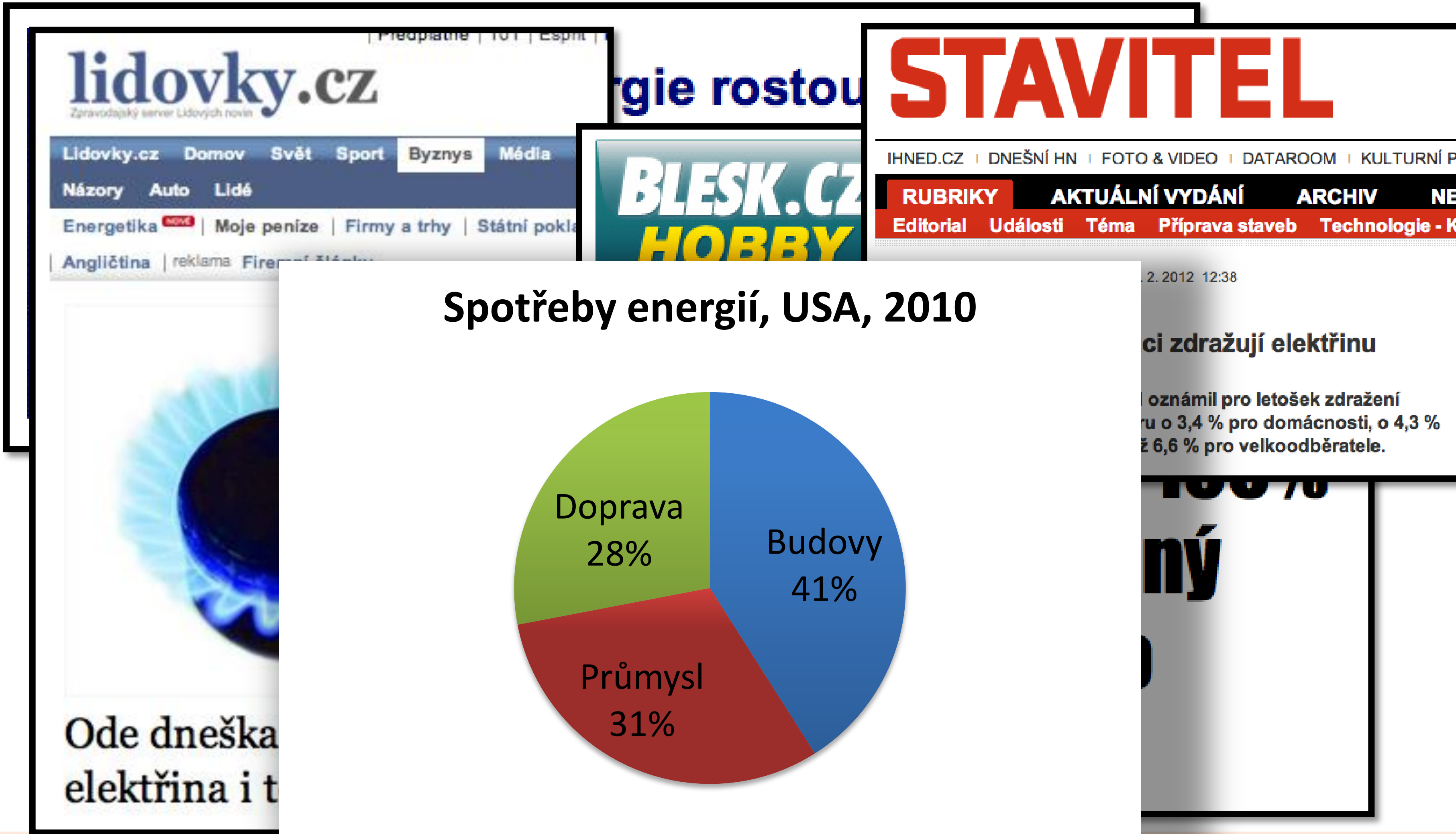
MOTIVACE

Motivace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u o 3,4 % pro domácnosti, o 4,3 %
ž 6,6 % pro velkoodběratele.
Spotřeby energií, USA, 2010
Doprava 28%
Budovy 41%
Průmysl 31%

Cíl

Ušetřit energii v budovách

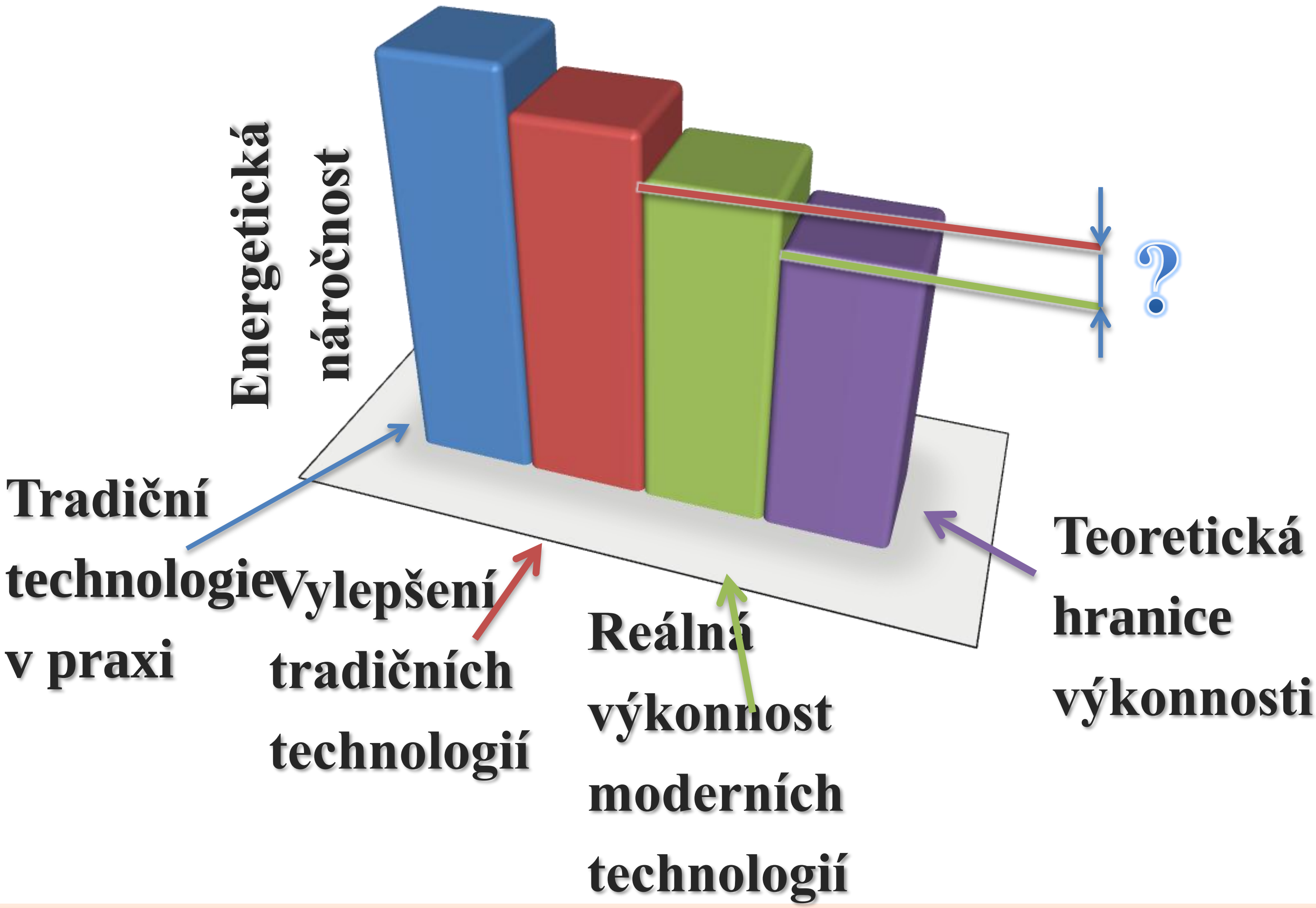
Jak ušetřit v budovách?

- Zateplení fasády
- Dobrá okna
- Snížení vnitřní teploty
- Alternativní zdroje energie
- **Lepší regulace**

Regulace v budovách

- Termostat
- Ekvitermní regulace
- Regulace podle referenční místnosti (např. PID)
- Podmínkové řízení – Rule Based Control (RBC)
- Něco “fajné”

Dilema praktické regulace



Strategie plnění cíle

- Chci ušetřit energii v budově
- Nasadím lepší regulaci
- Vyplatí se mi investice do pokročilé regulace
- Jakou pokročilou regulaci zvolit?
- Potřebuji minimalizovat energii při zachování tepelného komfortu, omezení na teplotu topné vody a se znalostí modelu budovy.
  - *Je vlastně slovní formulace* MPC *problému*

KONCEPCE REGULACE

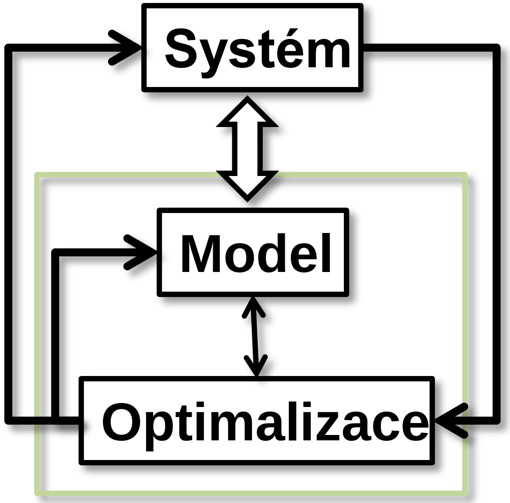
Řízení podle modelu

Mějme nějaký model systému (dopsat stav)

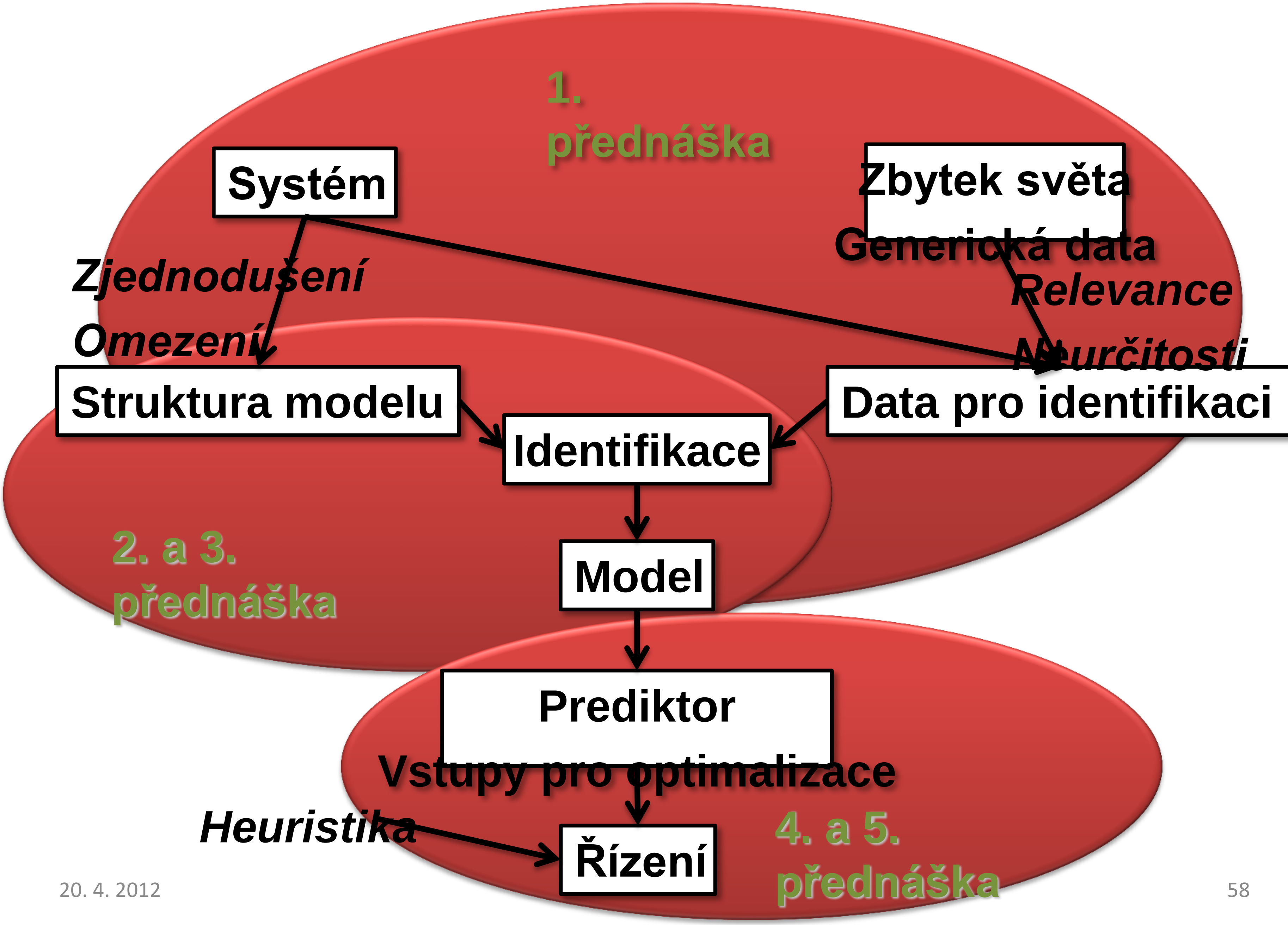
$$y = P(u, x, t)$$

Potom optimální řídicí vstupy jsou

$$u_{\text{optimal}} = \arg\min_u \mathcal{J}\left(P\left(u, x, t\right), u, t\right)$$



EVROPSKÁ UNIE


1.
přednáška
Systém
Zbytek světa
Generická data
Zjednodušení
Relevance
Omezení
Neurčitosti
Struktura modelu
Data pro identifikaci
Identifikace
2. a 3.
přednáška
Model
Prediktor
Vstupy pro optimalizace
Heuristika
Řízení
4. a 5.
přednáška

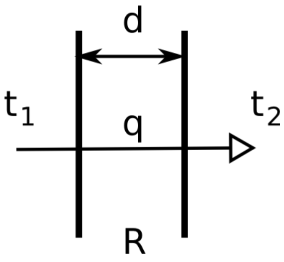
BUDOVA JAKO SYSTÉM

Co v budově bydlí?

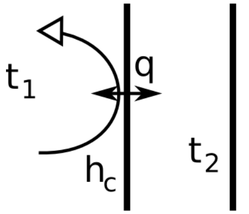
- Přenos tepla
- Tepelná rovnováha
- Tepelná pohoda
- Ventilace
- Energetická spotřeba
- Okrajové podmínky (bydlí okolo budovy)

Přenos tepla

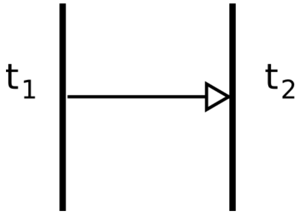
- Vedením (conduction)



- Prouděním (convection)

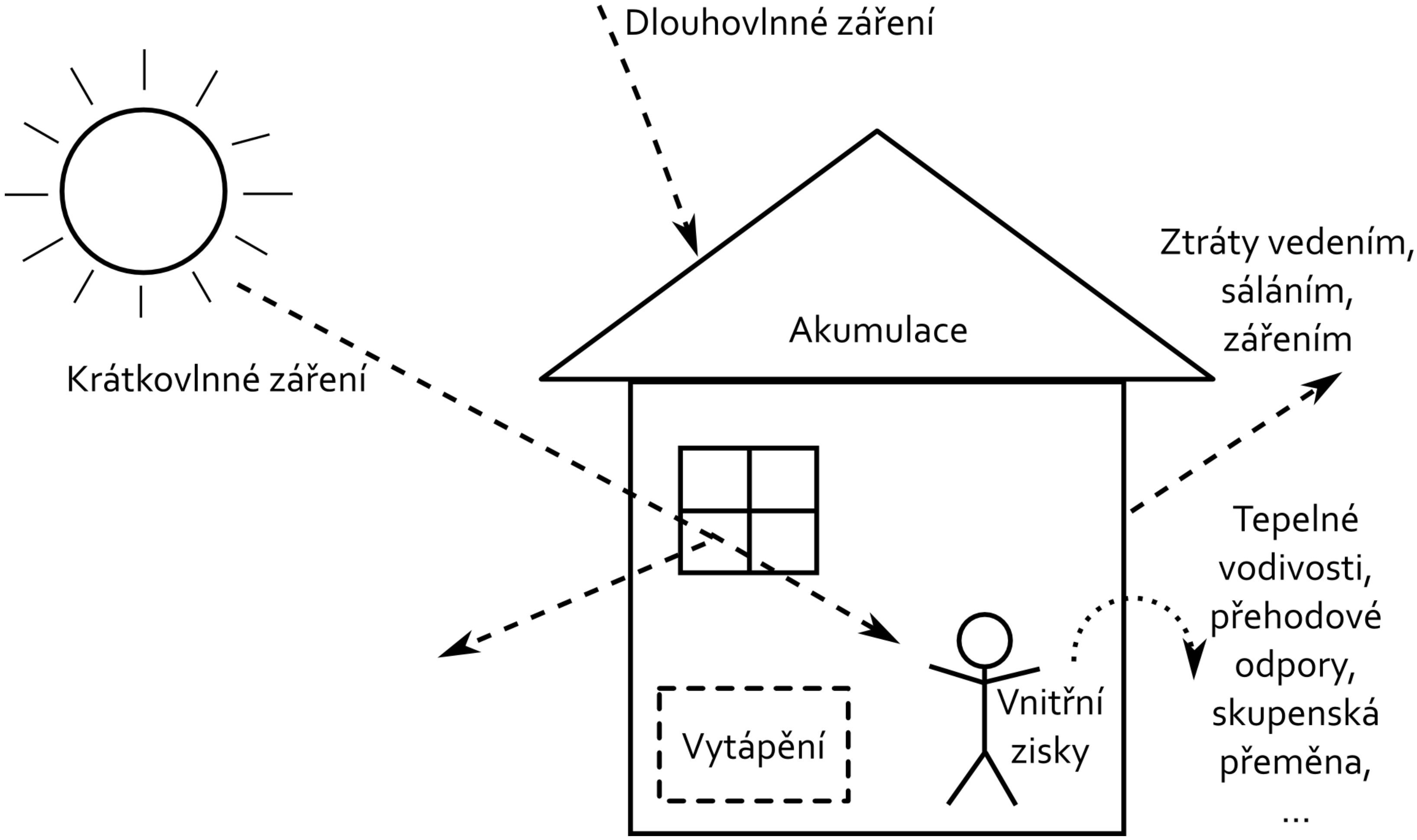


- Zářením (radiation)



$$q = U \Delta t$$

Přenos tepla v budově



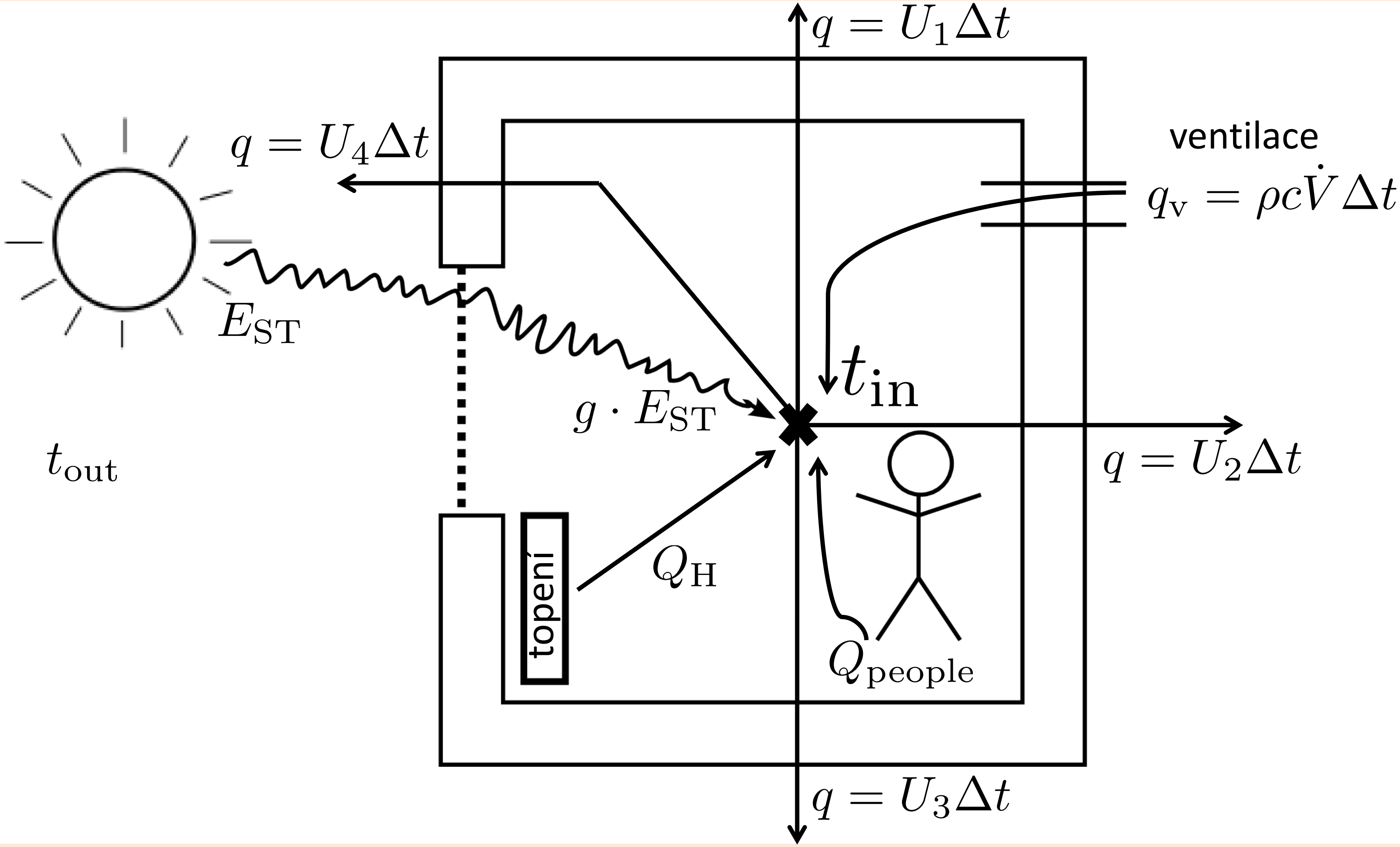
Tepelná rovnováha

- V ustáleném stavu platí:

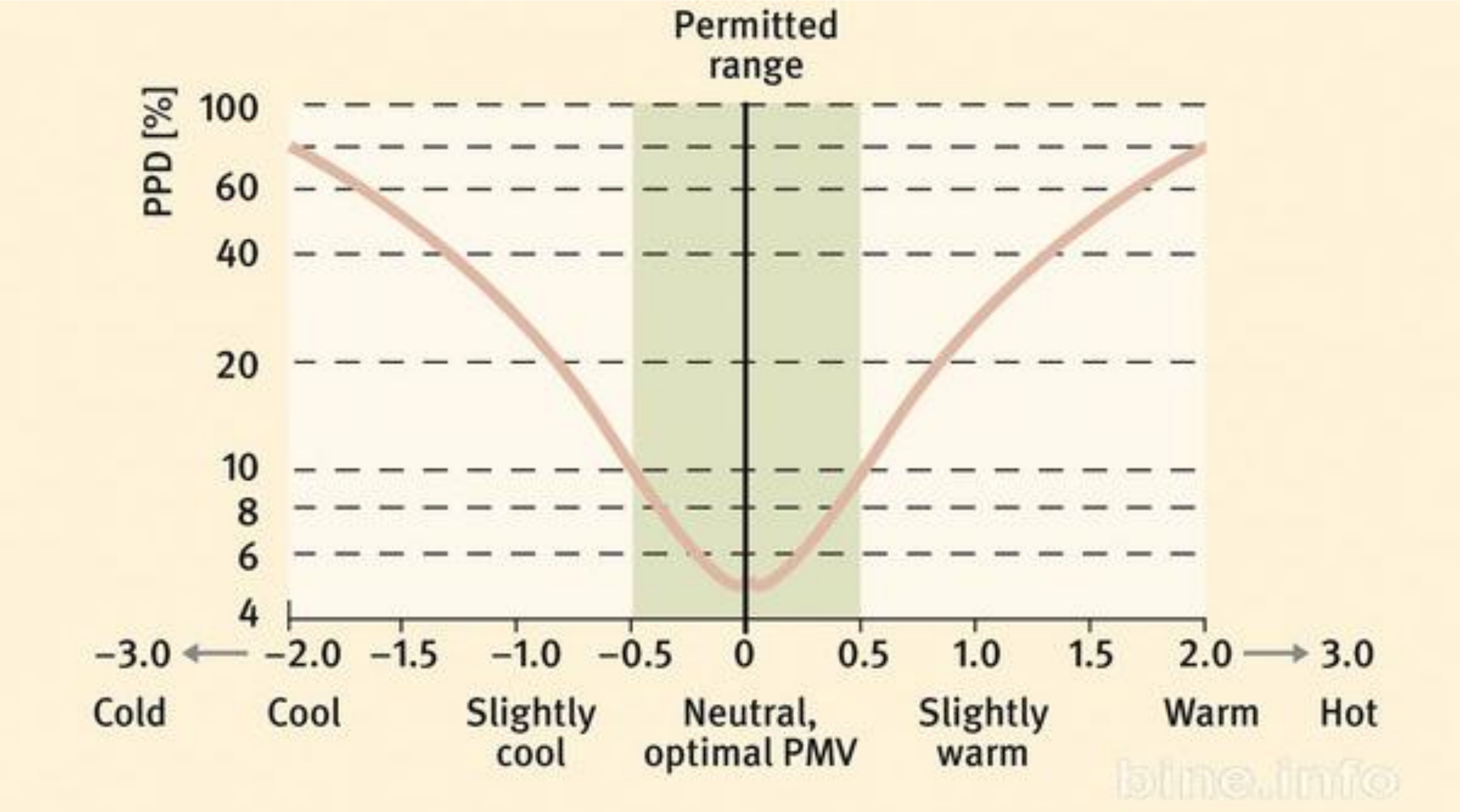
$$\sum Q = 0$$

$$g \cdot E_{\mathrm{S}} + Q_{\mathrm{people}} + Q_V + \sum_i U_i A_i \Delta t + Qh = 0$$

Tepelná rovnováha



Tepelná pohoda



Tepelná pohoda

- PMV závisí na:
  - Teplotě vzduchu
  - Sálavém teple
  - Relativní vlhkosti vzduchu
  - Rychlosti proudění vzduchu
  - “Oblečenosti” (clothing value) [clo]
  - Rychlosti metabolismu [met]

Energetická spotřeba

- Teplo předané topením
  – Emisivní efektivita
- Teplo předané rozvody
  – Ztráty vedení
  – Efektivita regulace
- Pomocné energie
  – Tepelné ztráty
- Výroba tepla
  – Efektivita výroby a distribuce

Budova

Společnost

Okrajové podmínky

- Venkovní teplota
- Sluneční svit
- Rozptýlená dlouhovlnná radiace
- Vítr (síla, směr)
- Vlhkost vzduchu
- Srážky
- Zastínění (stromy, okolními budovami)
- Orientace (S-J-V-Z)
- Geologické podloží
- atd.

PŘECHOD K MODELU BUDOVY

K čemu chci model?

- Cíl: Ušetřit energii za vytápění pomocí moderní regulace (MPC)
- Rozhoduju se statický X dynamický model podle Ts
- Budu tedy hledat LTI model ve tvaru:

$$x(t+1) = Ax(t) + Bu(t) + w(t)$$
$$y(t) = Cx(t) + Du(t) + v(t)$$

- Co bude v deterministické a co ve stochastické části?

Rozčlenění struktury modelu

- Deterministická část

- Vnitřní i vnější zdroje tepla
- Tepelné přenosy, ztráty
- Dynamika “stavební hmoty”
- Akumulace
- Některé okrajové podmínky
- Vliv slunečního záření

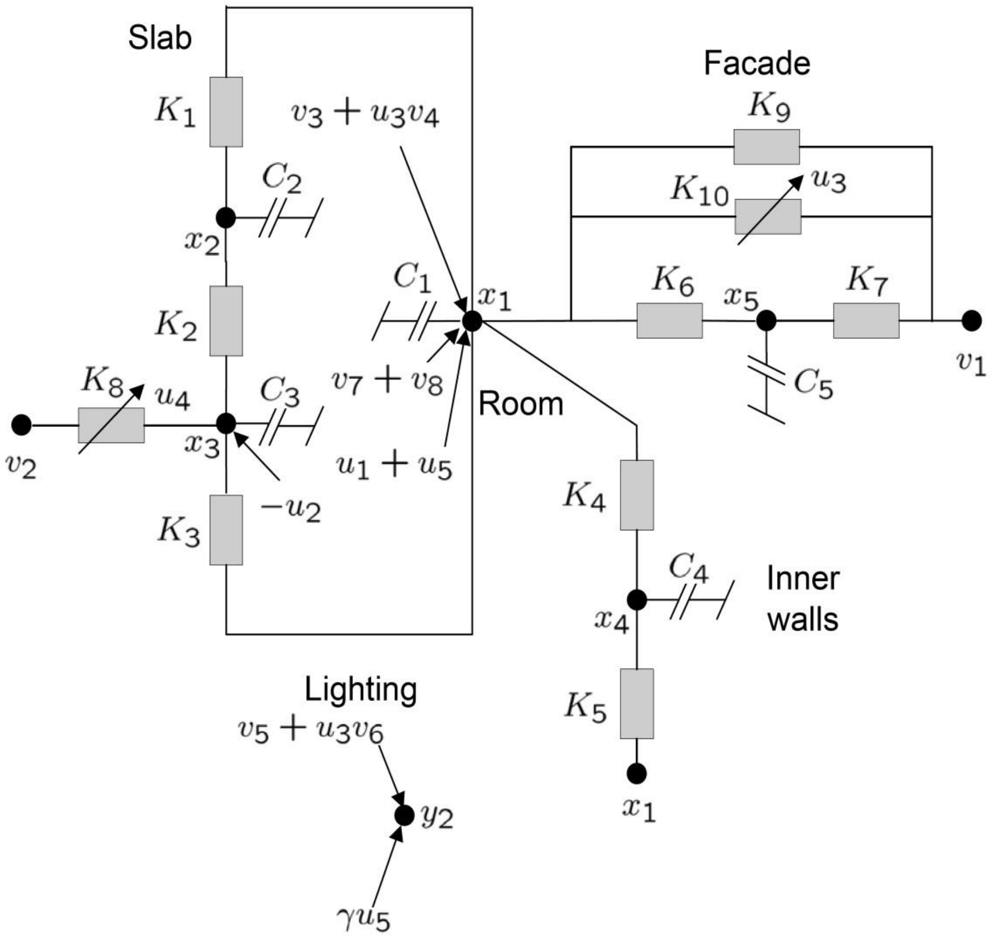
- Stochastická část

- Kolísání obsazenosti
- Některé okrajové podmínky
- Náhodné děje
- Vnitřní regulátory
- Individuální situace místností
- Otevírání oken, dveří

Variranty pro modelování

- White-box model
  - Podle fyzikálních rovnic, pro velké budovy velmi pracný
- Black-box model
  - Pouze obecný LTI model, identifikuji jednotlivé prvky matic bez jakýchkoliv znalostí fyziky budovy
- Gray-box model
  - V našem případě – znám strukturu modelu, ale nikoliv přesnou fyziku

Gray-box model



Black-box model

- Struktura:
  - Stavový LTI, ARX, ARMAX, OE, …
- Identifikace:
  - Nejmenší čtverce
  - Maximální věrohodnost
- Zvláštní případ – Subspace metody

Z čeho identifikovat?

- Reálná data
  - Potřebuji dostatečně “pestrá” měření
  - Identifikační experimenty jsou nákladné
- Specializovaný simulační software
  - Lidi z oblasti stavebnictví jsou zvyklí používat simulační softwary, které můžeme použít jako generátory dat
  - Velmi pracná tvorba simulátoru
  - Problematická validace

PRŮBĚŽNÉ SHRNUTÍ

Co zatím víme

- Chceme ušetřit energii v budovách pomocí pokročilé regulace
- Regulace bude založená na modelu budovy
- Jako systém je budova velmi komplexní
- Její model bude mít deterministickou a stochastickou část
- Ukážeme si subspace identifikaci modelu

Příště uvidíte…